Numerical Simulations of Reaching a Steady State: No Need to Generate Any Rational Expectations

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Abstract:

It is not easy to numerically simulate the path to a steady state because there is no closed form solution in dynamic economic growth models in which households behave generating rational expectations. In contrast, it is easy if households are supposed to behave under the MDC (maximum degree of comfortability)-based procedure. In such a simulation, a household increases or decreases its consumption according to simple formulae. In this paper, I simulate the path when households behave under the MDC-based procedure, and the results of simulations indicate that households can easily reach a stabilized (steady) state without generating any rational expectations by behaving according to their feelings and guesses about their preferences and the state of the entire economy.

Keywords: balanced growth path; economic growth model; government transfer; heterogeneity; simulation; steady state

JEL Classification: C60; D60; E10; H30; I30.

Introduction

The difficulty of simulating the path to steady state raises an important question. Can an ordinary household actually foresee the path to a steady state precisely in everyday life? The rational expectations hypothesis has been criticized for imposing substantial demands on economic agents. To generate rational expectations, households generally have to do something equivalent to computing complex large-scale non-linear dynamic macro-econometric models. Evans and Honkapohja (2001) argued that this problem can be solved by introducing a learning mechanism (see also, *e.g.,* Marcet and Sargent 1989; Ellison and Pearlman 2011), but this solution is not regarded as being sufficiently successful because the learning rules that they assumed are not necessarily thought to be sufficiently persuasive.

Harashima (2018, 2019) showed an alternative procedure for households to reach a steady state: the MDC (maximum degree of comfortability)-based procedure under which households keep their capital-wage ratio (CWR) at MDC. He showed that the behavior of households based on rational expectations (*i.e.*, the behavior under the RTP (rate of time preference]-based procedure) is equivalent to that under the MDC-based procedure (Harashima 2018, 2019, 2021, 2022a, 2022b).

Furthermore, Harashima (2018, 2019) showed that if preferences of households are heterogeneous under the MDC-based procedure, there is no guarantee of a steady state as with the case of the RTP-based procedure (Becker 1980; Harashima 2010, 2012, 2017, 2020). However, Harashima (2010, 2012, 2014, 2017, 2020) also showed that there is a state in which all optimality conditions of all heterogeneous households are satisfied (sustainable heterogeneity, SH) even if the abovementioned heterogeneities exist. Although SH cannot necessarily be naturally achieved, it can be achieved if a government intervenes appropriately.

An important difference between the MDC- and RTP-based procedures is that, unlike the RTP-based procedure, the MDC-based procedure does not require ordinary households to precisely and correctly calculate the exact amount of consumption in any period to reach a steady state. In contrast, the MDC-based procedure allows households to behave based on their vague feelings and guesses in each period, but for all this, households can reach a steady state. This means that unlike the case of the RTP-based procedure, the path to a steady state can easily be simulated if we suppose that households behave under the MDC-based procedure. The purpose of this paper is to simulate the path when households behave under the MDC-based procedure.

In the simulations, a household was set to increase or decrease its consumption according to simple formulae that are supposed to well capture and represent a household's behaviors under the MDC-based procedure. The results of simulations indicated that, in a homogeneous population, households naturally reach a stabilized (steady) state that is almost the same as that under the RTP-based procedure. This means that the equivalence between the MDC- and RTP-based procedure can be demonstrated not only theoretically (Harashima 2018, 2019, 2021, 2022a, 2022b) but also numerically. In a heterogeneous population, the results of simulations indicated that a stabilized (steady) state cannot be naturally achieved if households have different preferences and behave unilaterally, which completely matches the theoretical prediction (Harashima 2018, 2019, 2021, 2022a, 2022b). Furthermore, also as predicted theoretically, if a government appropriately intervenes, a stabilized (steady) state can be achieved although this stabilized (steady) state is not a "pure" SH; rather, it is an approximate SH.

Overall, the simulations showed that households can reach a stabilized (steady) state by behaving according to feelings and guesses about their CWRs and the state of the entire economy, which is in agreement with the theoretical predictions of Harashima (2018, 2019, 2021, 2022a, 2022b). Furthermore, this stabilized (steady) state can be interpreted to be equivalent to a steady state achieved by behaving based on rational expectations.

1. Sustainable Heterogeneity

In this section, I briefly explain the concept of SH following (Harashima 2010, 2012, 2014, 2017, 2020).

1.1. Sustainable heterogeneity

Here, three heterogeneities—RTP, degree of risk aversion (DRA), and productivity—are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for international transactions, but in this paper, this concept and the associated terminology are used even if each economy represents a group of identical households in a country.

The production function of Economy i (= 1, 2) is:

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \tag{1}$$

where: $y_{i,t}$ and $k_{i,t}$ are the production and capital of Economy i in period t, respectively; A_t is technology in period t; α ($0 < \alpha < 1$) is a constant and indicates the labor share.

All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance: $\int_0^t \tau_s ds$ mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}}$ $\left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ is returns on investments, $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds$ and $\frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ represent income receipts or payments on the assets that an economy owns in the other economy. Hence, $\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ is the balance on goods and services of Economy 1, and $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$ is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that:

$$\tau_t = \kappa\left(k_{1,t}, k_{2,t}\right) . \tag{2}$$

This two-economy model can be easily extended to a multi-economy model. Suppose that a country consists of H economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2,…, Economy H). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy i in period t, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t} u_i''}{u_i'}$, ω_i, and u_i are the RTP, DRA, productivity, and utility function of a household in Economy i, respectively (I = 1, 2, …, H). The production function of Economy i is:

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad . \tag{3}$$

In addition, $\tau_{i,j,t}$ is the current account balance of Economy i with Economy j, where i, j = 1, 2, … , H and $i \neq j$.. Harashima (2010, 2017) showed that if, and only if:

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right\} \tag{4}$$

for any i (= 1, 2, … , H), all the optimality conditions of all heterogeneous economies are satisfied, where m, v, and ϖ are positive constants. Furthermore, if, and only if, equation (4) holds:

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}} = \lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}} = \lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} = \lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_{i,j,s}ds}{dt}}{\int_0^t \tau_{i,j,s}ds} \tag{5}$$

is satisfied for any i and j ($i \neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (4) holds is SH by definition.

1.2. Sustainable heterogeneity with government intervention

As shown above, SH is not necessarily naturally achieved, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ (H – 1) be the combined economy consisting of Economies 1, 2, …, and (H – 1). The population of Economy 1+2+… + (H – 1) is therefore (H – 1) times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+…+ (H – 1) in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ (H – 1) to households in Economy H, and $\bar{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is:

$$g_t = \bar{g}_t k_{1+2+\cdots,+(H-1),t} \ . \tag{6}$$

where: $\bar{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2010, 2017) showed that if:

$$\lim_{t\to\infty}\bar{g}_t = \left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\omega_H}\right)^{-1}\left\{\frac{\varepsilon_H\sum_{q=1}^{H}\omega_q - \sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\varepsilon_H\sum_{q=1}^{H}\theta_q\omega_q - \theta_H\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\right\} \tag{7}$$

is satisfied for any i (= 1, 2, …, H) in the case that Economy H is replaced with Economy i, then equation (4) is satisfied (*i.e.*, SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty}\bar{g}_t$ = constant.

Note that the amount of government transfers from households in Economy 1+2+ … + (H – 1) to a household in Economy H at SH is:

$$(H-1)g_t = (H-1)\,k_{1+2+\cdots+(H-1),t}\lim_{t\to\infty}\bar{g}_t \ . \tag{8}$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy 1+2+ · · · + (H – 1) and vice versa.

2. Maximum Degree of Comfortability (MDC) based Procedure

The MDC-based procedure is briefly explained following Harashima (2018, 2019, 2021, 2022a, 2022b).

2.1. "Comfortability" of CWRs (capital-wage ratios)

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ where $\tilde{k}_t$ and $\tilde{w}_t$ are household k_t and w_t, respectively. Let Γ be the subjective valuation of $\frac{\tilde{w}_t}{\tilde{k}_t}$ by a household and Γ_i be the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ of household i (i = 1, 2, 3, … , M). Each household assesses whether it feels comfortable with its current Γ (*i.e.*, its combination of income and capital expressed by CWR). "Comfortable" in this context means "at ease," "not anxious," and other similar feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC). MDC therefore indicates the state at which the combination of revenues and assets is felt most comfortable. Let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$

is household i's Γ_i when it is at $\tilde{s}_i$.

2.2. Homogeneous population

I first examine the behavior of households in a homogeneous population (*i.e.*, all households are assumed to be identical).

Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.

Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (*i.e.*, the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state under the RTP-based procedure; that is, it is the steady state in a Ramsey-type growth model in which households behave based on rational expectations generated by discounting utilities by θ, where θ (> 0) is the RTP of a household. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: See (Harashima 2018, 2019).

Proposition 1 indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$. This means that both the MDC-based and RTP-based procedures can function equivalently and that CWR at MDC can be substituted for RTP as a guide for household behavior.

2.3. Heterogeneous population

In actuality, however, households are not identical - they are heterogeneous - and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker 1980; Harashima 2010, 2012, 2017, 2020). However, (Harashima 2010, 2012, 2017, 2020) has shown that SH exists under the RTP-based procedure. In addition, (Harashima 2018, 2019) has shown that SH also exists under the MDC-based procedure, although Rules 1-1 and Rules: 1-2 have to be revised, and a rule for the government should be added in a heterogeneous population.

Suppose that households are identical except for their MDCs (*i.e.*, their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (*i.e.*, SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ and several other related values. Specifically, let $\Gamma_{R,i}$ be Γ_R of household i, T be the net transfer that a household receives from the government with regard to SH, and T_i be the net transfer that household i receives (i=1,2,3, …,M).

Revised and additional rules

Household i should act according to the following rules in a heterogeneous population:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.

Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

At the same time, the government should act according to the following rule:

Rule 3: The government adjusts T_i for some i if necessary, so as to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases.

Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (an approximate SH), and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure, that is, in a Ramsey-type growth model in which households that are identical except for their θs behave generating rational expectations by discounting utilities by their θs. Furthermore, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any *i* under the RTP-based procedure in an economy where households are identical except for their θs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.
Proof: See (Harashima 2018, 2019).

Proposition 2 indicates that we can interpret $\tilde{S}_{MDC,SH,ap}$ as being equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of *T*, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

3. Simulation Method

3.1. Simulation assumptions

Environment

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are *H* economies in a country, the number of households in each of economy is identical, and households within each economy are identical.

Production

The production function of Economy *i* ($1 \le i \le H$) is:

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha}, \tag{9}$$

where: ω_i is the productivity of a household in Economy *i*. Because α indicates the labor share, I set $\alpha = 0.65$. In addition, I set $A_t = 1$ and $\omega_i = 1$ for any *t* and *i*. The initial capital a household owns is set at 1 for any household. The setting of $\omega_i = 1$ for any *i* will be relaxed in Section 4.4.

With $A_t = 1$ and $\omega_i = 1$, by equation (9), the production of a household in Economy *i* in period *t* ($y_{i,t}$) is calculated, for any *i*, by:

$$y_{i,} = k_{i,t}^{1-\alpha} \; . \tag{10}$$

Capitals

Because the marginal productivity is kept equal across economies within the country through arbitrage in markets, the amount of capital used (not owned) by each household (*i.e.*, $k_{i,t}$) is kept identical among households in all economies in any period; that is, $k_{i,t}$ is identical for any *i* although the amount of capital each household owns (not uses) can be heterogeneous. Hence, by equation (10), the amount of production ($y_{i,t}$) is always identical across households and economies regardless of how much capital a household in Economy *i* owns, when $\omega_i = 1$. In addition, for any *i*,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H} \quad , \tag{11}$$

where: $\check{k}_{i,t}$ is the amount of capital a household in Economy *i* owns (not uses). As shown above, I set the initial capital of a household owns to be 1 (*i.e.,* $\check{k}_{i,0} = 1$ for any *i*) throughout simulations in this paper.

Incomes

The capital income of a household in Economy *i* in period *t* ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \check{k}_{i,t} \quad , \tag{12}$$

where: r_t is the real interest rate in period *t* and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ . \tag{13}$$

Hence, by equations (9) and (13), the real interest rate r_t is calculated by:

$$r_t = (1-\alpha) k_{i,t}^{-\alpha} = (1-\alpha)\left(\frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H}\right)^{-\alpha} \ . \tag{14}$$

The labor income of a household in Economy *i* in period *t* ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H} \ . \tag{15}$$

Because the amount of capital used and the amount of labor inputted by a household is identical for any household in any economy when $\omega_i = 1$, household labor income is identical across economies. Note that if productivity ($\omega_{i.t}$) is heterogeneous among economies, production and labor income differ in proportion to their productivities, as will be shown in Section 4.4. Note also that in a homogeneous population, the labor income becomes equal to $\alpha y_{i,t}$ for any household.

Savings

Household savings in Economy *i* in period *t* ($s_{i,t}$) are calculated by:

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} \ . \tag{16}$$

In period *t* + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\check{k}_{i,t+1} = \check{k}_{i,t} + s_{i,t} \ . \tag{17}$$

3.2. Consumption formula

Consumption formula in a homogeneous population

For a simulation to be implemented, the consumption formula that describes how a household adjusts its consumptions needs to be set beforehand. However, under the MDC-based procedure, there is no strict consumption formula for households. A household just has to behave roughly feeling and guessing (*i.e.*, not exactly calculating) its CWR and CWR at MDC in each period. It increases its consumption somewhat if it feels that $\Gamma(\tilde{s}_i)$ is larger than $\Gamma_{i,t}$ and decreases its consumption somewhat if it feels the opposite way. The amount of the increase/decrease will differ by period. In this sense, the actual formula of consumption under the MDC-based procedure is lax and vague; therefore, it is difficult to set a strict consumption formula with a mathematical functional form.

Nevertheless, if we consider the average consumption over some periods (*i.e.*, moving averages), it will be possible to describe a mathematical form of the consumption formula because households will behave in a similar manner on average. Considering this nature, I introduce the following simple consumption formula because it seems to simply but correctly capture the behavior of households under the MDC-based procedure on average. Please note that that this consumption formula is not the only possible choice. Other, possibly more complex and subtle, functional forms could be chosen.

Consumption formula 1: The consumption of a household in Economy *i* in period *t* is:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} , \tag{18}$$

where: $\Gamma_{i,t}$ is the CWR of household in Economy *i* in period *t;* γ is a parameter. Because:

$$\theta_i = \left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i), \tag{19}$$

as shown in (Harashima 2018, 2019, 2021, 2022a, 2022b), by equation (18), equation (19) is equal to:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\frac{1-\alpha}{\alpha}}\right)^{\gamma} \ . \tag{20}$$

Although a household is set to precisely follow equation (18) in the simulations, in reality, they do not behave by calculating equation (18). Furthermore, they are not even aware of Consumption formula 1 itself and cannot know the exact numerical value of each $\Gamma(\tilde{s}_i) = \theta_i \alpha/(1-\alpha)$. Instead, households feel and guess whether they should increase or decrease consumption considering their income and wealth.

That is, Consumption formula 1 is set only for the convenience of calculation in the simulation. It seems to well capture the essence of household behavior in that it increases or decreases consumption depending on a household's feelings with regard to $\Gamma_{i,t}$ and $\Gamma(\tilde{s}_i)$. In this context, the value of parameter γ represents the average adjustment velocity of increase or decrease in consumption.

Consumption formula 1 means that a household's consumption is roughly equal to the sum of its incomes ($x_{L,i,t} + x_{K,i,t}$). The reason for this equality is that there is no technological progress and capital depreciation, so savings stay around zero at the stabilized (steady) state. As mentioned above, the adjustment velocity of consumption in each period is determined by the value of γ in equation (5). As the value of γ is larger, a stabilized (steady) state can be achieved more quickly (if it can be achieved). In this paper, I set the value of γ to be 0.5.

Consumption formula in a heterogeneous population

As shown in (Harashima 2018, 2019, 2021, 2022a, 2022b), in a heterogeneous population, a household behaving under the MDC-based procedure does not use its CWR ($\Gamma_{i,t}$) to make decisions about its consumption. Instead, it uses an adjusted value of CWR considering the behaviors of other heterogeneous households and the government because the entire economic state of the country depends on these heterogeneous behaviors in a heterogeneous population. Accordingly, in a heterogeneous population, Consumption formula 1 has to be modified to accommodate the adjusted CWR. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy *i* in period *t* and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate capital-wage ratio).

Consumption formula 2

Unilateral behavior implies that a household behaves supposing that other households must behave in the same manner as it does. In other words, it assumes that other households' preferences are almost identical to its preferences, or at least, its preferences are not exceptional but roughly the same as the preferences of the average household (Harashima 2018, 2019). If all households behaved in the same manner as a household in Economy *i* did, the real interest rate (r_t) would be equal to the household's $\Gamma_{R,i,t}(1-\alpha)/\alpha$ and eventually converge at its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. Hence, if a household in Economy *i* behaves unilaterally in a heterogeneous population, it feels and guesses that its $\Gamma_{R,i,t}\ (1-\alpha)/\alpha$ is roughly identical to the real interest rate (r_t). That is, the real interest rate will be used as $\Gamma_{R,i,t}(1-\alpha)/\alpha$, and $r_t \alpha/(1-\alpha)$ will be used as its adjusted CWR ($\Gamma_{R,i,t}$).

Therefore, even if a unilaterally behaving household's raw (unadjusted) CWR is accidentally equal to its CWR at MDC, the household does not feel that it is at its MDC unless at the same time r_t is accidentally equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household will instead feel that the value of r_t will soon change, and accordingly, its raw (unadjusted) CWR will also change soon. That is, it feels and guesses that the entire economic state of the country is not yet stabilized because r_t is not equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. As a result, the household will still continue to change its consumption to accumulate or diminish capital (see Lemma 2 in Harashima 2018, 2019).

Considering the above-shown nature of the adjusted CWR, *Consumption formula 1* can be modified to Consumption formula 2 to use in simulations with a heterogeneous population.

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy *i* in period *t* is

$$c_{i,t} = (x_{L,i,t} + x_{K,i,t})\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma} = (x_{L,i,t} + x_{K,i,t})\left(\frac{\Gamma(\tilde{s}_i)}{r_t\frac{\alpha}{1-\alpha}}\right)^{\gamma} = (x_{L,i,t} + x_{K,i,t})\left(\frac{\Gamma(\tilde{s}_i)\frac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma} \tag{21}$$

and equivalently, by equations (19) and (21),

$$c_{i,t} = (x_{L,i,t} + x_{K,i,t})\left(\frac{\theta_i}{r_t}\right)^{\gamma} . \tag{22}$$

As with $\Gamma_{i,t}$ in Consumption formula 1, the use of r_t in equation (21) does not mean that households always actually behave by paying attention to r_t. What Consumption formula 2 means is that, on average, unilaterally behaving households will feel and guess that r_t represents their adjusted CWRs.

Under the RTP-based procedure, a household changes its consumption according to:

$$\frac{\dot{c}_{i,t}}{c_{i,t}} = \varepsilon^{-1}(r_t - \theta_i) \tag{23}$$

where: ε is the degree of relative risk aversion.

That is, a household changes its consumption by comparing r_t and its $\theta_i = \Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household changes consumption as r_t increasingly differs from $\theta_i = \Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. This household's behavior under the RTP-based procedure is very similar to that according to Consumption formula 2, which means that the formula is basically consistent with a household's behavior under the RTP-based procedure.

In addition, in a homogeneous population, r_t is always equal to a homogenous household's $\Gamma_{i,t}(1-\alpha)/\alpha$ because all households behave in the same manner. Hence, equation (18) is practically identical to equation (21) (*i.e.*, Consumption formula 1 is practically identical to Consumption formula 2) because $\Gamma_{i,t}$ in equation (18) can be replaced with $r_t \frac{\alpha}{1-\alpha}$.

Consumption formula 2-a

In Consumption formula 2, a household is supposed to feel that its preferences are not exceptional and almost the same as the preferences of the average household, but it may not actually feel that way. It may instead feel that its preferences are different from those of the average household. In this case, the household will not only feel its preferences are different, but it will also have to guess how far its preferences are from the average (*i.e.*, by how much its adjusted CWR is different from the real interest rate).

For example, a household in Economy *i* may feel and guess that its adjusted CWR is:

$$\Gamma_{R,i,t} = \frac{\alpha}{1-\alpha}(r_t + \chi_i) \tag{24}$$

instead of $\Gamma_{R,i,t} = r_t \frac{\alpha}{1-\alpha}$ in Consumption formula 2, where χ_i is a constant and $\chi_i \neq \chi_j$ for any *i* and *j*. χ_i represents the magnitude of how much a household in Economy *i* feels it is different from the average household. I refer to a modified version of Consumption formula 2 in which $r_t \frac{\alpha}{1-\alpha}$ is replaced with $\frac{\alpha}{1-\alpha}(r_t + \chi_i)$ shown in equation (24) as Consumption formula 2-a. In this case, a household in Economy *i* behaves feeling that

$$\Gamma_{R,i,t} = \frac{\alpha}{1-\alpha}(r_t + \chi_i) = \Gamma_{i,t} \tag{25}$$

holds at a stabilized (steady) state that will be realized at some point in the future.

Consumption formula 2-b

In both Consumption formulae 2 and 2-a, the raw (unadjusted) CWR is not included and therefore plays no role. Nevertheless, a household may utilize a piece of information derived from its raw (unadjusted) CWR because past behaviors may contain some useful information for guiding future behavior. As indicated in Section 3.2.2.2, χ_i is a parameter that indicates how far a household is from the average household. In general, the value of the parameter should be adjusted if households obtain any new and additional pieces of information. This implies that a piece of information derived from the raw (unadjusted) CWR may be used to adjust the value of parameter χ_i.

For example, a household in Economy *i* may use its raw (unadjusted) CWR ($\Gamma_{i,t}$) to adjust the value of χ_i such that:

$$\chi_{i,t} = \chi_{i,t-1} + \zeta_i \left(\Gamma_{i,t} \frac{1-\alpha}{\alpha} - r_{t-1} - \chi_{i,t-1} \right) \tag{26}$$

where: $\chi_{i,t}$ is χ_i in period *t*, ζ_i is a positive constant and its value is close to zero.

Equation (26) means that a household in Economy *i* increases the value of $\chi_{i,t}$ a little if its raw (unadjusted) CWR is higher than its adjusted CWR ($r_{t-1} + \chi_{i,t-1}$) in the previous period and vice versa. It fine-tunes $\chi_{i,t}$ in this manner because it feels that equation (9) will eventually hold at some point in the future, as shown in Section 3.2.2.2. The value of ζ_i is close to zero because $\Gamma_{i,t}$ is highly likely to be almost equal to $\Gamma_{i,t-1}$, and therefore, the guess of $\chi_{i,t}$ in period *t* will not change largely from that of $\chi_{i,t-1}$ in period $t-1$. I refer to the modified version of Consumption formula 2-a in which χ_i is replaced with $\chi_{i,t}$ shown in equation (10) as Consumption formula 2-b.

3.3. Rule of government transfer

Although governments implement transfers among households in complex and subtle manners, a simple bang-bang control is adopted in simulations in this paper as the rule of government transfer for simplicity. In addition, government transfers in each period are assumed to be added to or extracted from the capital of each relevant household in the next period.

In simulations with government transfers, it is assumed for simplicity that there are two economies (Economies 1 and 2) in a country, the economies are identical except for each $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$, and all households in each economy are identical. Let κ be the $\check{k}_{1,t}$ that a government aims for to force a household in Economy 1 to own capital at a stabilized (steady) state (*i.e., κ* is the target value set by the government). Under these conditions, the bang-bang control of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy 1 to a household in Economy 2 in period *t* is T_{low} if $\check{k}_{1,t}$ is lower than κ and T_{high} if $\check{k}_{1,t}$ is higher than κ, where T_{low} and T_{high} are constant amounts of capital predetermined by the government.

In the simulations, I set T_{low} to be -0.1 and T_{high} to be 0.5. The value of κ is varied in each simulation depending on what stabilized (steady) state the government is aiming to achieve. Note that because of the discontinuous control signal in bang-bang control, flow variables may show discontinuous zigzag paths but stock variables can move relatively smoothly. These zigzag paths may look unnatural, but they are generated only because of the bang-bang control method that is adopted for simplicity.

Even if a household knows about the existence of government transfers, it still behaves based on Consumption formula 2 (or 2-a and 2-b) with no government transfer. That is, a household uses $x_{L,i,t} + x_{K,i,t}$, not $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}), as the "base" consumption in determining whether it should increase or decrease its consumption. This behavior superficially may mean that a household does not consider government transfers in the process of adjusting its CWR. However, it is implicitly assumed that a household knows that government transfers exist and that they are an exogenous factor.

Therefore, the household feels that the transfers should be removed from the elements that it can change or control freely. Furthermore, it is implicitly assumed that a household correctly knows the exact amount of government transfers.

However, these assumptions may be oversimplifications, and they will be relaxed in Section 4.3.3 to allow for incorrect guesses on the amount of government transfers. This relaxation enables a household to use $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}) instead of $x_{L,i,t} + x_{K,i,t}$ in determining its consumption.

4. Results of Simulations

4.1. Homogenous households

Naturally realized stabilized (steady) state

First, I simulate the case of a homogeneous population. In this case, households behave according to Consumption formula 1. I set the households' common CWR at MDC ($\theta_i \alpha/(1-\alpha)$) to be $0.04 \times 0.65/(1-0.65) = 0.0743$ (*i.e.,* their common RTP is 0.04). Here, by equations (9) and (13), the amount of capital when $r_t = \Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$ (*i.e.*, at steady state under the RTP-based procedure) is

$$k_{i,t} = \left(\frac{\Gamma(\tilde{s}_i)}{\alpha}\right)^{-\frac{1}{\alpha}} . \quad (27)$$

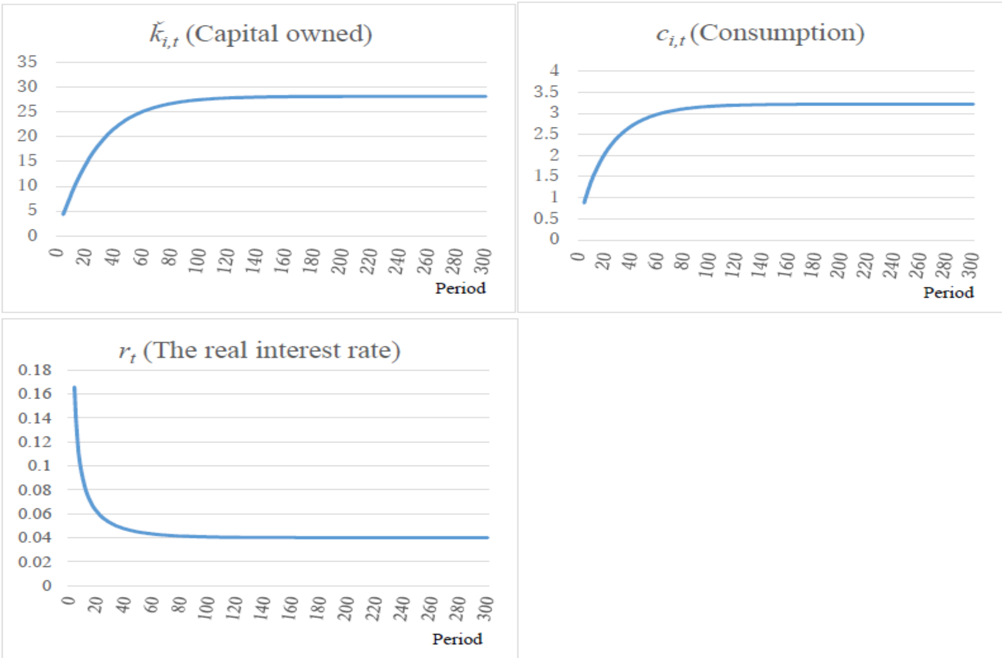
Here, under the RTP-based procedure, if households reach a steady state *(i.e.*, the real interest rate becomes equal to the RTP (0.04)), the amount of household capital is 28.13, and the amounts of production and consumption are equally 3.22 ($= k_{i,t}{}^{(1-0.65)}$) by equation (9) *(i.e.*, by the production function with α = 0.65).

The results of the simulation for this case are shown in Figure 1. Note that all variables in the figures are 10-period moving averages throughout this paper considering the effects of the bang-bang control of government transfers that are explained in Section 3.3.

Figure 1 shows the paths of household's capital, consumption, and real interest rate. Clearly, households reach a stabilized (steady) state. The amount of capital and consumption initially increase rapidly but eventually stabilize at amounts that are almost equal to those at the steady state under the RTP-based procedure (*i.e.*, 28.13 and 3.22, respectively). The real interest rate eventually stabilizes at the rate that is equal to their common $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$ (*i.e.*, 0.04). Hence, this stabilized (steady) state can be interpreted to be equivalent to the steady state reached under the RTP-based procedure as theoretically predicted (Harashima 2018, 2019). That is, the equivalence between the MDC- and RTP-based procedures is supported not only theoretically but also numerically.

Figure 1. Simulation of capital owned by each household ($\check{k}_{i,t}$), consumption ($c_{i,t}$), and the real interest rate (r_t) for

homogenous households



An important point is that Figure 1 clearly shows that a household never needs to generate any rational expectations to reach a stabilized (steady) state.

Difference between the two procedures

Although the stabilized (steady) state can be interpreted to be equivalent between the MDC- and RTP-based procedures, the paths to it differ between the two procedures. Under the RTP-based procedure, a unique exact amount of consumption in each period is strictly predetermined or needs to be perfectly foreseen. As a result, there is only one unique path to the steady state under the RTP-based procedure. On the other hand, under the MDC-based procedure, the amount of consumption in each period is laxly determined depending on the feelings and guesses of a household about its CRW at MDC and CWR in each period. Hence, many different paths are possible; in fact, an infinite number of paths to a stabilized (steady) state can exist.

4.2. Households with heterogeneous CWRs (capital-wage ratios) at MDC (maximum degree of comfortability): The case of unilateral behavior

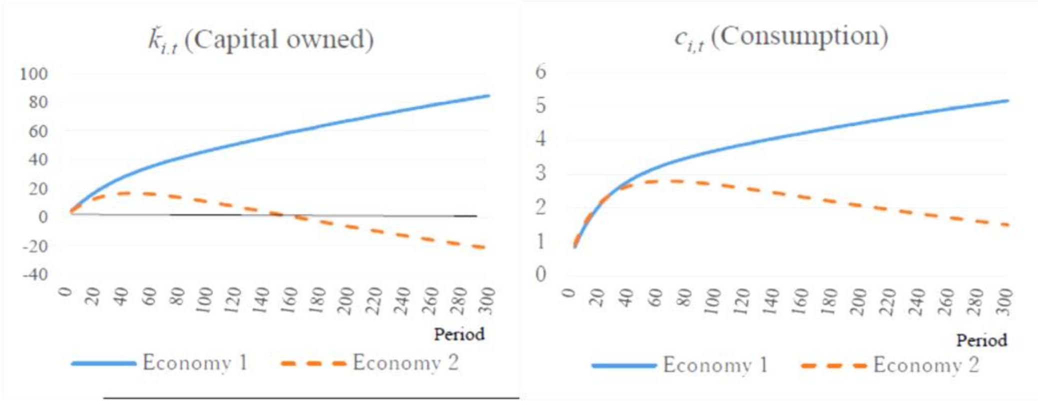
Next, I simulate the case of a heterogeneous population. Under the RTP-based procedure, if households are heterogeneous, there is no guarantee they will reach a steady state as (Becker 1980) indicated; Harashima (2018, 2019, 2021, 2022a, 2022b) theoretically showed that this is also true under the MDC-based procedure. As shown in the Section 3.2.2, in a heterogeneous population, a household behaves according to *Consumption formula* 2.

Here, a two-economy case (Economies 1 and 2) is simulated. The two economies are identical except for CWR at MDC $(\theta_i\alpha/(1-\alpha))$. I set the CWR at MDC of household in Economy 1 to be $0.035×0.65/(1-0.65) = 0.065$ (*i.e.*, its RTP is 0.035) and that of Economy 2 to be $0.045×0.65/(1-0.65) = 0.0836$ (*i.e.*, its RTP is 0.045). The average $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$ of the two economies is therefore 0.04 (i.e., the average CWR at MDC is 0.0743).

No stabilized (steady) state

First, I examine the case where a household behaves unilaterally in the sense that it does not consider the consequences of other households and the government does not intervene. The results of the simulation are shown in Figure 2. Clearly, neither economy can reach a stabilized (steady) state. A household in Economy 1 continues to accumulate capital, and a household in Economy 2 continues to lose capital, eventually owing debt to households in Economy 1. These results match the theoretical prediction of (Becker 1980) in models based on the RTP-based procedure and also the predictions of (Harashima 2018, 2019, 2021, 2022a, 2022b) in the model based on the MDC-based procedure.

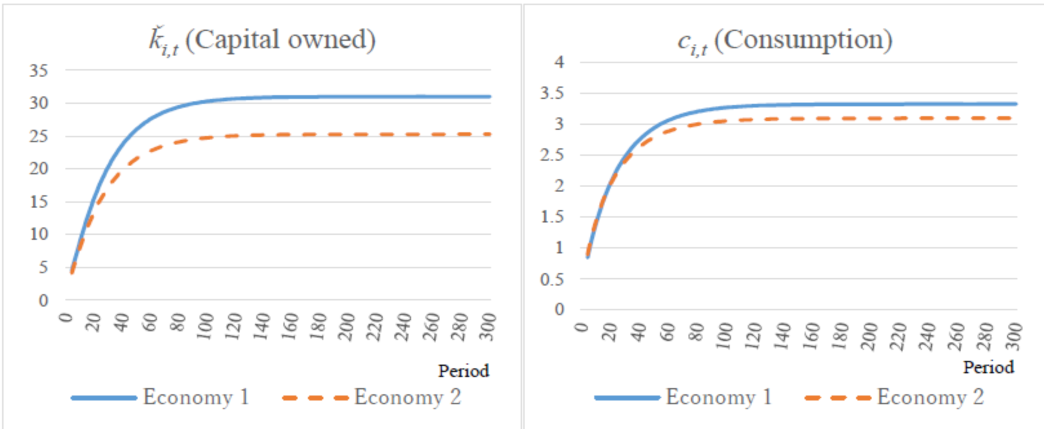
Figure 2. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in the case of heterogeneous CWRs at MDC: unilateral behavior



Exceptional case

There is an exceptional case in which even if households behave unilaterally and the government does not intervene, households can reach a stabilized (steady) state. If households behave according to Consumption formula 2-a and correctly know the value of each CWR at MDC from the average CWR at MDC, they can reach a stabilized (steady) state.

Figure 3. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in the case of exceptional case of heterogeneous CWRs at MDC

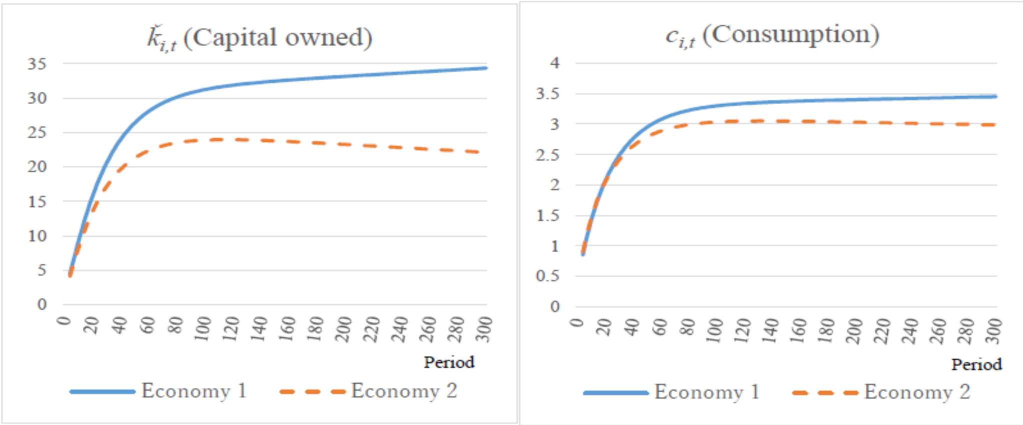


Suppose that a household in Economy 1 correctly knows that its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$ (*i.e.*, 0.035) is lower than the average (*i.e.*, 0.04) by 0.005 and that a household in Economy 2 correctly knows that its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$ (*i.e.*, 0.045) is higher than the average (*i.e.*, 0.04) by 0.005. In this case, the adjusted CWR of a household in Economy 1 will be the real interest rate minus 0.05 (*i.e.*, 0.035) and that in Economy 2 will be the real interest rate plus 0.05 (*i.e.*, 0.045). Figure 3 shows the results for this case. Clearly, a stabilized (steady) state is achieved.

If households' feelings and guesses about the distances from the average are even a little incorrect, however, no stabilized (steady) state can be achieved. For example, suppose that a household in Economy 1 guesses that its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$ is lower than the average by 0.0047 (*i.e.*, 0.0353) and that a household in Economy 2 guesses that its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$ is higher than the average by 0.0047 (*i.e.*, 0.0447).

Figure 4 indicates the results of this case, and it clearly shows that no stabilized (steady) state is achieved.

Figure 4. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in the case of heterogeneous CWRs at MDC when households make a small incorrect guess on the distance of their $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$ from the average



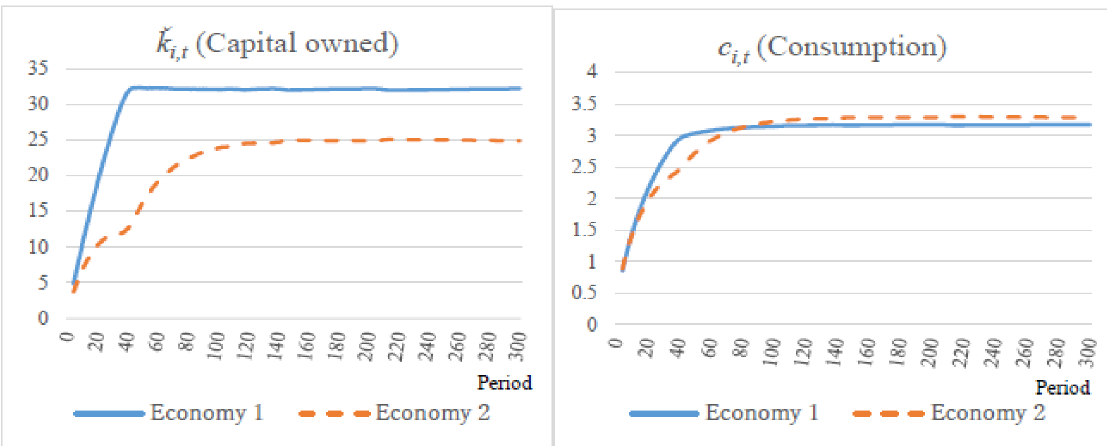
Because it seems highly unlikely that all households can always correctly know the amounts they vary from the average, this exceptional stabilized (steady) state cannot generally be achieved. In this sense, we can say that no stabilized (steady) state exists if households behave unilaterally and the government does nothing.

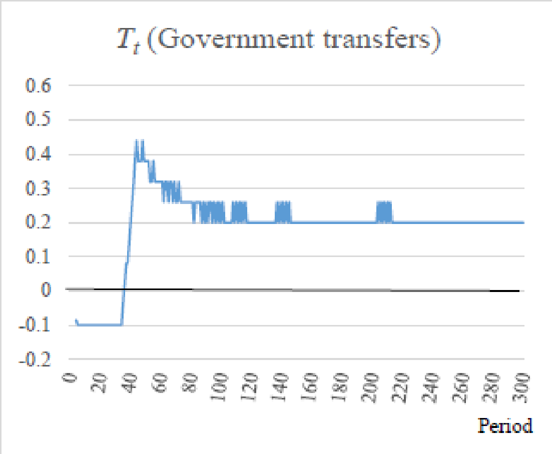
4.3. Households with heterogeneous CWRs (capital-wage ratios) at MDC (maximum degree of comfortability): The case of sustainable heterogeneity

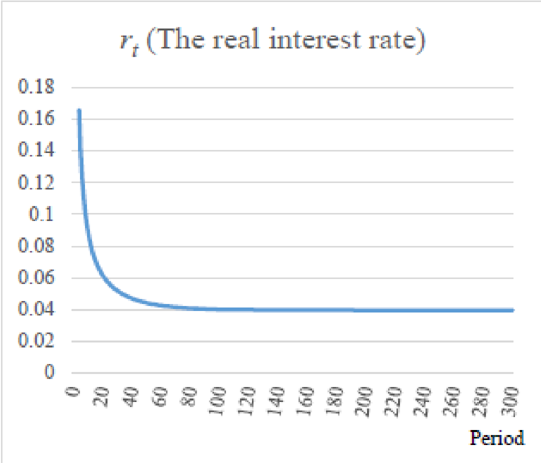
Sustainable heterogeneity by government intervention

The same two-economy model as that used in Section 4.2 is also used in this section, but in this case, government intervention is included. Figure 5 shows the result of government interventions that are implemented according to the Transfer rule shown in Section 3.3. Figure 5 indicates that capital, consumption, government transfers, and the real interest rate all eventually stabilize. This stabilized (steady) state can be interpreted to be equivalent to a SH with government interventions under the RTP-based procedure, see Harashima (2018, 2019, 2021, 2022a, 2022b). Nevertheless, this stabilized (steady) state is merely one of many other possible stabilized (steady) states that vary depending on the pattern of government interventions (*i.e.,* different values of κ). This nature will be further examined. Note that because the values of all variables in Figure 5 are 10-period moving averages, the amount of government transfers in period t (T_t) indicates the weighted sum of T_{low} and T_{high} during the 10 periods surrounding period t.

Figure 5. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and the real interest rate (r_t) in the case of heterogeneous CWRs at MDC: SH with government intervention







In Figure 5, the household consumptions in Economies 1 and 2 stay around 3.17 and 3.29, respectively, at the stabilized (steady) state. That is, the stabilized household consumption in Economy 1 is smaller than that in Economy 2, which is consistent with the theoretical prediction in models in which households behave under the RTP-based procedure as shown in (Harashima 2010, 2012, 2014, 2017, 2020).

After stabilization, the average CWR of all households is identical to the average CWR at MDC (*i.e.*, $0.04 \times 0.65/(1-0.65) = 0.0743$), which corresponds to an average RTP of 0.04. In addition, the average capital and consumption of all households are 28.6 and 3.23, respectively, which are almost identical to those in the case of a homogeneous population with $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i = 0.04$. Therefore, if we estimated the value of the RTP in a country that consisted of two heterogeneous economies based on a model that assumes a homogeneous population using the RTP-based procedure, the estimated RTP would be 0.04.

Note that in Figure 5, the raw (unadjusted) CWRs of households in Economies 1 and 2 are eventually kept around their respective CWRs at MDC (*i.e.*, the respective values of $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$ are around 0.035 and 0.045), but this is true because the value of κ has been intentionally set to stabilize at these values. As mentioned above, there can be many stabilized (steady) states at which the raw (unadjusted) CWRs are not equal to their CWRs at MDC.

Multilateral steady state

As shown in Harashima (2012, 2014, 2020), under the RTP-based procedure, SH can be achieved without government intervention if households behave multilaterally in the sense that they behave intentionally to make all optimal conditions of all households satisfied. This type of SH is called a multilateral steady state. Only one multilateral steady state exists for each heterogeneous population, and in the two-economy model, $\breve{k}_{1,t} < \breve{k}_{2,t}$ holds at the multilateral steady state. Harashima (2012, 2014, 2020) showed that a multilateral steady state is identical to a SH achieved by government intervention when government transfers after stabilization are controlled such that $\breve{k}_{1,t} = \breve{k}_{2,t}$ holds.

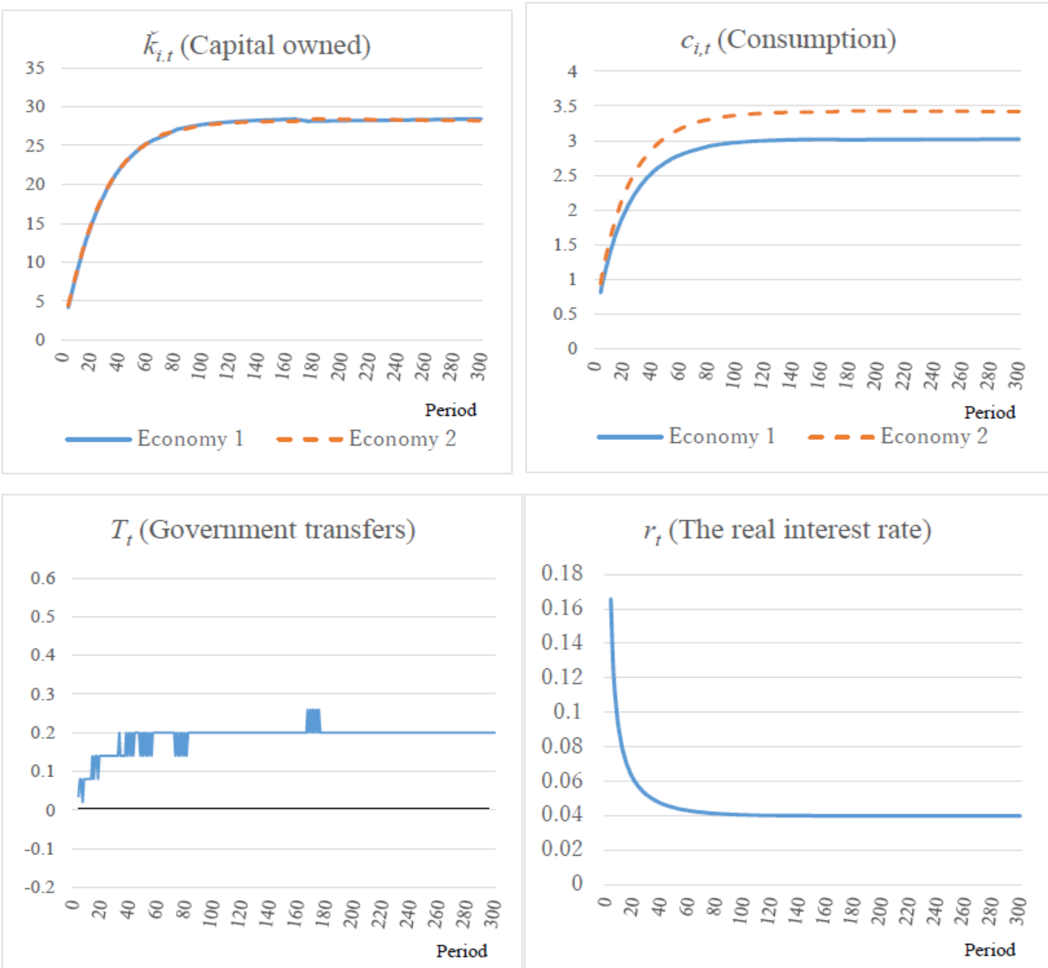
A state that corresponds to a multilateral steady state under the RTP-based procedure will be also achieved under the MDC-based procedure. Figure 6 shows a stabilized (steady) state that is achieved by government intervention when $\breve{k}_{1,t} = \breve{k}_{2,t}$ holds. This state is equivalent to a multilateral steady state under the MDC-based procedure. Note that the state shown in Figure 9 in Section 4.3.3 is also equivalent to a multilateral steady state because $\breve{k}_{1,t} = \breve{k}_{2,t}$ holds there as well. Nevertheless, the other stabilized (steady) states in the other simulations in this paper do not correspond to the multilateral steady state because $\breve{k}_{1,t} \neq \breve{k}_{2,t}$ after stabilization in these simulations.

Approximate sustainable heterogeneity

It is important to note that even if a stabilized (steady) state is compulsorily achieved by a government, households in both economies do not feel comfortable. That is, they cannot feel that they are at MDC because their adjusted CWRs are not equal to their CWRs at MDC. Therefore, even if a government feels that the achieved stabilized state is SH, households do not. In a pure sense, therefore, this stabilized (steady) state is not a SH; rather, it is a state compulsorily stabilized by the government. Harashima (2018, 2019) calls this type of state an "approximate SH."

As shown in Harashima (2018, 2019), a reason why only an approximate SH is possible under the MDC-based procedure is that both households and the government cannot correctly know the effects of government interventions and heterogeneity in households on the entire economy of the country. As a result, the heterogeneous households and the government feel and guess these effects differently, usually incorrectly; therefore, each household's adjusted CWR is not guaranteed to be equal to its CWR at MDC. As a result, a stabilized (steady) state at which all households simultaneously feel comfortable will not be achieved.

Figure 6. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and the real interest rate (r_t) in the case of heterogeneous CWRs at MDC: SH that is equivalent to a multilateral steady state

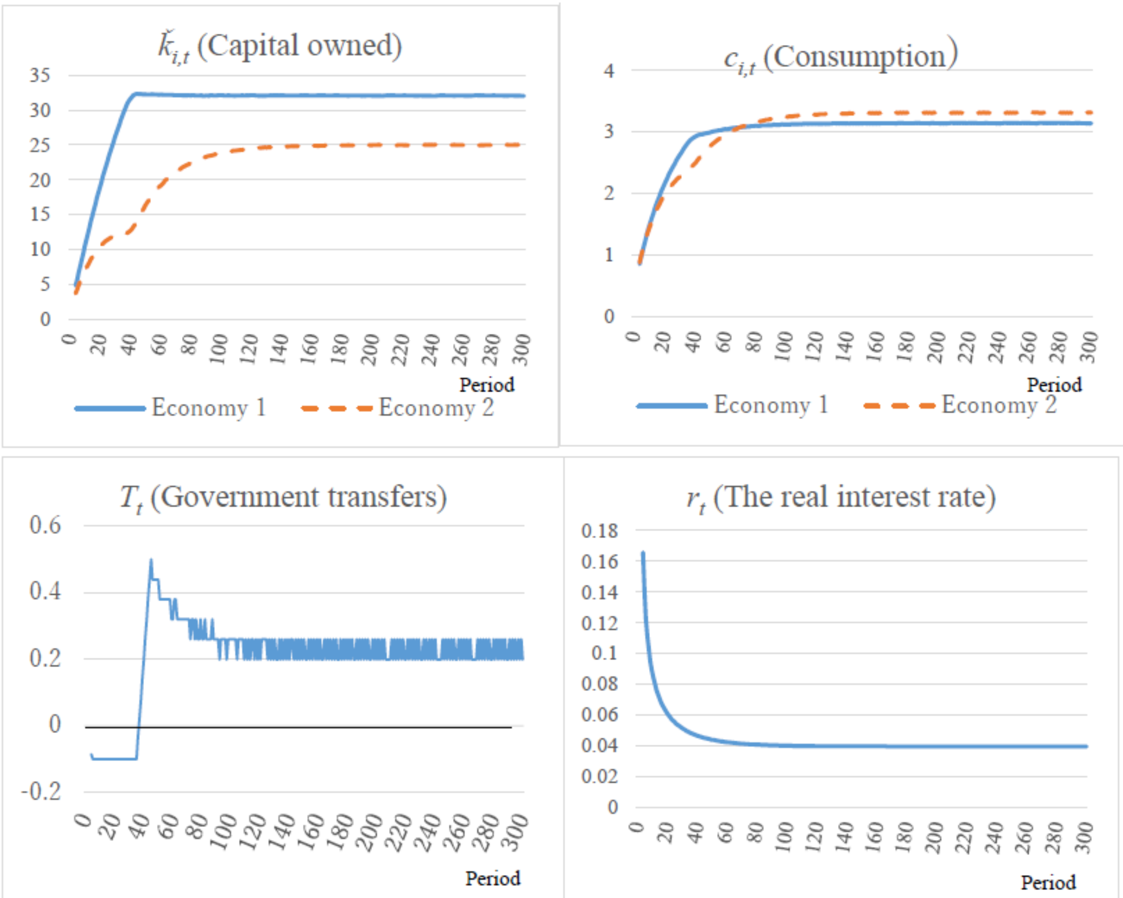


On the other hand, as indicated in Section 4.3.1, many stabilized (steady) states can be achieved by a government under the MDC-based procedure, depending on how it intervenes. This means there can also be many approximate SHs, each of which corresponds to one of the stabilized (steady) states achieved by the government under the MDC-based procedure. Nevertheless, even under the RTP-based procedure, many stabilized (steady) states can be achieved by a government (Harashima 2014). Hence, as indicated theoretically in (Harashima 2018, 2019), each of the many stabilized (steady) states achieved by a government under the MDC-based procedure can be interpreted to be equivalent to an SH with government intervention under the RTP-based procedure. The meaning of this equivalence will be further examined in Section 4.3.5.

To investigate the nature of approximate SHs numerically, I implement two simulations in which households incorrectly estimate the government interventions. In the first simulation, households wrongly estimate the amount of government transfers to be 10% smaller than the actual amount; in the second simulation, the amount is 50% smaller. Because of these incorrect estimates, households consume larger or smaller amounts than they would in the case of correct estimates. Figure 7 presents the results of the first simulation. Clearly, even in this case, the government can achieve an approximate SH. Furthermore, even in the second simulation in which households make a larger error, the government can achieve an approximate SH (Figure 8). These results indicate that even if households wrongly estimate the amount of government transfers, a government can still achieve an approximate SH.

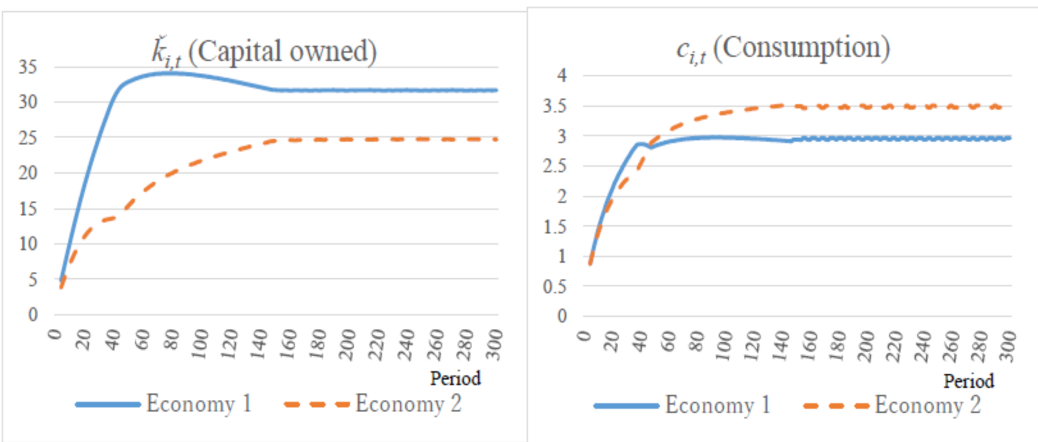
Figure 7. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and the real

interest rate (r_t) in the case of heterogeneous CWRs at MDC: SH when households make a small error estimating the government's interventions (10%)



Governments can also make a mistake in guessing the values needed to reach a stabilized (steady) state. Similar to the previous cases for households, I simulate the case in which a government wrongly guesses that households in Economies 1 and 2 have the same $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$ = 0.04, when in fact, a household in Economy 1 has a value of 0.035 and a household in Economy 2 has a value of 0.045. In this case, even when a government makes a mistake, an approximate SH can be achieved (Figure 9).

Figure 8. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and the real interest rate (r_t) in the case of heterogeneous CWRs at MDC: SH when households make a large error estimating the government's interventions (50%)



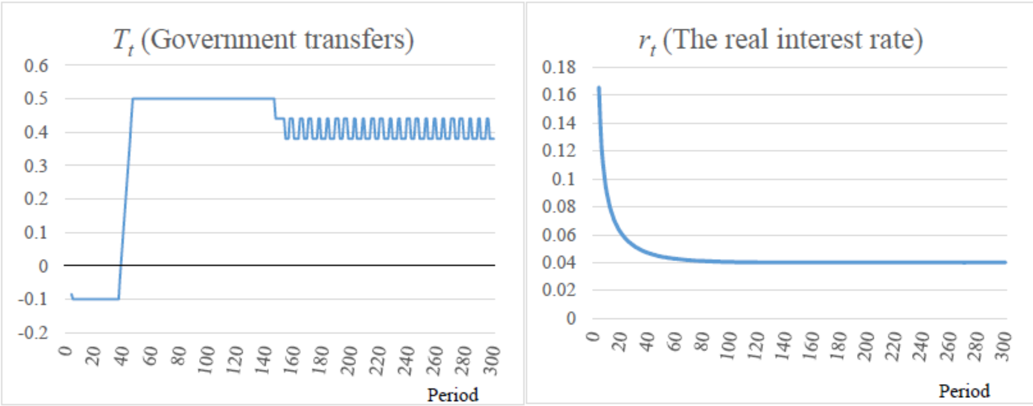
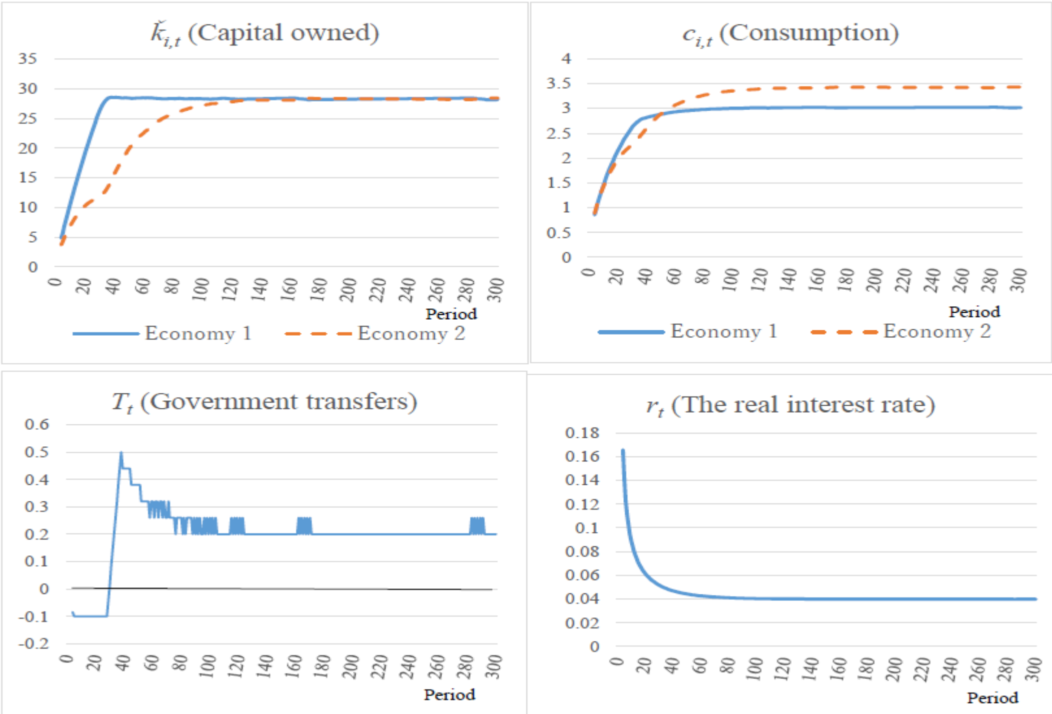


Figure 9. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and the real interest rate (r_t) in the case of heterogeneous CWRs at MDC: SH when the government makes a mistake in estimating households' CWRs at MDC



The meaning of approximate sustainable heterogeneity

As shown in previous section, households do not satisfy and feel comfortable with stabilized (steady) states under the MDC-based procedure. In this sense, these stabilized (steady) states may not be interpreted to be "optimal." If so, no "optimal" stabilized (steady) state can exist under the MDC-based procedure in a heterogeneous population.

Furthermore, even at a multilateral steady state under the RTP-based procedure, the raw (unadjusted) CWR of households are not equal to their $\theta_i \alpha/(1-\alpha)$ (i.e., $\Gamma_i(1-\alpha)/\alpha$ is not equal to θ_i) even though households behave strictly according to their own intrinsic RTPs and all of their optimality conditions are satisfied. That is, at a multilateral steady state under the RTP-based procedure,

$$\theta_i = \Gamma_i \left(\frac{1-\alpha}{\alpha}\right) \tag{28}$$

is held only aggregately and collectively, but not individually, in a heterogeneous population. In other words, the relation expressed by equation (19) holds only aggregately and collectively under the RTP-based procedure in a heterogeneous population. The same can be said of the SH that is achieved by government transfers under the RTP-based procedure.

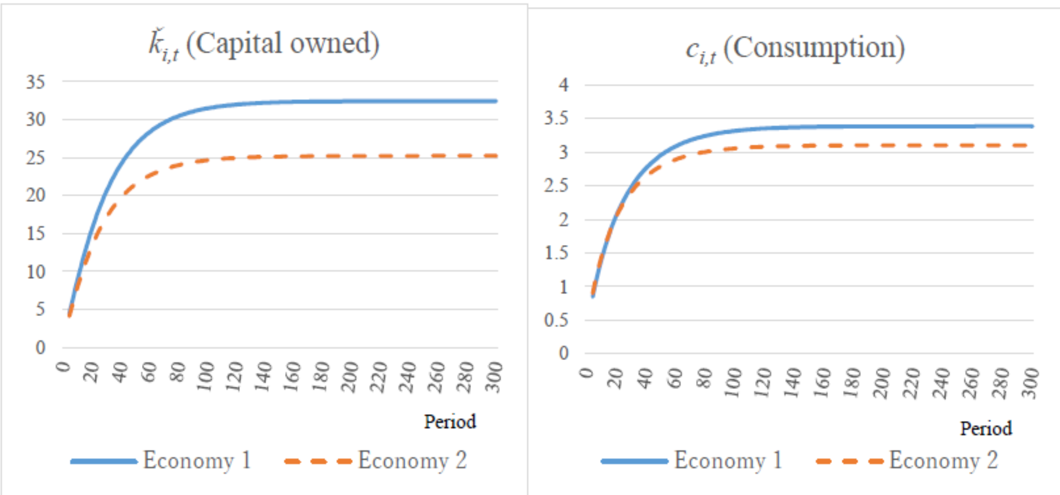
Whether an approximate SH under the MDC-based procedure and a multilateral steady state under the RTP-based procedure can be called an "optimal" state depends on the definition of optimality. Nevertheless, approximate SHs and multilateral steady states have the following two common features. First, they are stabilized (steady) states, and second, all households can behave thoroughly according to their intrinsic preferences (*i.e.,* their CWRs at MDC or RTP). In other words, heterogeneous households can reach a stabilized (steady) state by behaving according to their own intrinsically given heterogeneous preferences throughout all periods. If optimality is defined in this sense, both an approximate SH and a multilateral steady state can be seen to be optimal.

"Irrational" stabilized (steady) states

A completely different type of stabilized (steady) state can also exist under the MDC-based procedure. Suppose that even though households are heterogeneous, they behave according to Consumption formula 1 (*i.e.*, behave only depending on their raw (unadjusted) CWRs). This means that a household behaves "irrationally" because it does not use the information about other households' behaviors and the economic conditions of the country even though it knows that these factors have important effects on its CWR. In this case, I use the term "rational" simply in the sense of a household using all available information and not to refer to the rational expectations hypothesis. Therefore, "irrational" means to deliberately not use some pieces of available information. Figure 10 shows the results of this case. Clearly, a stabilized state is achieved without government intervention.

There is no SH under the RTP-based procedure that corresponds to this irrational one. As Harashima (2010, 2012, 2014, 2017, 2020) showed, under the RTP-based procedure, if a government does not intervene, only one SH can exist (*i.e.*, the multilateral steady state). However, the irrational stabilized (steady) state shown in Figure 10 is completely different from this multilateral steady state because $\breve{k}_{1,t} < \breve{k}_{2,t}$ is not held. The reason a corresponding SH does not exist is that all SHs generated under the RTP-based procedure are the results of households' rational behaviors, particularly those according to the rational expectations hypothesis. In this sense, it is quite natural that no SH under the RTP-based procedure corresponds to an irrational stabilized (steady) state.

Figure 10. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in the case of irrational stabilized (steady) state for heterogeneous households that do not use all available information



Note that the irrational stabilized (steady) state shown in Figure 10 is different from the exceptional case shown in Figure 3 because the raw (unadjusted) CWR is not equal to equation (8) (with the correct value of χ_i) before reaching the stabilized (steady) state, although they are equal after reaching it.

4.4. Households with heterogeneous productivities

Harashima (2010, 2012, 2014, 2017, 2020) theoretically showed that even if productivities (ω_i) are heterogeneous across households, SH is naturally achieved. In addition, (Harashima 2018, 2019, 2021, 2022) showed that this is also true under the MDC-based procedure. In this section, I examine whether this phenomenon can be also observed in a simulation.

I set the productivities of households in Economies 1 and 2 to be 1.05 and 0.95, respectively, and all households in the two economies are assumed to possess the same $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$, which is set to be 0.04. As indicated in (Harashima 2010, 2012, 2014, 2017, 2020), if productivities are heterogeneous, the production of household i is calculated by:

$$y_{i,t} = \left(\frac{\omega_i}{\sum_{i=1}^{H} \omega_i} \sum_{i=1}^{H} k_{i,t} \right)^{1-\alpha} \tag{29}$$

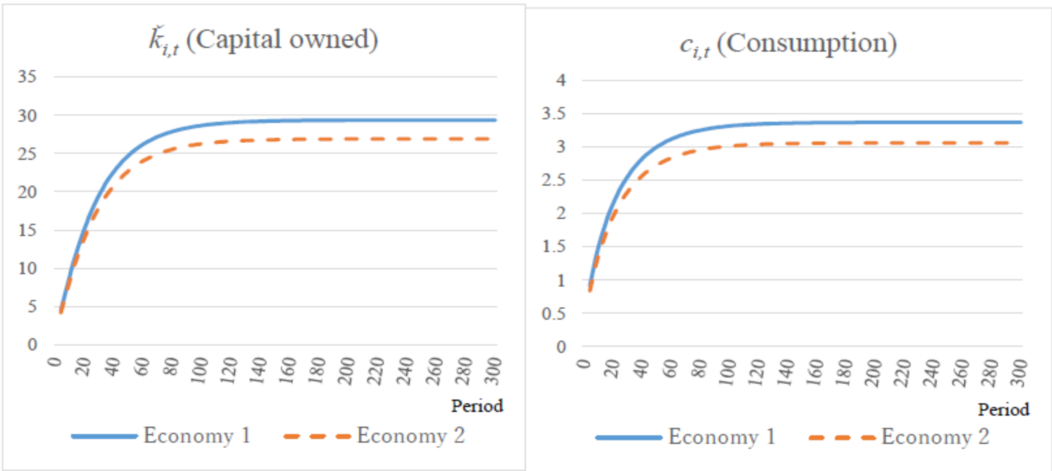
and its labor income is calculated by

$$x_{L,i,t} = \frac{\omega_i}{\sum_{i=1}^{H} \omega_i} \left(\sum_{i=1}^{H} y_{i,t} - \sum_{i=1}^{H} x_{K,i,t} \right) . \tag{30}$$

Households commonly behave unilaterally according to Consumption formula 1. Note that in this case, even if households behave according to Consumption formula 2 without government intervention, the result is the same as if they behave according to Consumption formula 1 because $\Gamma_{i,t} = r_t \alpha/(1-\alpha) = \Gamma_{R,i,t}$ always holds for all households.

Figure 11 shows the result of the simulation in this case. Clearly, a stabilized (steady) state is naturally achieved without any government intervention, as predicted theoretically. At this stabilized (steady) state, their CWRs are almost equal to their common CWR at MDC, and equivalently, the values of $\Gamma_{i,t}(1-\alpha)/\alpha$ of households in the two economies equally converge at almost 0.04, which is identical to their common $\theta_i = \Gamma(\tilde{s}_i)(1-\alpha)/\alpha$.

Figure 11. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) in the case of households with heterogeneous productivities



Because of the difference in productivity, the amount of capital a household in Economy 1 owns is 9.2% larger than that in Economy 2 and the amount of consumption of a household in Economy 1 is 10.0% larger than that in Economy 2 after stabilization. Such magnitudes of differences are quite consistent with the approximate 10% difference in their productivities (*i.e.*, 1.05 and 0.95).

Conclusion

It is not easy to numerically simulate the path to a steady state in dynamic economic models in which households behave generating rational expectations because there is no closed form solution in these models. As an alternative and approximate method, therefore, a log-linearization in the neighborhood of the steady state has been usually used (*e.g.*, Blanchard and Kahn 1980; Kydland and Prescott 1982; Uhlig 2001). The difficulty of simulating the path to a steady state raises an important and serious question: can an ordinary household actually have the foresight to see the path to a steady state precisely in everyday life?

Harashima (2018, 2019, 2021, 2022a, 2022b) showed an alternative procedure for households to reach a steady state (*i.e.*, the MDC-based procedure). In this paper, I simulated paths to a steady state that are reached under the MDC-based procedure in several different situations. In the simulations, a household is set to increase or decrease its consumption according to simple formulae that are presumed to capture the essence of household behavior under the MDC-based procedure.

The results of the simulations indicate that households can reach a stabilized (steady) state by generally behaving according to their feelings and guesses about their CWRs and the state of the entire economy. In a homogeneous population, households naturally reach a stabilized (steady) state that is almost the same as that under the RTP-based procedure. On the other hand, in a population with heterogeneous CWRs at MDC, no stabilized (steady) state is achieved if households behave unilaterally and the government does not intervene, as predicted theoretically. Nevertheless, also as predicted theoretically, if a government appropriately intervenes, approximate SHs can be achieved.

Furthermore, in a population with heterogeneous productivities, households naturally reach a stabilized (steady) state without any government intervention, also as predicted theoretically. These results mean that the equivalence between the MDC- and RTP-based procedure is proved not only theoretically (Harashima 2018, 2019, 2021, 2022a, 2022b) but also numerically.

References

[1] Becker, R.A. (1980). On the Long-run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households, *The Quarterly Journal of Economics*, 95(2): 375–382. https://doi.org/10.2307/1885506

[2] Blanchard, O.J. and Kahn, Ch.M. (1980). The Solution of Linear Difference Models under Rational Expectations, *Econometrica*, 48(5): 1305-1311. https://doi.org/10.2307/1912186

[3] Ellison, M., and Pearlman, J. (2011). Saddlepath Learning, *Journal of Economic Theory*, 146(4): 1500-1519. https://doi.org/10.1016/j.jet.2011.03.005

[4] Evans, G.W., and Seppo, H. (2001). *Learning and Expectations in Macroeconomics*, Princeton and Oxford, Princeton University Press, 424 pp. ISBN: 978-0691049212

[5] Harashima, T. (2010). Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population, *MPRA Paper* No. 24233. https://mpra.ub.unimuenchen.de/22521/1/MPRA_ paper_22521.pdf

[6] Harashima, T. (2012). Sustainable Heterogeneity as the Unique Socially Optimal Allocation for Almost All Social Welfare Functions, *MPRA Paper* No. 40938. https://mpra.ub.unimuenchen.de/40938/1/MPRA_paper _40938.pdf

[7] Harashima, T. (2014). Sustainable Heterogeneity in Exogenous Growth Models: The Socially Optimal Distribution by Government's Intervention, *Theoretical and Practical Research in Economic Fields*,5(1):73-100.

[8] Harashima, T. (2017). Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population, *in Japanese, Journal of Kanazawa Seiryo University*, 51(1): 31-80.

[9] Harashima, T. (2018). Do Households Actually Generate Rational Expectations? "Invisible Hand" for Steady State, MPRA Paper No. 88822. https://core.ac.uk/download/pdf/ 214008545.pdf

[10] Harashima, T. (2019). Do Households Actually Generate Rational Expectations? "Invisible Hand" for Steady State, *in Japanese, Journal of Kanazawa Seiryo University*, 52(2): 49-70.

[11] Harashima, T. (2020). Sustainable Heterogeneity as the Unique Socially Optimal Allocation for Almost All Social Welfare Functions, *in Japanese, Journal of Kanazawa Seiryo University*, 54(1): 71-95. https://mpra.ub.uni-muenchen.de/40938/1/MPRA_paper_40938.pdf

[12] Harashima, T. (2021) Consequence of Heterogeneous Economic Rents under the MDC-based Procedure, *Journal of Applied Economic Sciences,* Volume XVI, Summer, 2(72): 185-190. https://mpra.ub.uni-muenchen.de/105765/1/ MPRA_paper_105765.pdf

[13] Harashima, T. (2022a). A Theory of Inflation: The Law of Motion for Inflation under the MDC-based Procedure, MPRA Paper No. 113161. https://mpra.ub.uni-muenchen.de/113161/4/ MPRA_paper_113161.pdf

[14] Harashima, T. (2022b). A Theory of Inflation: The Law of Motion for Inflation under the MDC-based Procedure, *in Japanese, Journal of Kanazawa Seiryo University*, 54(1).

[15] Kydland, F.E., and Prescott, E.C. (1982). Time to Build and Aggregate Fluctuations, *Econometrica*, 50(6): 1345-1370. https://doi.org/10.2307/1913386

[16] Marcet, A.I., and Sargent, T.J. (1989). Convergence of Least Squares Learning Mechanisms in Self-referential Linear Stochastic Models, *Journal of Economic Theory*, 48(2): 337-368. https://doi.org/10.1016/ 00220531(89)90032-X

[17] Uhlig, H. (2001). A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily, in *Computational Methods for the Study of Dynamic Economies*, Ch. 3, Marimon, R. (Ed.), Scott, A. (Ed.), Oxford University Press. https://www.sfu.ca/~kkasa/uhlig1.pdf