Journal of Applied Economic Sciences
Volume XIX, Spring, Issue 1(83), 2024

Electronic Trading and Financial Crisis Effects in the e-MID Interbank Market: A Multivariate Multiplicative Error Model Analysis

Vahidin JELESKOVIC
https://orcid.org/0000-0002-1510-2048
Humboldt University of Berlin, Germany
vahidin.jeleskovic@hu-berlin.de

Markus ENGLER
Independent researcher, Germany
markusengler@gmx.net

Article's history:

Received 1st of March, 2024; *Received in revised form* 17th of March, 2024; *Accepted* 25th of March, 2024; *Available online*: 27th of March, 2024.
Published: 30th of March, 2024 as article in the Volume XIX, Spring, Issue 1(83), 2024.

Suggested citation:

Jeleskovic, V., & Engler, M. (2024). *Electronic Trading and Financial Crisis Effects in the e-MID Interbank Market: A Multivariate Multiplicative Error Model Analysis*. *Journal of Applied Economic Sciences*, Volume XIX, Spring, Issue 1(83), 63 – 76. https://doi.org/10.57017/jaes.v19.1(83).04

Abstract:

We employ a multivariate multiplicative error model (MMEM) to explore the interplay between high-frequency return volatilities, trading volume, and trading intensities within the Italian Electronic Interbank Credit Market (e-MID). Our research question also aims at analysing the e-MID and, more specifically, the behaviour of traders in relation to the financial crisis. For this purpose, we consider four time periods: the first period before the first intervention by the ECB, the second period before the collapse of Lehman Brothers, and finally, the last intervention by the ECB. Utilising five-minute intervals, our analysis reveals a robust causal relationship among volatilities, volumes, and trading intensities in this electronic market, yielding highly significant coefficient estimates of the multivariate multiplicative error model (MMEM). However, these relationships change qualitatively, showing a shift in e-MID market behaviour before, during, and after the outbreak of the financial crisis. Notably, we find evidence that trading was in a Pareto optimum in the first period, but as soon as uncertainties hit the market, this Pareto optimum becomes unstable and breaks down completely in the last period of an extremely illiquid market state. To the best of our knowledge, this paper represents the first and inaugural empirical application of MMEM to an interbank credit market, contributing valuable insights into its intricate dynamics.

Keywords: multivariate multiplicative error model (MMEM); interbank markets; intraday trading process.

JEL Classification: C15; C32; C52; C55; E43; G01.

Introduction

The increasing availability of high-frequency financial data has brought the empirical analysis of trading behaviour and the modelling of trading processes to the forefront of financial econometrics. Numerous empirical studies have demonstrated a strong contemporaneous relationship between daily aggregated trading volume and volatility across various financial markets. This observation aligns with the mixture-of-distribution hypothesis (MDH) introduced by Clark (1973). The MDH relies on central limit arguments, assuming that daily returns consist of the sum of intra-daily (logarithmic) price changes associated with intraday equilibria.

Several studies have extended this relationship analysis to intraday financial market data. Building on this body of research, the primary objective of this work is to investigate the interbank credit market. Specifically, it aims to explore the connections between price volatilities, trading volume, trading intensities. Additionally, factors such as bid-ask spreads and market depth, as represented by an open limit order book (Harris (1994), Hautsch (2007), Hautsch and Jeleskovic (2008)), may also be considered in this analysis.

We employ a multivariate multiplicative error model (MMEM) to explore the interplay between high-frequency return volatilities, trading volume, and trading intensities within the Italian Electronic Interbank Credit Market (e-MID). Utilizing five-minute data from the e-MID, our analysis reveals a robust causal relationship among volatilities, volumes, and trading intensities in this electronic market, yielding highly significant MMEM coefficient estimates.

Engle and Russell (1997, 1998) introduced the Autoregressive Conditional Duration (ACD) model, which is designed to model autoregressive duration processes in financial data. It does so through a multiplicative error process and a GARCH-type parameterization of the dependent duration mean. The term "MEM" (Multiplicative Error Model) was initially introduced by Engle (2002). In his work, he approached the "standard" MEM as a general framework for modelling positive-valued dynamic processes.

Manganelli (2005) extended Engle's work by proposing a multivariate MEM to jointly model high-frequency volatilities, trading volume, and trading intensities (measured in terms of the duration between two trades). Manganelli's Multivariate MEM (MMEM) is particularly appealing because it allows for the analysis of simultaneous, instantaneous, and direct effects among microstructure variables in electronic trading. However, it requires an assumption of weak exogeneity, meaning there must be a predefined causal relationship between these variables in the very current period, which necessitates theoretical assumptions about their causal connections.

Hautsch (2007) extends the basic MEM structure by introducing a common latent dynamic factor that acts as a subordinated process driving the various components. This model combines elements of both GARCH-type and stochastic volatility (SV) models and is known as the Stochastic MEM. Engle and Gallo (2006) employ various MEM specifications to jointly model different volatility indicators, such as absolute returns, daily range, and realized volatility. Cipollini et al. (2007) further extend the MEM by incorporating a copula specification to capture contemporaneous relationships between the variables under consideration.

Given the significance of Multiplicative Error Models (MEMs) in modelling high-frequency trading processes, liquidity dynamics, volatility, and other market phenomena, we aim to demonstrate the practical application of MEM in capturing the multivariate dynamics of volatility, trade sizes, and trading intensities. This application is based on transaction data obtained from the Italian interbank market, specifically e-MID. The interbank credit market is characterized by a high level of trust among market participants, resulting in very low uncertainties and, consequently, very low risk premia. As a result, banks tend to prioritize liquidity over profits, as they can deploy this liquidity to earn higher interest rates on other markets. The other reason is that the interbank credit market is typically used to address banks' liquidity shortages of any kind. Several studies have already explored the interbank market, particularly e-MID, focusing on various research topics. Gabbi et al. (2012) investigated market microstructure, bank behaviour, and interbank spreads using daily aggregated data spanning from 1999 to 2009. During the same time frame, Raddant (2012) examined trade flow, absolute volume, and preferred lending relationships within the market. Additionally, Politi et al. (2010) and Demertzidis (2020) provided a comprehensive market overview by employing basic statistical measures. They also introduced variable computations for both pre and post-financial crisis periods. Furthermore, Jeleskovic and Demertzidis (2019) and Demertzidis and Jeleskovic (2020) conducted an analysis of intraday yield curves within the context of the e-MID. Their work introduced a novel concept of intraday yield curves.

In addition to network analysis, another focal point of market studies lies in the examination of interest rates and the micro- and macroeconomic determinants of these rates. This area has been explored by Kapar et al. (2012), Angelini et al. (2009), and Gabrielli (2010). These studies utilize transaction data from both overnight loans and longer maturities. Baglioni and Montecini (2008b) construct a hypothetical market for one-hour interbank loans to calculate the "intraday price of money" for the Italian interbank market.

The trading volume represents another significant aspect of research on the Italian interbank market. Porzio et al. (2010) employ an autoregressive model with multiple predictors to forecast the traded volume in the market. Liquidity distribution during the crisis was investigated by Vento (2010). Brunetti et al. (2009) combine their study on volume with an examination of the effects of Central Bank decisions on the market. Fricke (2012) also conducts a combined analysis of market volume and the trading strategies of market participants. All these studies consistently observe a continuous decrease in market volume during the crisis, which eventually leads to a market breakdown.

However, to the best of our knowledge, there has been no prior research analysing the microstructure of an interbank credit market using multivariate multiplicative error models. Consequently, the primary objective of our paper is to fill this gap and provide initial findings within the framework of multiplicative multivariate error models applied to an interbank credit market. In accordance with this, we focus on two main research questions in our paper. Firstly, we examine the suitability of an MMEM for modelling the dynamics of microstructure variables in electronic trading in the interbank credit market. In our paper, we rely on Manganelli's MMEM as the basis for our empirical analysis, specifically using the version proposed by Hautsch and Jeleskovic (2008). Secondly, we investigate the consequences of the financial crisis on these dynamics. Regarding the second research question, we draw inspiration from Manganelli's (2005) findings. Manganelli (2005) notably discovered that the results concerning the joint dynamics of microstructure variables for infrequently traded stocks lack the robustness observed in frequently traded stocks. We consider this evidence highly important for the purposes of analysing our research questions because liquidity on the e-MID significantly decreased overall after the financial crisis.

Furthermore, we conduct multiple estimations to scrutinize shifts in e-MID market behaviour during the recent financial crisis. Our findings underscore the effectiveness of high-frequency data models in deciphering interbank credit market dynamics. Notably, we observe changes in market behaviour during the crisis, with liquidity variables having a negative impact on volatility before the crisis, in contrast to the period following the outbreak of financial turmoil. Hence, we can assert that liquidity may have a stabilizing effect on the price dynamics of credits in an undisturbed interbank credit market. However, after the outbreak, it is reasonable to assume a positive effect of liquidity on the volatility in the e-MID at an intraday frequency. To the best of our knowledge, this paper represents the first empirical application of MMEM to an interbank credit market, providing valuable insights into its complex dynamics.

The structure of the paper is as follows: Section 2 outlines the fundamental principles of MEM and introduces a multivariate specification of MEM. Section 3 provides insights into the data used and the mechanics of the Italian interbank market. Section 4 offers an overview of variable computation, model specifications, and the results of estimation. Finally, Section 5 offers concluding remarks.

1. Multiplicative Multivariate Error Models (MMEM)

In this section of the study, we give an overview about the econometric model and we present the model specification used for our analysis.

The univariate MEM

Let $\{Y_t\}$, $t = 1, \dots, T$ denote a non-negative random variable. Then, the univariate multiplicative error model (MEM) for Y_t is given by

$$Y_t = \mu_t \varepsilon_t \tag{1}$$

$$\varepsilon_t | \mathcal{F}_{t-1} \sim \mathrm{i.i.d.}\, D(1, \delta^2) \tag{2}$$

where $\mathcal{F}_t$ denotes the information set up to t, μ_t is a non-negative conditionally deterministic process given $\mathcal{F}_{t-1}$ and ε_t is a unit mean, i.i.d. variate process defined on non-negative support with variance δ^2.Than the following shall apply:

$$E[Y_t | \mathcal{F}_{t-1}] \stackrel{\text{def}}{=} \mu_t \tag{3}$$

$$\mathrm{Var}[Y_t | \mathcal{F}_{t-1}] = \delta^2 \mu_t^2 \tag{4}$$

The major idea of the MEM is to parameterize the conditional mean μ_t in terms of a function of the information set $\mathcal{F}_{t-1}$ and parametersθ. The basic linear MEM (p, q) than can be written as,

$$\mu_t = \omega + \sum_{j=1}^{p} \alpha_j Y_{t-j} + \sum_{j=1}^{q} \beta_j \mu_{t-j} \tag{5}$$

with $\omega > 0$, $\alpha_j \geq 0$ and $\beta_j \geq 0$.

This basic linear MEM was introduced by Manganelli (2005) and Engle (2002). For a closer look at the specifications and variations of the model of our interest, see Hautsch (2007), Hautsch and Jeleskovich (2008) and Engle et al. (2012).

The linear MEM specification is extended to an often-used logarithmic specification of a MEM. This specification ensures the positivity of μ_t without implying parameter constraints. This is especially important whenever the model is augmented by explanatory variables or when the model has to accommodate negative cross correlations or autocorrelations in a multivariate setting. Two versions of the logarithmic MEM, already introduced by Bauwens and Giot (2000), are given (with p = q = 1) by

$$\log \mu_t = \omega + \alpha g(\varepsilon_{t-1}) + \beta \log \mu_{t-1} \tag{6}$$

where $g(\cdot)$ is given either by $g(\varepsilon_{t-1}) = \varepsilon_{t-1}$ or $g(\varepsilon_{t-1}) = \log \varepsilon_{t-1}$. The process is covariance stationary if $\beta < 1$, $E[\varepsilon_t \exp\{\alpha g(\varepsilon_t)\}] < \infty$ and $E[\varepsilon_t \exp\{2\alpha g(\varepsilon_t)\}] < \infty$. For more details, see Bauwens and Giot (2000).

The multivariate vector MEM

For a k-dimensional positive-valued time series, denoted by $\{Y_t\}$, t = 1…T, with $Y_t \stackrel{\text{def}}{=} (Y_t^{(1)}, \dots, Y_t^{(k)})$, the vector MEM (VMEM) for Y_t is defined by:

$$Y_t = \mu_t \odot \varepsilon_t = \operatorname{diag}(\mu_t)\varepsilon_t \quad (7)$$

where: $\odot$ denotes the Hadamard product (element-wise multiplication) and ε_t is a k- dimensions vector of reciprocal and serially innovation processes where the j -th element is given by

$$\varepsilon_t^{(j)} | \mathcal{F}_{t-1} \sim \text{i.i.d. } D(1, \delta_j^2), \; j = 1, \dots, k. \quad (8)$$

The extension of the linear MEM proposed by Manganelli (2005) and Engle (2002) is than given by:

$$\mu_t = \omega + A_0 Y_t + \sum_{j=1}^{p} A_j Y_{t-j} + \sum_{j=1}^{q} B_j \mu_{t-j} \quad (9)$$

where ω is a (k × 1) vector and A_0 , A_j and v are (k × k) parameter matrices. The A_0 matrix captures the relationships between the elements of Y_t and only its upper triangular elements are non-zero.

This structure implies that $Y_t^{(i)}$ is predetermined for all variables $Y_t^{(j)}$ with j $< i$. So Y_t^{i} is conditionally i.i.d. given $\{Y_t^{(j)}, \mathcal{F}_{t-1}\}$ for j $< i$. With this specification the relationship between the variables is taken into account without requiring multivariate distributions for ε_t and eases the estimation of the model. The ordering of the variables in the Y_t matrix is typically chosen in accordance with the research objective or follows economic reasons.

In correspondence to the univariate logarithmic MEM, we obtain a logarithmic VMEM specification by

$$\log \mu_t = \omega + A_0 \log Y_t + \sum_{j=1}^{p} A_j \, g(\varepsilon_{t-j}) + \sum_{j=1}^{q} B_j \, \log(\varepsilon_{t-j}) \quad (10)$$

where: $g(\varepsilon_{t-j}) = \varepsilon_{t-j}$ or $g(\varepsilon_{t-j}) = \log \varepsilon_{t-j}$, respectively.

2. Data and e-MID

In this section, we explain the main characteristics of the e-MID interbank market and the dataset we used in this study. We also describe the computation of the used variables and show some statistical facts and descriptive statistics.

e-MID

The e-MID interbank market is a fully automatic platform for interbank loan, managed by the e-MID company (Italy) and supervised by the Bank of Italy. Credit institutions and investment companies can participate in the market system if their total asset size is about 10 million US-Dollar (or its equivalent in another currency) or 300 million Euros (or equivalent in another currency).

One main difference to other interbank markets is that it is almost fully transparent to all participants. Buy and sell proposals appear on the market platform together with the identity (bank-ID) of the market member. In sell transactions, the money flows from the aggressor bank to the quoter bank. In this case, the quoter bank is borrowing money and the aggressor is lending money. In buy transactions, the money flows from the quoter bank to the aggressor bank which indicates that the aggressor is borrowing and the quoter bank is lending. The aggressor bank is the bank placing the order in the market or the bank which actively chooses an existing order. The trading takes place during the time period from 8:00 a.m. to 6:00 p.m. ECT.

The e-MID market does not offset any counter party risk because the participants always know the opponent by its bank-ID. Also, the search costs for all platform participants are identical. In the market, each bank can actively choose any counter party present in the order book to start a trade. The two parties than can negotiate about the trade, change the volume or price (credit rate) or deny the transaction. This transparency in the market can bring a disadvantage during a financial crisis. If there is a high uncertainty in financial markets and banks also give importance to reputation game, the banks avoid trading in such transparent markets. Given this behaviour, the transaction volume and the number of participants in the market consequently decrease during the financial crisis.[1]

[1] For further details about e-MID, see e.g., Baglioni and Monticini (2008b) or Gabbi et al. (2012).

Dataset

We use a dataset which includes all transactions in the e-MID interbank market from 1.10.2005 to 31.03.2010[2]. For each transaction we have information about the date, the time of the trade, the quantity, the interest rate, the type of the transaction (sell or buy) and the Bank-ID of the quoter and the aggressor. Table 1 shows an example of the transaction data.

In this study we concentrate on the overnight (ON) and the overnight large (ONL) transactions, because these transactions cover more than 90% of the total transaction volume in the market. The overnight large trades have a size over 100 million euros. We also compute the volume-weighted mean rate as

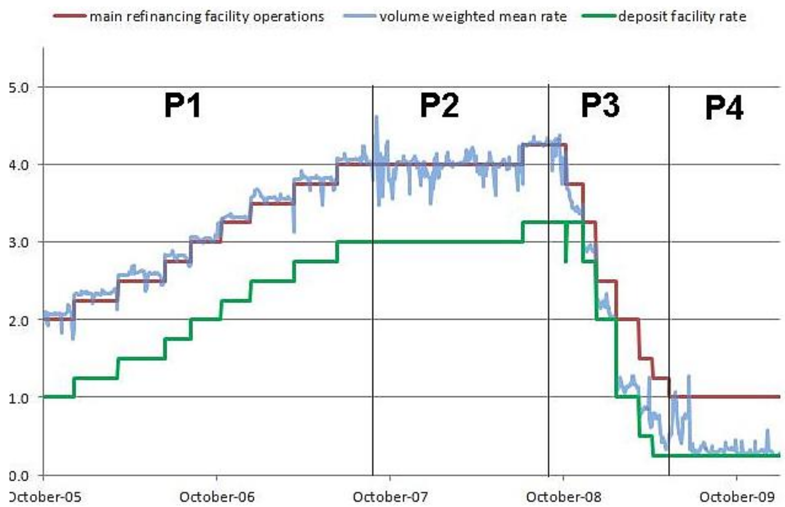
$$\bar{r}_w^t = \frac{\sum_{i=1}^{N_t} r_i v_i}{\sum_{i=1}^{N_t} v_i} \tag{11}$$

where: N_t is the number of transactions in an interval and v_i and r_i represent the volume and the rate of the i^{th} transaction in the interval. This computation is necessary, because we only have information about the executed orders and information about the whole order back of the market.

According to Politi (2010), Gabrieli (2011), Masi et al. (2006), Gaspar et al. (2007), Iori et al. (2007), Iori & Precup (2007), Iori et al. (2008), Gabbi et al. (2012) and Raddant (2012), who use a similar dataset for their investigations on the Italian interbank market, this computed rate (and also its volatility) can be used in statistic models or for statistical analyses. In addition, Brunetti (2009) and Baglioni & Monticini (2008a, 2008b) used the data for intra-daily analyses of the interest rate, intraday volatility and intraday market behaviour.

To investigate a change in the market behaviour before, during and after the crisis we divide the total data set in four sub periods. For the length of each period, we orientate ourselves by the definition of Gabbi et al. (2012), Gabrieli (2011) & Politi et al. (2010) for their analyses. In addition, we split the third period into two sub periods, because we recognise that the preferred classification of the former studies accords to the EZB key rate (RPS[3] and DEP[4]) decisions. So, we split the third period by 13. March 2009 (latest change of the RPS) to see if there are any changes in the market behaviour based on the rate decisions of the EZB. Figure 1 shows the e-MID rate, the RPS and the DEP.

Figure 1. Volume weighted mean rate of the e-MID, the main refinancing facility operations (rate) and the deposit facility rate on daily base with the four sub periods



Note: The e-MID rate and the main refinancing facility operations (rate) correspond before and during the crisis (till Lehman brothers collapse in September 2009), then the e-MID rate starts to correspond to the Deposit facility rate. Based on this behaviour, we decide to split the post Lehman period in two sub periods for our analysing purpose.

[2] We use the German date notation (dd.mm.jjjj).

[3] Main refinancing facility operations (RPS) are regular liquidity-providing reverse transactions with a frequency and maturity of one week. They are executed by the NCBs on the basis of standard tenders and according to a pre-specified calendar. The main refinancing operations play a pivotal role in fulfilling the aims of the Euro system's open market operations and normally provide the bulk of refinancing to the financial sector.

[4] Deposit facility rate (DEP): counterparties can use the deposit facility to make overnight deposits with the NCBs. The interest rate on the deposit facility normally provides a floor for the overnight market interest rate.

The analysis covers four distinct periods: the first period spans from 01.10.2005 to 08.08.2007 (P1); the second period spans from 09.08.2007 to 14.09.2008 (P2); the third period spans from 15.09.2008 to 12.05.2009 (P3); and the fourth period spans from 13.05.2009 to 31.03.2010 (P4). Notably, two key events during the financial crisis are the initial intervention of the ECB on 09.08.2007, and the Lehman Brothers' collapse in the USA on 15.09.2008. Furthermore, the date of 13.05.2009, holds significance in the European interbank markets, marking the latest reduction of key interest rates by the ECB.

3. Main Results

In this section, we will describe the variables and their computations, show some descriptive statistics of the Italian interbank market and illustrate an application of the VMEM to model jointly return volatilities, average trade sizes and the number of trades for intra-day trading in the interbank market.

Variables and descriptive statistics

Based on the early findings of Karpoff (1987) and Harris (1994) and in line with Hautsch (2007) and Hautsch & Jeleskovic (2008), the main purpose of our study is to analyse the influence of the volume and the trade intensity on the volatility in the interbank market. We want to examine if there is a similar link between these variables as the authors above have figured out for other financial markets. For our analysis we use five-minute intervals for aggregate the variables like Hautsch (2007) uses for his (S)MEM study on US Blue Chips or like Brunetti et al. (2009) for their study on the News Effect on the e-MID market. This aggregation helps to reduce the complexity of the model and allows us to compute usable data even in periods with very low trading intensity.

For applications of MEMs to irregularly spaced data, see Manganelli (2005) or Engle (2000). Table 2 shows the descriptive statistics of the variables for each period (including all zero value intervals).

In order to reduce the impact of opening and closure effects, we decide to use only the observations from 9:00 a.m. to 5:30 p.m., because in the opening and closing hours the market participants typically analyse the information of the former day to plan their activities and so the market volume and market activity is very low and not representative for the relationships between the variables.[5]

A typical feature of high-frequency intraday data is the strong influence of intraday seasonality, which is shown by several empirical studies. For closer look see Bauwens & Giot (2001) or Hautsch (2004). According to empirical findings from other markets (for example, Hautsch & Jeleskovic, 2008), we observe that the liquidity demand (volume per trade and trade intensity) follows a U-shape pattern with a period of low trading activity around noon. Like Hautsch & Jeleskovic (2008), we also observe the highest volatility after the opening of the market and before the closure, which is an indication of information processing during the first minutes of trading similar to the most other financial markets.

One possibility to face intraday seasonality is to augment the specification of μ_t by appropriate regressors. An alternative way is to adjust to seasonality in a first step. For this possibility the effect of a pre-adjustment on the final parameter estimates is controversially discussed in the literature (see Veredas et al., 2001). Like most empirical studies prefer the two-stage method since it reduces the model complexity and the number of parameters to be estimated in the final step, we also follow this proceed in pre-adjust the variables. For the seasonal adjustment we use a moving average (MA) with the total length of 97 intervals to compute the seasonal factor of each interval on every day[6]. In the next step we compute the average of the seasonal components for each interval over each sub period. Then, we divide the variable by the seasonal component of the corresponding interval to get the adjusted variables. At last, we divide the variable by the seasonal factor to get an estimate for the seasonal component of each interval[7]. The resulting seasonality patterns are shown in Figure 2 to Figure 5.

In the first two periods the main volatility occurs at the end of the trading time. In the third period the volatility decreases in the evening hours 4 p.m. to 6 p.m. and increases again in the morning hours and increases in the morning hours 8 a.m. to 9 a.m. However, as a contrast, in the fourth period the main volatility occurs in the morning hours, the opening of the market. The volatility in the evening hours is still present in period four (and higher compared to the previous periods). The intraday seasonality of the volume variable has the typical U shape with huge orders in the morning and evening hours. A small spike is also visible around the lunch time (1 p.m. to 3 p.m.),

[5] We based our decision on the work of Brunetti (2009) and Baglioni and Montichini (2008b). Brunetti identifies a time interval from 8:30 a.m. to 5:00 p.m. as representative in his study of Central Bank intervention effects. Baglioni and Montichini (2008b) identify the time interval from 9:00 a.m. to 18:00 p.m. as representative in their study of the hypothetical intraday loan market.

[6] Due to 97 intervals per day.

[7] Hautsch (2007) uses cubic spline function for seasonal adjustment. For our case of equal-length time intervals the both methods lead to the same results.

which represents a period with only a few but huge orders in the market. In period one, two and four the mean volume drops rapidly for the last hour of trading. The intraday seasonality pattern for the number of trades per five-minute interval also shows that the main trading activity takes place between 9:30 a.m. to 4:30 p.m., with a dip around noon. The pattern is similar for all periods.

For the estimation purpose we use the autocorrelation and cross correlation of the computed variables as indication for the ordering of the variables. All variables are significantly auto correlated at lag one for all four periods, except the volume variable in period four, which is highest auto correlated at lag four. The cross correlation for all variables is highest (or lowest) at lag zero, expect for the cross correlation of the squared returns and the number of trades in the third period. The two variables are significantly cross correlated at lag 13, which is an indication of the change of the trading behaviour in the crisis period[8]. Due to the autocorrelations and cross correlations of the variables and in respect to the subject of this work, we chose the volatility variable (squared log returns as the dependent variable.[9] Furthermore, we assume the volume variable dependents on the number of trades per five-minute interval. The rationale behind that is that the higher the number of trades the higher the volume,

Similar to Hautsch (2007) and Hautsch & Jeleskovic (2008), we make the following assumptions for the process of squared returns, denoted as:

$Y_t^{(1)} = r_t^2,$ we assume:

$Y_t^{(1)} | Y_t^{(2)}, Y_t^{(3)}, \mathcal{F}_{t-1} \sim N(0, \mu_t^{(1)})$ and for $Y_t^{(j)}$, $j \in \{2, 3\}$, we assume:

$Y_t^{(j)} | Y_t^{(j+1)}, \ldots, Y_t^{(3)}, \mathcal{F}_{t-1} \sim \mathrm{Exp}(\mu_t^{(j)})$.

Here, we define $j = 2$ for trading volume and $j = 3$ for the number of trades. The rationale behind these assumptions is that trading volume is often proportional to the number of trades. Traders typically break down their trades into smaller pieces to reduce the market impact and develop more effective trading strategies. Both of these variables then influence changes in returns and, consequently, volatility. While it is well-known that both the normal and exponential distributions may not fully capture the distributional properties of high-frequency trading processes, they do provide a basis for Quasi-Maximum Likelihood (QML) estimation of the model.

Due to the difficulty of handling non-positive valued variables in the VMEM, which can lead to pitfalls and potential artifacts, we have opted to exclude all intervals containing zero values for at least one of the three variables. This decision is based on the belief that these intervals with zero values do not provide valuable information and can therefore be disregarded[10]. Additionally, following the approach of Hautsch & Jeleskovic (2008), we have normalized the variables by dividing each of them by its mean value.

Finally, we estimate a three-dimensional Log-VMEM for squared log returns, trade sizes and the number of trades by their corresponding seasonality components. For simplicity and to keep the model tractable, we restrict our analysis to a specification of the order p = q = 1 and fully parameterized matrix A_1 and diagonal B_1 matrix.[11] The innovation term is chosen as $g(\varepsilon_t) = \varepsilon_t$.

Estimation results for multivariate VMEM

Table 3 presents the estimation results for the four periods, while Table 4 provides descriptive statistics for the variables used in the estimation and the VMEM residuals, as per the explained specification. However, it is essential to delve deeper into the descriptive statistics to gain a better understanding of the state of e-MID. Firstly, it's worth noting that the mean of squared returns (and, consequently, returns themselves) is extremely low in the first two periods, indicating low interest rates and consequently low risk premia. This situation undergoes a dramatic shift in the last two periods following the outbreak of the financial crisis. During these latter periods, there is a noticeable surge in uncertainty. Therefore, prior to the crisis, e-MID predominantly functioned as a market for liquidity. However, after the outbreak, interest rates assumed greater significance in trading strategies. Likewise,

[8] The higher lag in the cross correlation of the volatility variable and the number of trades indicates a low level of market activity in the third period combined with still remaining high volatility.

[9] Depends on volume and number of trades.

[10] Manganelli (2005) argues that during these intervals, inactivity in trading could be attributed to the possibility of bad news. However, whether there is good or bad news in the vicinity, it is expected to have some impact on trading. Therefore, it is more likely that the absence of new information is the underlying reason for the inactivity in trading dynamics.

[11] We also adhere to the approach of Manganelli (2005) and Hautsch and Jeleskovic (2008) in this regard.

both trading volume and the average number of trades per time interval witnessed a substantial decline in the last period, signifying a shift towards a less liquid market.

In summary, based on the estimation results provided in Table 3, the key findings can be summarized as follows. Firstly, we obtain significant parameter estimates, signifying a notable and meaningful interrelationship among all variables across all periods. Additionally, LB statistics reveal a substantial decrease in their values in the residuals in comparison to the original variables, suggesting that a significant portion of their dynamics is accounted for by the MMEM. Nevertheless, the LB statistics are still relatively high, indicating that the model may not be perfectly specified. Nonetheless, we can conclude that the MMEM is well-suited for capturing the joint dynamics of microstructure trading variables.

Secondly, as evidenced by the sum of the diagonal elements in A_1 and the elements in B_1, all trading components exhibit strong positive autocorrelation in both the short and long term. However, the degree of persistence in these autocorrelations varies across periods. Notably, persistence in dependent variables experienced a significant decrease in the last period by extreme reduce of coefficients estimates in B_1. This decline can be attributed to the concurrent reduction in liquidity supply, including volume and market participants, during the final period of the crisis. This finding aligns with the conclusions drawn by Brunetti (2009), who investigated a crowding-out effect on the interbank market resulting from increased ECB activities. Similar qualitative results were observed by Jeleskovic & Demertzidis (2020) as well as Demertzidis & Jeleskovic (2021).

Thirdly, concerning the impact of past events, past volatility is seen as an opportunity, motivating traders to optimize and increase their trade sizes and the number of trades in the following time interval when volatility has previously risen. This relationship remains stable, as expected from a theoretical standpoint. Trade sizes are significantly positively influenced by past trading intensities during all periods, and vice versa, as one would anticipate from a theoretical standpoint as well. This finding indicates that a higher trading speed tends to increase trade sizes over time and vice versa. However, in the last period, market participants observing a low liquidity supply increase trade sizes in dependency of previous number of trades even though they trade much less. A possible explanation for this finding is that market participants demand larger loan sizes to have an amortization reserve to eventually pay back outstanding loans, even if the interbank market is short on liquidity supply.

However, we observe a change in the character of this causal relationship due to the effects of the crises and decreasing liquidity regarding volatility influenced by liquidity variables from the previous time interval. Specifically, in the last period, the sign of the coefficients governing the influence of trade numbers on volatility changes and becomes negative. During this exceptionally illiquid period, a lower number of trades per time interval is associated with higher volatility, while a higher number of trades is associated with lower volatility. The rationale behind this observation is that a higher number of trades introduces more information into the market, thus dampening volatility. Conversely, with a low number of trades, each individual trade can have a more significant impact on the market, relatively increasing volatility. Additionally, the impact of trading volume changes its nature beginning in the second period, becoming negative and maintaining this negativity for the subsequent periods. This indicates that, following the first ECB intervention on 09.08.2007, changes in the behaviour of participating banks on e-MID are recognizable in trading volume, which negatively affects volatility in the next time interval. We interpret these findings as follows: in a well-functioning market during the first period, higher previous volumes suggest a greater willingness of banks to provide liquidity (whether buying or selling credits), thereby increasing trading activity and volatility. However, after the first ECB intervention, higher volume becomes a signal of reduced uncertainty in the next time interval, consequently leading to lower volatility. Therefore, the volume, as a microstructure variable in electronic trading on e-MID, proves to be more sensitive and informative regarding market uncertainties than the number of trades and volatility per time interval.

To gain a deeper understanding of these relationships, we must consider the dynamics in the most recent time interval. Let's take a closer look at the estimates in matrix A_0, which models the mutual correlation between these three variables in terms of weak exogeneity. This time interval corresponds to the execution of trades based on traders' strategies, which are optimized based on available information from the previous interval. It's not surprising that the sign of the coefficients for trading size and the number of trades influencing volatility in the current time interval is opposite to that of the previous time interval, as this reflects the basis in trading strategy. However, the underlying reason for this observation lies in game theory. Recall that e-MID, as an interbank credit market, is characterized by very low uncertainty (risk) and, consequently, very low risk premia. The primary objective is to obtain liquidity, with the interest rate being the instrument to achieve this. When a trading partner seeks or provides liquidity, they signal the volume with a specific interest rate. The counterparty takes note of the offer and considers the interest rate as a fixed instrument. Ultimately, the deal is completed, and the interest rate is silently accepted as constant. Hence, this Pareto optimum is stable as long as risks are negligible.

Conclusion

In summary, our analysis reveals robust dynamic interdependencies and causal relationships among high-frequency volatility, liquidity supply, and liquidity demand on e-MID. These relationships remain robust in terms of estimates but exhibit instability in terms of the sign of estimated coefficients, indicating qualitatively different mutual influences under different market states. Our study highlights the enduring influence of liquidity variables on market volatility in the interbank market.

We also find evidence that traders behave in a manner resembling a Pareto optimum before the financial crisis, where stable interest rates are used as an instrument to achieve their goals. However, during financial crises and heightened risks, this Pareto optimum breaks down. These changes may signal deficiencies in the e-MID and potential market failures or inefficiencies during and after financial crises, aligning with observations made by other researchers (e.g., Porzio et al., 2009; Jeleskovic & Demertzidis, 2020; Demertzidis & Jeleskovic, 2021).

Hence, we demonstrate that econometric models designed for high-frequency financial data, such as the MMEM, can be effectively applied to analyse the interbank market. These findings can offer valuable insights for the development of trading strategies and the implementation of (automated) trading algorithms. Moreover, our results have implications for policy and financial decision-making, as they provide guidance and estimations of potential effects on interbank markets. They can also aid in timing macroeconomic-driven decisions more effectively. In such cases, it may be advisable to compute simplified variables that align with conventional financial market metrics, such as deriving interest rates without the need for complete order book information.

Credit Authorship Contribution Statement

Markus Engler and Vahidin Jeleskovic jointly contributed to the conception and design of the study. Markus Engler conducted the data collection and analysis, while Vahidin Jeleskovic provided critical input and oversight throughout the research process. Both authors contributed equally to the interpretation of the results and drafting of the manuscript. Markus Engler and Vahidin Jeleskovic have reviewed and approved the final version of the paper for submission.

Acknowledgments

Authors disclose any financial or other substantive conflict of interest that could be seen to influence your results or interpretations.

Conflict of Interest Statement

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

[1] Angelini P., Nobili A. and Picillo M. C., (2009). The Interbank Market After August 2007: What has Changed, and Why? *Journal of Money, Credit and Banking*, 43(5), 923–958. http://www.jstor.org/stable/20870085

[2] Baglioni, A., Monticini, A., (2008a). The Intraday Interest Rate under a Liquidity Crisis: The Case of August 2007. Quaderni dell'Istituto di Economia e Finanza IEF0083, Dipartimenti e Istituti di Scienze Economiche (DISCE), Universit`a Cattolica del Sacro Cuore, Milano. Available at SSRN: https://ssrn.com/abstract=1330895 or http://dx.doi.org/10.2139/ssrn.1330895

[3] Baglioni, A., & Monticini, A., (2008b). The intraday price of money: Evidence from the e-MID interbank market. *Journal of Money, Credit and Bankin*g, 40(7), 1533–1540. https://doi.org/10.1111/j.1538-4616.2008.00171.x

[4] Bauwens, L., & Giot, P. (2001). *Econometric Modelling of Stock Market Intraday Activity*. Springer NY. 978-0-7923-7424-4 https://doi.org/10.1007/978-1-4757-3381-5

[5] Bauwens, L., & Giot, P. (2000). The Logarithmic ACD Model: An Application to the Bid/Ask Quote Process of two NYSE Stocks. *Annales d'Economie et de Statistique*, 60, 117–149. https://doi.org/10.2307/20076257

[6] Brunetti, C., di Filippo, M., & Harris, J. H. (2011). Effects of Central Bank Intervention on the Interbank Market During the Subprime Crisis. *Review of Financial Studies*, 24, 6, 2053-2083. https://doi.org/10.1093/rfs/hhq123

[7] Cipollini, F., Engle, R. F. & Gallo, G. M. (2007). Vector Multiplicative Error Models: Representation and Inference. *Technical Report, NBER.* https://www.nber.org/system/files/working_papers/w12690/w12690.pdf

[8] Clark, P. (1973). A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. Econometrica, 41(1), 135–155. https://doi.org/10.2307/1913889

[9] Demertzidis, A. (2022). Interbank Transactions on the Intraday Frequency: Different Market States and the Effects of the Financial Crisis. *International Journal of Economics, Business and Management Research*, 6, 09, 140-175. http://dx.doi.org/10.51505/ijebmr.2022.6911

[10] Demertzidis, A. & Jeleskovic, V. (2021). Empirical Estimation of Intraday Yield Curves on the Italian Interbank Credit Market e-MID. *Journal of Risk and Financial Management*, 14(5), 212. https://doi.org/10.3390/jrfm14050212

[11] Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*: 50(4), 987-1008. https://doi.org/10.2307/1912773

[12] Engle, R. F. (2000). The Econometrics of Ultra-High-Frequency Data. *Econometrica*, 68(1), 1–22. https://doi.org/10.1111/1468-0262.00091

[13] Engle, R. F. (2002). New frontiers for ARCH models. *Journal of Applied Econometrics*, 17(5), 425-446. https://doi.org/10.1002/jae.683

[14] Engle, R. F., Fleming, M. J., Ghysels, E. & Nguyen, G. (2012). Liquidity, volatility, and flights to safety in the US Treasury market: Evidence from a new class of dynamic order book models. Staff Report, No. 590, Federal Reserve Bank of New York, New York, NY. https://www.econstor.eu/bitstream/10419/93594/1/733627927.pdf

[15] Engle, R. F. & Russell, J. R. (1997). Forecasting the Frequency of Changes in Quoted Foreign Exchange Prices with the Autoregressive Conditional Duration Model. *Journal of Empirical Finance*, 4(2-3), 187-212. https://doi.org/10.1016/S0927-5398(97)00006-6

[16] Engle, R. F. & Russell, J. R. (1998). Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data. *Econometrica*: 66, 1127-1162. http://dx.doi.org/10.2307/2999632

[17] Engle, R. F., & Gallo, G. M. (2006). A Multiple Indicators Model for Volatility Using Intra-Daily Data. *Journal of Econometrics*: 131, 3–27. https://web-static.stern.nyu.edu/rengle/EngleGallo.pdf

[18] Fricke, D. (2012). Trading Strategies in the Overnight Money Market: Correlations and Clustering on the e-MID Trading Platform. *Physica A*. https://www.files.ethz.ch/isn/141704/KWP1766.pdf

[19] Gabbi, G., Iori, G., Germano, G., Hatzopoulos, V. & Politi, M. (2012). Market microstructure, bank's behaviour and interbank spreads. (Report No. 12/06); London; UK: Department of Economics; City University London. http://dx.doi.org/10.2139/ssrn.2345185

[20] Gabrieli, S. (2011). The functioning of the European interbank market during the 2007–08 financial crisis. *CEIS Working Paper Series*, Nr. 158. http://dx.doi.org/10.2139/ssrn.1525463

[21] Gaspar, V., Perez-Quiros, G., & Mendizabal, H.R. (2007). Interest rate determination in the interbank market. *Working Paper Series* No. 351/April 2004. http://dx.doi.org/10.2139/ssrn.533013

[22] Harris, L. E. (1994). Minimum Price Variations, Discrete Bid-Ask Spreads, and Quotation Sizes. The Review of Financial Studies: 7(1), 149-178. http://www.jstor.org/stable/2962289

[23] Hautsch, N. (2004). Modelling Irregularly Spaced Financial Data - Theory and Practice of Dynamic Duration Models, Vol. 539 of Lecture Notes in Economics and Mathematical Systems. Springer, Berlin. https://doi.org/10.1007/978-3-642-17015-7

[24] Hautsch, N. (2007). Capturing common components in high frequency financial time series: a multivariate stochastic multiplicative error model, SFB 649 Discussion Paper, No. 2007, 052, Humboldt University of Berlin, Collaborative Research Center 649 - Economic Risk, Berlin. https://www.econstor.eu/bitstream/10419/25224/1/558613772.PDF

[25] Hautsch, N., & Jeleskovic, V. (2008). Modelling High-Frequency Volatility and Liquidity Using Multiplicative Error Models. SFB 649DP2008047, Humboldt Univärsität zu Berlin, Germany. http://dx.doi.org/10.2139/ssrn.1292493

[26] Iori, G., Masi, G. D., Precup, O. V., Gabbi, G., & Caldarelli, G. (2008). A network analysis of the Italian overnight money market. *Journal of Economic Dynamics & Control,* 32(1), 259–278. https://doi.org/10.1016/j.jedc.2007.01.032

[27] Iori, G., & Precup, O. V. (2007). Weighted network analysis of high-frequency cross-correlation measures. *Physical Review* E: 75(3), 036110, 1-7. https://doi.org/10.1103/PhysRevE.75.036110

[28] Iori, G., Ren`o, R., Masi, G.D., & Caldarelli, G. (2007). Trading strategies in the Italian interbank market. *Physica A*: 376(C), 467–479. https://doi.org/10.1016/j.physa.2006.10.053

[29] Jeleskovic, V., & Demertzidis, A. (2020). Comparing different methods for the estimation of interbank intraday yield curves. *Journal of Corporate Accounting & Finance,* 31(1), 1-19. https://onlinelibrary.wiley.com/doi/pdf/10.1002/jcaf.22438

[30] Kapar, B., Iori, G., & Olmo, J. (2012). The cross-section of interbank rates: A nonparametric empirical investigation. Department of Economics, City University London in its series Working Papers with number 12/03. http://openaccess.city.ac.uk/1613/

[31] Karpoff, J. M. (1987). The Relation between Price Changes and Trading Volume: A Survey. *Journal of Financial and Quantitative Analysis,* 22(1), 109-126. https://doi.org/10.2307/2330874

[32] Manganelli, S. (2005). Duration, Volume and Volatility Impact of Trades. *Journal of Financial Markets*, 8(4), 377-399. https://doi.org/10.1016/j.finmar.2005.06.002

[33] Masi, G.D., Iori, G., & Caldarelli, G., (2006). A fitness model for the Italian interbank money market, *Physical Review E*, 74, 066112, 1-5. https://doi.org/10.1103/PhysRevE.74.066112

[34] Mistrulli, P. E. (2011). Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns. *Journal of Banking & Finance*, 35(5), 1114-1127. https://doi.org/10.1016/j.jbankfin.2010.09.018

[35] Politi, M., Iori, G., Germano, G., & Gabbi, G., (2010). The overnight interbank market. Simple statistics and facts before and during the credit crisis. https://editorialexpress.com/cgi-bin/conference/download.cgi?db_name =CEF2010&paper_id=509

[36] Porzio, C., Battaglia, F., Meles, A. & Starita, M. G. (2010). Financial turmoil and asymmetric information theory: Evidence from the e-mid platform, In: Fiordelisi, F., Molyneux, P., Previati, D. (eds) *New Issues in Financial and Credit Markets.* Palgrave Macmillan Studies in Banking and Financial Institutions. Palgrave Macmillan, London. https://doi.org/10.1057/9780230302181_3

[37] Raddant, M. (2012). Structure in the Italian Overnight Loan Market. *Kiel Working Papers 1772*; Kiel Institute for the World Economy. https://www.econstor.eu/bitstream/10419/58277/1/716977656.pdf

[38] Taylor, S. J. (1982). Financial Returns Modelled by the Product of Two Stochastic Processes - a Study of Daily Sugar Prices, in O. D. Anderson (eds), Time Series Analysis: Theory and Practice, North-Holland, Amsterdam. https://www.research.lancs.ac.uk/portal/en/publications/financial-returns-modelled-by-the-product-of-two-stochastic-processes-a-study-of-daily-sugar-prices(dd713097-5087-4f05-ad7f-3ae82d019a92)/export.html

[39] Vento, G. (2010). "Interbank Market and Liquidity Distribution during the Great Financial Crisis: the e-MID Case", in New Issues in Financial Institutions and Markets (edited by F. Fiordelisi, P. Molyneux and D. Previati), Palgrave Macmillan, July, pp. 218

[40] Veredas, D., Rodriguez-Poo, J. & Espasa, A. (2002). On the (Intradaily) Seasonality, Dynamics and Durations Zero of a Financial Point Process. A semiparametric approach. *CORE Discussion Papers*; 2002/23 (2002). http://hdl.handle.net/2078.1/4267

Appendix

Table 1: Example of the used data set of the e-MID (Italian interbank market). Market shows the currency of the transaction. Duration shows the maturity of contract, which was traded (there are different maturities from overnight to one year). Date and Time are the date and the time when the transaction takes place. Rate is the interest rate of the contract in percent. The StartDate and the EndDate show the starting date of the contract. Quoter and Aggressor show the identity of the quoter and the aggressor bank. The quoter has placed the order in the order book and the aggressor has actively chosen the living order. Verb shows the kind of transaction from the aggressor side. So, the label Sell means that the aggressor lends money to the quoter and the label Buy means that the aggressor borrows money from the quoter.

Market	Duration	Date	Time	Rate	Amount	StartDate	EndDate	Quoter	Aggressor	Verb
TRAS_EUR	ONL	03.10.2005	09:39:04	2.08	400	03.10.2005	04.10.2005	DE0021	FR0005	Buy
TRAS_EUR	ON	03.10.2005	09:39:18	2.08	10	03.10.2005	04.10.2005	IT0162	IT0211	Sell
TRAS_EUR	ON	03.10.2005	09:45:42	2.085	50	03.10.2005	04.10.2005	IT0177	GB0005	Buy
TRAS_EUR	ON	03.10.2005	09:47:21	2.085	45	03.10.2005	04.10.2005	IT0257	IT0183	Sell
TRAS_EUR	ONL	03.10.2005	09:56:28	2.085	130	03.10.2005	04.10.2005	IT0257	IT0171	Sell

Table 2: Descriptive statistics of squared log returns (multiplied by 100), average volumes per trade as well as the number of transactions based on five minutes intervals for each period. The following descriptive statistics are shown: Number of observations, mean, standard deviation, minimum, maximum, 5%-, 10%-, 50%-, 90%-, as well as 95%-quantile, kurtosis, univariate and multivariate Ljung-Box statistic (computed for squared log returns, volumes and number of trades) associated with 20 lags.

	P1			P2			P3			P4		
	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades
Obs	45881	45881	45881	27257	27257	27257	16102	16102	16102	22213	22213	22213
Mean	0.0086	19.94	3.11	0.0092	17.36	2.98	0.5716	19.69	2.20	0.8513	13.68	1.71
S.D.	0.2041	19.82	2.99	0.0769	18.23	2.85	3.3016	45.16	2.46	3.5775	37.44	1.99
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Max	16.44	100.00	27.00	5.27	100.00	27.00	138.92	1000.00	28.00	134.67	2000.00	21.00
q05	0.0000	0.00	0.00	0.0000	0.00	0.00	0.0000	0.00	0.00	0.0000	0.00	0.00
q10	0.0000	0.00	0.00	0.0000	0.00	0.00	0.0000	0.00	0.00	0.0000	0.00	0.00
q50	0.0001	15.00	2.00	0.0004	12.78	2.00	0.0194	9.67	2.00	0.0377	7.50	1.00
q90	0.0023	45.00	7.00	0.0111	39.17	7.00	0.9931	41.71	5.00	1.8166	30.00	4.00
q95	0.0058	56.50	9.00	0.0279	52.00	8.00	2.2962	71.87	7.00	3.7374	45.07	6.00
Kurtosis	3323.14	3.85	3.98	1560.33	5.82	4.61	665.46	110.25	6.43	421.06	955.55	4.93
LB(20)	5287.78	491	2344.84	8802.05	284.01	1145.71	4422.61	277.25	1145.71	4274.57	1121.75	2290.71
MLB(20)	17876.31			21076.7			12209.94			13377.09		

Figure 2: Intraday seasonality pattern for squared log returns (left), mean trading volume (middle) and number of trades (right) for the first period.

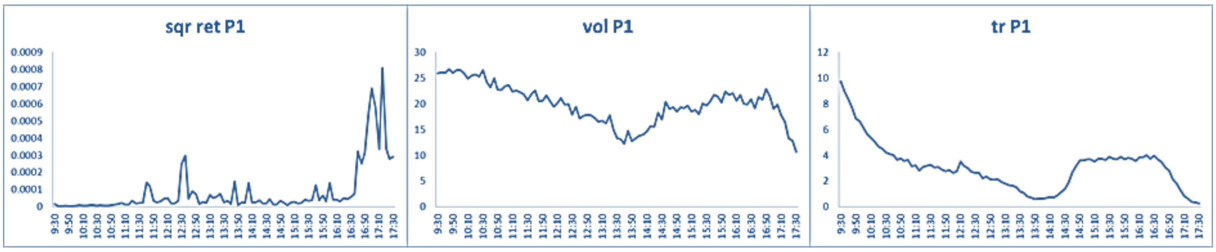


Figure 3: Intraday seasonality pattern for squared log returns (left), mean trading volume (middle) and number of trades (right) for the second period.

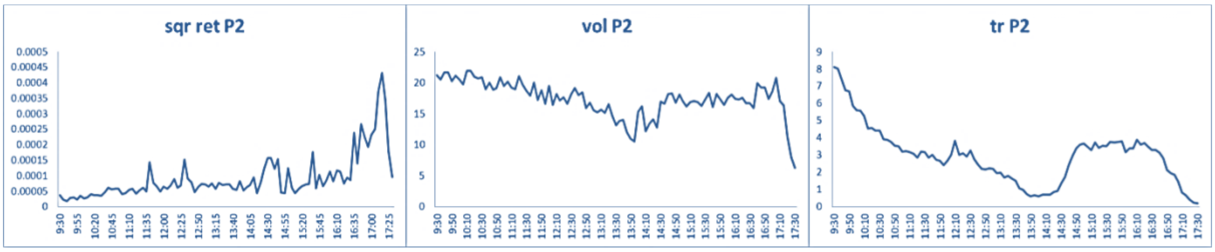


Figure 4: Intraday seasonality pattern for squared log returns (left), mean trading volume (middle) and number of trades (right) for the third period.

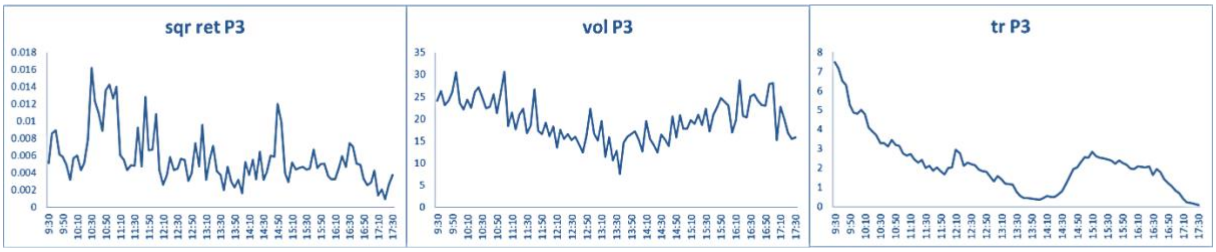


Figure 5. Intraday seasonality pattern for squared log returns (left), mean trading volume (middle) and number of trades (right) for the fourth period.

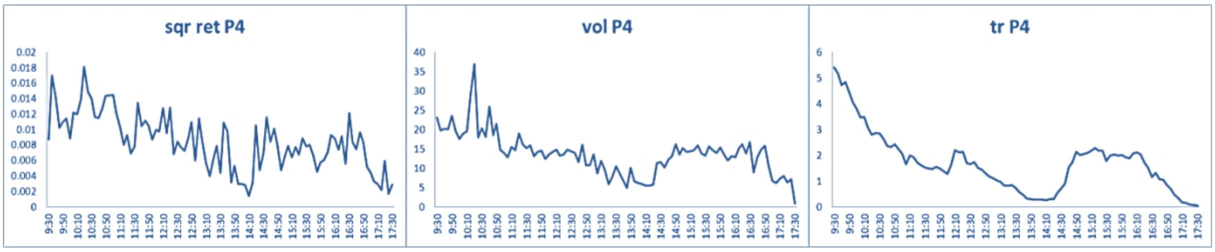


Table 3: Quasi-maximum likelihood estimation results of the MMEM for seasonally adjusted squared log returns, average trade sizes and number of trades per five-minute interval. Standard errors are computed based on the OPG covariance matrix. (Log likelihood function (LL), Bayes Information Criterion (BIC)).

	P1		P2		P3		P4	
	Coeff.	Std. err.	Coeff.	Std. err.	Coeff.	Std. err.	Coeff.	Std. err.
OM1	-0.7742	0.0105	-0.8279	0.0130	-0.4203	0.0127	-0.4553	0.0159
OM2	-0.1525	0.0186	-0.0690	0.0179	-0.0870	0.0115	-0.0496	0.0106
OM3	-0.2249	0.0132	-0.2697	0.0108	-0.2738	0.0169	-0.2779	0.0195
A0_12	-0.0398	0.0037	0.0449	0.0043	0.0114	0.0038	0.0434	0.0065
A0_13	-0.1527	0.0051	-0.1956	0.0068	-0.0859	0.0059	0.0597	0.0085
A0_23	0.0372	0.0105	0.0875	0.0093	0.2050	0.0070	0.2504	0.0054
A1_11	0.5489	0.0048	0.5315	0.0064	0.3716	0.0077	0.6064	0.0113
A1_12	0.0123	0.0043	-0.0187	0.0049	-0.0081	0.0029	-0.0130	0.0020
A1_13	0.2648	0.0086	0.3967	0.0094	0.1351	0.0089	-0.0051	0.0102
A1_21	0.0290	0.0100	0.0203	0.0088	0.0427	0.0057	0.0370	0.0058
A1_22	0.0871	0.0083	0.0658	0.0068	0.0570	0.0027	0.0322	0.0028
A1_23	0.0498	0.0120	0.0057	0.0112	0.0269	0.0082	0.0284	0.0072
A1_31	0.1051	0.0075	0.1154	0.0069	0.0929	0.0129	0.0689	0.0157
A1_32	0.0252	0.0054	0.0526	0.0042	0.0224	0.0035	0.0071	0.0055
A1_33	0.1252	0.0079	0.1318	0.0060	0.1791	0.0125	0.2071	0.0129
B1_11	0.9682	0.0008	0.9684	0.0009	0.9856	0.0008	0.8552	0.0046
B1_22	0.6030	0.0470	0.5540	0.0479	0.5833	0.0148	0.4722	0.0178
B1_33	0.7930	0.0161	0.7996	0.0114	0.7766	0.0222	0.6026	0.0331
LL	-48603.24		-57789.30		-32406.08		-41717.36	
BIC	-48692.37		-57878.43		-32489.91		-41802.81	

Table 4: Summary statistics of the standardized seasonality adjusted time series and the corresponding MEM residuals for the four periods. Ljung-Box statistics of the residuals (LB), squared filtered residuals (LB2) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

Descriptive statistics of raw data												
	P1			P2			P3			P4		
	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades
Mean	1.13	1.00	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S.D.	19.24	0.89	0.77	6.08	0.95	0.74	4.75	2.17	0.93	4.05	2.32	1.02
LB(20)	5287.78	491.00	2344.84	8802.05	284.01	1767.76	4422.61	277.25	1145.71	4274.57	121.75	2290.71
LB2(20)	4122.97	486.70	1944.57	963.93	125.19	831.00	624.25	42.11	41.20	2373.77	21.64	1580.96
MLB(20)	17876.31			21076.71			12209.94			13377.09		
Summary Statistics MEM Residuals												
	P1			P2			P3			P4		
	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades	Sq. ret.	Avg. vol.	Trades
Mean	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S.D.	7.34	0.85	0.67	3.84	0.92	0.68	2.38	1.78	0.75	2.46	2.11	0.81
LB(20)	6.15	19.53	59.10	24.21	25.42	45.00	241.95	17.94	23.67	123.43	29.39	24.78
LB2(20)	0.01	21.99	35.57	0.01	15.38	23.35	7.73	5.61	10.39	4.16	0.59	19.17
MLB(20)	196.87			602.18			383.97			84.00		