How Many Innovations Need to Be Produced in the Process of Endogenous Growth with Fluid Intelligence

Taiji HARASHIMA
Department of Economics, Kanazawa Seiryo University, Japan
harashim@seiryo-u.ac.jp

Article's history:

Received 7th of May, 2022; *Received in revised form* 31st of May, 2022; *Accepted* 17th of June, 2022;
Published 30th of June, 2022.

Suggested citation:

Harashima, T. 2022. How Many Innovations Need to Be Produced in the Process of Endogenous Growth with Fluid Intelligence, *Journal of Applied Economic Sciences*, Volume XVII, Summer, 2(76): 107 – 121.

Abstract:

In innovation-based endogenous (Schumpeterian) growth theory, the production of innovations is constrained basically by the finite nature of the labor supply. In this paper, I show that innovations are constrained because (1) the amount of fluid intelligence of researchers in an economy is limited and (2) the returns on investments in technologies and in capital are kept equal through arbitrage in markets. With these constraints, equilibrium values of the number of researchers and their average productivity in an economy exist, and the equilibrium value of average productivity determines the amount of innovation production in each period. Distributions of fluid intelligence among researchers are most likely heterogeneous across economies, but if economies are open to each other, an economy with a smaller number of researchers with a high level of fluid intelligence can grow at the same rate as an economy with more of them.

Keywords: endogenous growth; fluid intelligence; innovation; production of innovation; researchers.

JEL Classification: O31; O40.

Introduction

Innovations (new technologies) have to be produced continuously for an economy to endogenously grow steadily. The greater the number of innovations, the higher the growth rate. Because people will prefer higher economic growth rates, the rate could become very high if an economy produces as many innovations as possible. However, the average growth rates in industrialized economies in the long run have historically been only about 2% or less, which implies that there is the upper bound of the number of innovations produced in each period.

In the literature of innovation-based endogenous (or Schumpeterian) growth theory (*e.g.,* Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992, 1998, Aghion *et al.* 2014), economic growth is mostly determined by firms' behaviors with regard to research and development activities, and the constraint on innovation production basically originates from the finite nature of the labor supply. In response to criticism about scale effects in innovation-based endogenous models (Jones 1995a), these models have been modified in various ways (*e.g.,* Jones 1995b, Kortum 1997, Segerstrom 1998, Young 1998, Peretto 1998; Dinopoulos and Thompson 1998, Eicher and Turnovsky 1999, Peretto and Smulders 2002), but the basic nature the innovation constraint has remained unchanged.[1]

Innovations are the fruits of intellectual activities, and it therefore seems highly likely that the constraint originates in the innovative talents or intelligence of workers, not simply in the fact there is a finite number of workers. In addition, investments in technologies compete with those in capital, and thereby the return on investments in technologies will be eventually kept equal to that in capital through market arbitrage, which means that innovation production is substantially constrained by activities not only in the labor market but also in various other markets, particularly financial ones. Because of this constraint, even if there is a large amount of innovative talent, the number of innovations produced may be less than that the talent could produce under some ideal condition.

The purpose of this paper is to examine whether it is the number of workers or the amount of intelligence that workers possess that is the origin of the constraint on the number of innovations produced, and how this constraint affects innovation production in an environment where investments in technologies and capital compete. I also examine what happens if the distributions of intelligence among researchers are heterogeneous across economies.

I examine these questions by combining three different types of models: an asymptotically non-scale

[1] Although these modified models have been criticized for adopting *ad hoc* assumptions to solve the problem of scale effects (Jones 1999).

endogenous growth model, a "maximum degree of comfortability" (MDC) model, and a total factor productivity (TFP) model based on fluid intelligence. The asymptotically non-scale endogenous growth model was presented by Harashima (2010, 2013, 2017, 2019b), in which technologies are generated in the same manner as consumption goods and capital (*i.e.,* by inputting technologies, capital, and labor). In addition, investments in technologies compete with those in capital.

In this model, the scale effects found in endogenous growth models (Jones 1995a) asymptotically disappear as the population increases because of increases in uncompensated knowledge spillovers and the arbitrage between investments in capital and technologies. An extension of this model is the model of sustainable heterogeneity (SH) in an economy with a heterogeneous population (Harashima 2010, 2017). The MDC model was presented by Harashima (2018a, 2019a). In this model, households reach the same steady state as they do in a Ramsey-type growth model, without generating rational expectations using the rate of time preference. The TFP model based on fluid intelligence was presented by Harashima (2009, 2012, 2016, 2020a), in which the fluid intelligence of ordinary workers is an important element in TFP. Fluid intelligence is the ability to solve novel problems by thinking logically without only depending on knowledge previously acquired and is an essential element in the ability to generate innovations (*e.g.,* Cattell 1963, 1971, Lord and Novick 1968; van der Linden and Hambleton 1997).

As Harashima (2020b) showed, the decreasing rate of the marginal utility of households has to be kept constant on balanced growth paths in both the RTP and MDC models, and this constancy constrains the endogenous growth rate. The reason for this constancy is most likely that the number of innovations produced in each period is constrained by factors related to the process of innovation production, not by factors related to the utilities or preferences of households. In this paper, I show that innovation production is constrained because (1) the amount of fluid intelligence of researchers is limited in an economy and (2) the returns on investments in technologies and in capital are kept equal through arbitrage in markets. The number of researchers and their average productivity in an economy is determined through interrelations between the abovementioned constraint factors (1) and (2). That is, a unique equilibrium number of researchers and level of average productivity exist in an economy, and innovations are produced in each period at the level that is consistent with that equilibrium.

Distributions of researchers' fluid intelligence are most likely heterogeneous across economies, and therefore the number of researchers and their average productivity will be also heterogeneous. Hence, the decreasing rates of marginal utility and growth rates of technologies are also heterogeneous if the economies are closed to one another. However, if these economies are open to each other, the decreasing rate of marginal utility and the growth rate of technologies are equalized between them because produced innovations can be used equally in any economy, and therefore an economy with a smaller number of researchers with a high level of fluid intelligence can grow at the same rate as an economy with more researchers.

1. Endogenous Growth

1.1. The Model of Reaching Steady State Using the Rate of Time Preference (RTP)

I first examine the production of innovations in the conventionally assumed model of reaching steady state by generating rational expectations using the rate of time preference (the RTP model), in which a household reaches the steady state or balanced growth path by generating rational expectations with its expected utilities discounted by RTP (hereafter, the "RTP-based procedure"). I use the RTP model presented by Harashima (2013, 2019b), which is an endogenous growth model version of a Ramsey-type growth model. It avoids both scale effects and the influence of population growth, which are serious problems in many other endogenous growth models (Jones, 1995a).

Note that in this paper, "innovation" specifically means the economically valuable knowledge or technology that is newly produced to be used for production and is "accumulated." Harashima (2009, 2012, 2016, 2020a) showed that, in addition to accumulated innovations, there are many minor temporary "non-accumulated" innovations. These non-accumulated innovations are mostly produced by ordinary workers (*i.e.,* not by researchers), but they make up an indispensable element in TFP. Nevertheless, this paper focuses only on innovations that are produced by researchers and accumulated; non-accumulated innovations produced by ordinary workers are only implicitly assumed.

1.1.1. The Model

Outputs (Y_t) are the sum of consumption (C_t), the increase in capital (K_t), and the increase in technology (A_t) in period *t* such that:

$$Y_t = C_t + \dot{K}_t + v\dot{A}_t,$$

where: $v(>0)$ is a constant, and a unit of K_t and v^{-1} of a unit of A_t are equivalent; that is, they are produced using the same quantities of inputs (capital, labor, and technology).

Thus,

$$\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t,$$

where: n_t is the population growth rate. It is assumed for simplicity that $n_t = 0$, and thereby L_t is constant such that $L_t = L$ for any *t*. The production function is:

$$y_t = A_t^{\alpha} k_t^{1-\alpha} , \tag{1}$$

where: α is a constant and indicates the labor share. For any period,

$$m = \frac{M_t}{L_t},$$

where: M_t is the number of firms (all of which are assumed to be identical) and m (> 0) is a constant. In addition, through arbitrage between investments in k_t and A_t in markets,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t} \tag{2}$$

is always kept, *where*: $\varpi(> 1)$ is a constant and indicates the effect of patent protection. As a result,

$$A_t = \frac{\varpi\alpha}{mv(1-\alpha)}k_t$$

always holds, and therefore,

$$\dot{A}_t = \frac{\varpi\alpha}{mv(1-\alpha)}\dot{k}_t \qquad y_t = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t,$$

and

$$\dot{k}_t = \frac{mL(1-\alpha)}{mL(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t - c_t\right].$$

Suppose that L is sufficiently large and thereby:

$$\frac{mL(1-\alpha)}{mL(1-\alpha)+\varpi\alpha} = 1. \tag{3}$$

On the other hand, the utility function $u(c_t)$ is the familiar power utility function:

$$u(c_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon} \quad \text{if } \varepsilon \neq 1 \qquad u(c_t) = \ln c_t \quad \text{if } \varepsilon = 1, \tag{4}$$

where: ε is a positive parameter representing the degree of risk aversion (DRA) and $\varepsilon = -\frac{c_t\frac{d^2u}{dc_t^2}}{\frac{du}{dc_t}}$.

Harashima (2013, 2019b) showed that the optimal growth rate of consumption is:

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right], \tag{5}$$

and that this path is the balanced growth path in the RTP model.

1.1.2. The Decreasing Rate of the Marginal Utility

By equation (4),

$$\frac{\dot{c}_t}{c_t} = -\varepsilon^{-1}\frac{\dot{\upsilon}_t}{\upsilon_t} ,$$

where: $\upsilon_t = \frac{du(c_t)}{dc_t}$ is the marginal utility. On a balanced growth path, $\frac{\dot{c}_t}{c_t}$ is constant; thereby, the decreasing rate of marginal utility $-\frac{\dot{\upsilon}_t}{\upsilon_t}$ is also constant because ε is constant. Harashima (2020b) showed that, on a balanced growth path, the marginal utility decreases at a constant rate:

$$-\frac{\dot{\upsilon}_t}{\upsilon_t} = \left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\ ;$$

that is, the decreasing rate of the marginal utility ($-\frac{\dot{\upsilon}_t}{\upsilon_t}$) is equal to the marginal productivity of capital ($\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha}$) minus RTP (θ).

Here, let G_t be the growth path of an economy in period t, and $\Psi(G_t)$ be the average growth rate of the economy on G_t. In addition, let $\tilde{G}_{RTP}$ be the balanced growth path in the RTP model, and $\Psi(\tilde{G}_{RTP})$ be $\Psi(G_t)$ for $G_t = \tilde{G}_{RTP}$.

1.2. The Model of Reaching Steady State on the Basis of the Maximum Degree of Comfortability (MDC)

I next examine the production of innovations in the model of reaching steady state on the basis of the "maximum degree of comfortability" (the MDC model), in which a household behaves according to the "MDC-based procedure." Before examining the production of innovations, I briefly explain the MDC-based procedure following Harashima (2018a, 2019a).

1.2.1 "Comfortability" of the Capital-wage Ratio (CWR)

Let k_t and w_t be per capita capital and wages (labor income), respectively, in period *t*. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\breve{w}_t}{\breve{k}_t}$, where $\breve{k}_t$ and $\breve{w}_t$ are the k_t and w_t of the household, respectively. Let *Γ* be the household's subjective valuation of $\frac{\breve{w}_t}{\breve{k}_t}$ and Γ_i be the value of $\frac{\breve{w}_t}{\breve{k}_t}$ of household i (i = 1, 2, 3, … , M). Each household should next assess whether it feels comfortable with its current *Γ*, that is, its combination of income and capital. "Comfortable" in this context means at ease, not anxious, and other similar related feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. A higher value of DOC indicates a household feels more comfortable with its Γ. For each household, there will be a most comfortable value of the capital-wage ratio (CWR) because a household will feel less comfortable if its CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be the state at which a household's DOC is at its maximum (the "maximum degree of comfortability," MDC), and let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ therefore, indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is the Γ_i of household *i* at $\tilde{s}_i$.

On the other hand, even under the MDC-based procedure, utilities from consumption are felt by households in a similar manner as under the RTP-based procedure. Under the MDC-based procedure, the utility of a household (μ) is a function of c_l, which is the level of current or future consumption estimated by a household. Suppose a usual power utility function such that:

$$\mu = \frac{c_l^{1-\delta}}{1-\delta} \quad \text{if } \delta \neq 1$$

$$\mu = \ln c_l \quad \text{if } \delta = 1,$$

where: δ (≥ 0) is a parameter. Therefore,

$$\delta = -\frac{c_l \frac{d^2\mu}{dc_l^2}}{\frac{d\mu}{dc_l}} (> 0) .$$

In addition,

$$\frac{\dot{c}_l}{c_l} = -\delta^{-1}\frac{\dot{\upsilon}_{\mu,t}}{\upsilon_{\mu,t}}, \qquad (6)$$

where: $\upsilon_{\mu,t} = \frac{d\mu(c_l)}{dc_l}$ is the marginal utility.

1.2.2. Steady State

Suppose that all households are identical (*i.e.,* the population is homogeneous).

Rules

Household *i* should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any *i*.

Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any *i*.

Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t, and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (*i.e.,* the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state in the RTP model. That is, it is the one derived in a Ramsey-type growth model in which households behave by discounting utilities by θ and generating rational expectations, where θ (> 0) is household RTP. Finally, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ in the RTP model in which θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: See Harashima (2018a, 2019a).

Proposition 1 indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$, which means that both procedures can function equivalently and that CWR at MDC is substitutable for RTP as a guide for household behavior.

In addition, Harashima (2018a, 2019a) showed that the essential result is the same in a heterogeneous population.

1.2.3. Response to Technological Progress

Harashima (2018a, 2019a) showed how a household responds to technological progress under the MDC-based procedure through two channels:

a) If a new version of a product with better performance is introduced at almost the same price as the old version, a household will buy the new version instead of the old version while its MDC remains unchanged.

b) If a household's income unexpectedly and permanently increases, the household begins to feel that its current Γ is unexpectedly higher than $\Gamma(\tilde{s})$. However, because of the increase in income, its capital also gradually increases unexpectedly, and the household will maintain this accumulation of capital until its Γ returns to its $\Gamma(\tilde{s})$.

Through these household responses, the economy grows with technological progress.

Let $\tilde{G}_{MDC}$ be the growth path in which households behave according to the MDC-based procedure as well as responses (a) and (b), and $\Psi(\tilde{G}_{MDC})$ be $\Psi(G_t)$ when $G_t = \tilde{G}_{MDC}$. In addition, let c_g be the average $\frac{\dot{c}_l}{c_l}$ on $\tilde{G}_{MDC}$.

1.2.4. Production of Innovations

If technologies are only given exogenously, how sensitively or quickly households respond to new technologies through channels (a) and (b) will not affect economic growth rates. However, if technologies are endogenously generated, economic growth rates will be significantly affected because firms have to make decisions on investments in new technologies fully considering how households will respond.

If households respond less sensitively or quickly, they will purchase fewer products with new technologies in a unit period. Firms therefore will be more cautious about investments in new technologies because they may not obtain sufficient returns from the investments or, even worse, suffer losses. As a result, if households in an economy respond less sensitively or quickly on average, the speed of technological progress and thereby the growth rate of the economy will be lower.

1.3. Household Behavior

Under the MDC-based procedure, $\frac{A_t}{k_t}$ is kept constant on $\tilde{G}_{MDC}$ and thereby $\tilde{G}_{MDC}$ is a balanced growth path (*i.e.*, $\frac{\dot{c}_l}{c_l}$ is constant, Harashima 2020b), so the decreasing rate of marginal utility $-\frac{\dot{v}}{v}$ on $\tilde{G}_{MDC}$ is also constant by equation (6) if δ is constant. Let Υ be this constant decreasing rate of marginal utility (*i.e.*, $-\frac{\dot{v}}{v}$ on $\tilde{G}_{MDC}$).

Proposition 2: Assign Υ the value that satisfies

$$\Upsilon = \left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\,, \tag{7}$$

where: the values of ϖ, α, m, v, and θ are all the same as those in the RTP model. If all households are identical and behave according to the MDC-based procedure, and if the value of δ that is calculated based on the value of c_g on $\tilde{G}_{MDC}$ and the assigned value of Υ is used as the value of ε in the RTP model, $\Psi(\tilde{G}_{MDC}) = \Psi(\tilde{G}_{RTP})$.

Proof: See Harashima (2020b).

Proposition 2 indicates that we can interpret that $\tilde{G}_{MDC}$ is equivalent to $\tilde{G}_{RTP}$. That is, the RTP- and MDC-based procedures can function equivalently and are substitutable, not only for reaching steady state (Harashima 2018a, 2019a) but also for endogenous economic growth.

Although the two procedures are equivalent, *section 1.2.3* indicates that with respect to responding to technological progress, the MDC-based procedure is far easier to use than the RTP-based procedure because the RTP-based procedure requires a household to do something equivalent to computing a complex macro-econometric model with the "correct" and "true" value of RTP to generate rational expectations. Hence, it is far more likely that the MDC-based procedure is actually used (Harashima 2018a, 2019a).

2. Productivity of Researchers

2.1. Constant Decreasing Rate of Marginal Utility

Proposition 2 indicates that the economic growth in the RTP and MDC models can be interpreted to be equivalent, but it is very difficult or maybe impossible to judge which model is the "true" or "correct" model because we cannot know the "true" and "correct" values of Υ and θ. It is possible that the two models are equally the "true" or "correct"; that is, they are two sides of the same coin.

If they truly are two sides of the same coin, however, a question arises: if Υ in the MDC model and $\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta$ in the RTP model are determined independently, the probability that equation (7) holds will be quite low. For equation (7) to always hold, at least one variable in equation (7) has to be an endogenous variable. Harashima (2020b) showed that there are two possibilities: (a) Υ is determined intrinsically in a household's mind and is related to the household's preferences, and therefore, $\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta$ is endogenously determined so as to be consistent with the predetermined value of Υ; or (b) $\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta$ is bound by some factors that are not related to household behavior except for θ, and therefore, Υ is endogenously determined to be consistent with the predetermined value of $\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta$.

Historically, countries have experienced periods of both continuously very high and low growth rates. In particular, they have experienced continuously very high growth rates when new technologies could be continuously and abundantly introduced from foreign countries, for example, during the "catch-up period" in developing economies. This means that households prefer or allow growth rates that are as high as possible (*i.e.,* households have no predetermined upper limit). It seems highly likely that Υ is not constrained by household preferences, and therefore, possibility (b) seems far more likely to be true than possibility (a).

On the other hand, because innovations are produced by researchers, it seems highly likely that the production of innovations is bound by researcher productivity. Furthermore, the productivities of researchers are highly likely to depend on the fluid intelligence of researchers, which will be exogenously given. This implies that Υ is eventually bound by the "amount" of fluid intelligence that researchers possess in an economy.

2.2. Researcher Productivity

If the productivities of researchers are truly a source of possibility (b) and they actually depend on fluid intelligence, the fluid intelligence of researchers should be an important element in $\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta$. The question arises, however, how is $\left(\frac{\varpi\alpha}{m\mathrm{v}}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta$ related to fluid intelligence? α, m, and ϖ are parameters in the production function and market structure as indicated in equations (1) and (2), the values of which are basically determined by the given structure of the economy, and θ is a parameter that represents a household preference. On the other hand, the value of v is determined by the productivity of workers in the production of innovations because the same quantities of inputs (capital, labor, and technology) are used for producing a unit of K_t and v^{-1} of a unit of A_t, as shown in *section 1.1.1.* If labor inputs with regard to innovation production (*i.e.,* the work of researchers) are more productive, a larger amount of A_t can be produced with the same quantities of inputs (A_t, K_t, and L_t), that is, with the same number of researchers, which means that the value of v^{-1} indicates the productivity of workers in producing a unit of A_t.

A worker is inputted either as a labor input to produce consumption goods and services or capital (I call a worker who provides this kind of labor a "usual worker") or as a labor input to produce innovations (a "researcher"). If the productivities of workers are given exogenously and are constant, the average productivity of workers in an economy is constant, and furthermore, the average productivity of usual workers is almost constant no matter how many relatively higher productivity workers are inputted as researchers (equivalently, no matter how many relatively higher productivity workers are inputted as usual workers) because most workers in the economy are considered to be usual workers.

Researchers usually make up only a small portion of workers in an economy, and only workers who possess relatively higher productivities can be researchers. It seems highly likely that workers with higher fluid intelligences possess higher productivities (Harashima 2009, 2012, 2016, 2020a), and therefore it is assumed that only workers who possess relatively higher fluid intelligences can be researchers. Hence, unlike the case with usual workers, the average productivity of researchers in an economy will differ depending on how many workers are inputted as researchers because as the number of researchers is larger, workers with relatively lower intelligences (productivities) can or need to become researchers. The average productivity of researchers will therefore decrease as more researchers are included.

The number of workers that need be inputted as researchers in an economy is determined by how many new innovations need be produced for endogenous balanced growth. That is, the number of researchers is an endogenous variable that depends on how the economy grows endogenously on a balanced growth path. As the necessary number of researchers increases, relatively lower fluid intelligence (productivity) workers need to be inputted as researchers. As noted above, the average productivity of researchers in an economy thereby becomes lower. That is, the productivity v^{-1} is an endogenous variable that varies depending on how the economy grows endogenously.

Harashima (2009, 2012, 2016, 2020a) showed that the productivity of worker i(ω_i) is determined by the fluid intelligence of worker *i* and the production function with regard to worker *i* can be expressed as:

$$y_{i,t}=\omega_i A_t^{\alpha}k_t^{1-\alpha}\ , \tag{8}$$

where: $y_{i,t}$ is per capita production when the labor input is worker *i.*

3. Constraints on Innovation Production

3.1. Productivity of Researchers

3.1.1. *Distribution of Productivities of Researchers*

It seems highly likely that the problem-solving abilities (fluid intelligence) of people will approximately follow a normal distribution, and the fluid intelligence of researchers will correspond to the tail of distribution of all workers (researchers + usual workers) because researchers will require higher levels of fluid intelligences. Therefore, approximately, the fluid intelligence of researchers will increase exponentially from the lowest level to the highest, even if the differences among them are small because of an upper limit on human fluid intelligence. Following this line of thought, it is assumed that the fluid intelligence levels of researchers *(i.e.,* those on the tail of the normal distribution) increase exponentially from the lowest to the highest, and the number of researchers whose fluid intelligence is FI ($N_{R,FI}$) in an economy is modeled simply as:

$$N_{R,FI} = \frac{z}{\exp(FI)} \tag{9}$$

for $FI \geq FI_R$, where FI_R is the minimum level of fluid intelligence required to work as a researcher and $z(>0)$ is a parameter.

Note that all workers with $FI \geq FI_R$ need not be researchers, and some of them will actually do something else. It is likely that workers have a preference with regard to whether or not they want to be researchers. If the strength of this preference is the same for all workers, the ratio of researchers to all the other workers with the same FI is identical for any $FI \geq FI_L (\geq FI_R)$ in an economy, where FI_L is the lowest FI of any researcher in the economy. In this paper, it is assumed for simplicity that the strength of this preference is the same for all workers.

3.1.2. Fluid Intelligence of Researchers

On the basis of the theory of fluid intelligence and item response theory (*e.g.,* Cattell 1963, 1971, Lord and Novick 1968, van der Linden and Hambleton 1997). Harashima (2020c) showed that the probability of a worker's solving unexpected problems in a unit of time, $\hat{p}(FI)$, can be modeled as:

$$\hat{p}(FI) = \hat{\kappa} + \frac{1-\hat{\kappa}}{1+\exp[-\hat{\gamma}(FI-\hat{D})]} , \tag{10}$$

where: FI (∞ > FI > -∞) is a worker's fluid intelligence, $\hat{\gamma}$ (> 0) is a parameter that characterizes the slope of the function, $\hat{D}$ is a parameter that indicates the average difficulty of unexpected problems that the worker has to solve, and $\hat{\kappa}$ (1 ≥ $\hat{\kappa}$ ≥ 0) is the probability that unexpected problems are solved by chance (see also Harashima 2009, 2018b, 2012, 2016, 2020a).

Harashima (2012, 2020a) showed that the productivities of workers are positively correlated with their values of $\hat{p}(FI)$; that is, FI is positively correlated with productivity. Equation (10) indicates that the higher a worker's FI, the higher the probability of solving unexpected problems in a unit of time.

Solving difficult unexpected problems is basically delegated to highly educated and trained experts (*i.e.,* researchers) (Harashima 2009, 2012, 2016, 2020a). Unexpected problems delegated to researchers will be so difficult that the value of $\hat{D}$ (*i.e.,* the average difficulty of the problems) will be sufficiently larger than the value of researchers' fluid intelligences. Let the values of $\hat{D}$ and FI be standardized to be values that are far larger than unity in equation (10). With this standardization, if the value of $\hat{D}$ is sufficiently larger than FI, the value of $\exp[-\hat{\gamma}(FI-\hat{D})]$ is sufficiently larger than unity, and therefore,

$$\frac{1}{1+\exp[-\hat{\gamma}(FI-\hat{D})]} \cong \frac{\exp(\hat{\gamma}FI)}{\exp(\hat{\gamma}\hat{D})} \cong \frac{1+\hat{\gamma}FI}{\exp(\hat{\gamma}\hat{D})} = \bar{\gamma}_0 + \bar{\gamma}_1 FI$$

where: $\bar{\gamma}_0$ and $\bar{\gamma}_1$ are positive constants. Hence, by equation (10),

$$\hat{p}(FI) \cong \hat{\kappa} + (1-\hat{\kappa})(\bar{\gamma}_0 + \bar{\gamma}_1 FI) .$$

Here, it is assumed for simplicity that $\hat{\kappa} = 0$, and therefore,

$$\hat{p}(FI) \cong \bar{\gamma}_0 + \bar{\gamma}_1 FI \ . \tag{11}$$

Equation (11) means that:

$0 < \bar{\gamma}_0 + \bar{\gamma}_1 FI < 1$, for any $FI \geq FI_R$ because $\hat{p}(FI)$ is a probability.

3.1.3. Number of Researchers

Number of Researchers per Innovation

As the productivities of researchers increase, the same number of innovations can be produced with fewer researchers, which means that to produce a unit of innovation, a specific amount of summed talent (fluid intelligence) of researchers is needed, not a specific number of researchers.

Taking this property into consideration, suppose that an amount of talent *T* is required to produce v^{-1} of a unit of *A*, and *T* is supplied by researchers with fluid intelligences above $x(\geq FI_R)$ such that:

$$T = \int_x^{\infty} N_{R,FI}\hat{p}(FI)\,\mathrm{d}FI\,. \tag{12}$$

Equation (12) means that the amount of necessary talent T is expressed by the weighted sum of probabilities $\hat{p}(FI)$ of researchers whose FIs are above x. By equations (9), (11), and (12),

$$T = \int_x^{\infty} \frac{z}{\exp(FI)}(\bar{\gamma}_0 + \bar{\gamma}_1 FI)\mathrm{d}FI$$

and by partial integration,

$$T = \frac{z[\bar{\gamma}_0 + \bar{\gamma}_1(x-1)]}{\exp(x)}\,. \tag{13}$$

Hence, by equation (13),

$$\frac{dx}{dz} = \frac{1}{T\exp(x)}\left\{\frac{[\bar{\gamma}_0 + \bar{\gamma}_1(x-1)]^2}{[\bar{\gamma}_0 + \bar{\gamma}_1(x-2)]}\right\}. \tag{14}$$

Because the FIs of researchers are far larger than unity (remember that they are standardized in *section 3.1.2),*

$$\bar{\gamma}_0 + \bar{\gamma}_1(x-2) > 0, \tag{15}$$

and thereby, by equation (14) and inequality (15),

$$\frac{dx}{dz} > 0. \tag{16}$$

As larger numbers of workers possess higher levels of fluid intelligence (*i.e., z* becomes larger), the hurdle to become a researcher becomes higher (*i.e., x* increases).

Number of Researchers in an Economy

As an economy grows, the necessary number of innovations that has to be produced for successive growth increases (*i.e*., necessary units of new technologies increase) because the size of the economy is increasing. More units of innovations have to be produced in every period to sustain a constant growth rate. On the other hand, as equation (3) indicates, if the population is sufficiently large, the scale effects in endogenous growth (Jones, 1995a) disappear because of increases in uncompensated knowledge spillovers and the arbitrage between investments in capital and technologies (Harashima 2013, 2019b). Therefore, even if the economy grows, the number of researchers (labor inputs to produce innovations) is unchanged. Even if their number is unchanged, however, the amount of newly produced innovations in each period can increase as the economy grows because K and A increase as the economy grows.

However, the effect of increased *K* and *A* on the production of innovations is beyond the scope of this paper. The focus here is only on the amount of innovation, from which the effects of increased *K* and *A* on the production of new units of *A* are removed. The number of produced innovations after removing these effects is constant in each period on a balanced growth path because the number of researchers and their productivities are constant. In the following sections, therefore, I only focus on the amount of innovation after removing these effects.

Suppose that ηv^{-1} of a unit of A has to be newly produced in an economy in each period, where $\eta(> 1)$ is a parameter and this newly produced amount of A is the amount after removing the effects of increased *K* and *A*. By equation (13), therefore, the necessary amount of summed talent *(i.e.,* fluid intelligence) of researchers can be expressed as:

$$\eta T = \frac{z[\bar{\gamma}_0 + \bar{\gamma}_1(x-1)]}{\exp(x)}\,,$$

and thereby,

$$T = \left(\frac{z}{\eta}\right)\frac{[\bar{\gamma}_0+\bar{\gamma}_1(x-1)]}{\exp(x)}. \quad (17)$$

$\frac{z}{\eta}$ in equation (17) corresponds to *z* in equation (13), and $\frac{z}{\eta} < z$, which means that the value of *x* in equation (17) is smaller than that in equation (13) by inequality (16). Because the number of researchers is the sum of workers whose values of FI are above *x*, a smaller value of *x* indicates that workers with lower fluid intelligence levels need to be inputted as researchers. In any case, the number of researchers in an economy is determined by the value of *x* in equation (17).

3.1.4. The Average $\hat{p}(FI)$ of Researchers

The average $\hat{p}(FI)$ of researchers in an economy, $\hat{p}_A(FI)$, is given by:

$$\hat{p}_A(FI) = \frac{\int_x^\infty N_{R,FI}\hat{p}(FI)\mathrm{d}FI}{\int_x^\infty N_{R,FI}\mathrm{d}FI} = \frac{\int_x^\infty \frac{\bar{\gamma}_0+\bar{\gamma}_1 FI}{\exp(FI)}\mathrm{d}FI}{\int_x^\infty \frac{1}{\exp(FI)}\mathrm{d}FI}. \quad (18)$$

By partial integration,

$$\int_x^\infty \frac{\bar{\gamma}_0+\bar{\gamma}_1 FI}{\exp(FI)}\mathrm{d}FI = \frac{\bar{\gamma}_0+\bar{\gamma}_1(x-1)}{\exp(x)}, \quad (19)$$

and

$$\int_x^\infty \frac{1}{\exp(FI)}\mathrm{d}FI = \frac{1}{\exp(x)}. \quad (20)$$

Therefore, by equations (18), (19), and (20), the average $\hat{p}(FI)$ of researchers, $\hat{p}_A(FI)$, is:

$$\hat{p}_A(FI) = \frac{\int_x^\infty N_{R,FI}\hat{p}(FI)\mathrm{d}FI}{\int_x^\infty N_{R,FI}\mathrm{d}FI} = \bar{\gamma}_0 + \bar{\gamma}_1(x-1). \quad (21)$$

3.1.5. Productivity of Researchers

It seems highly likely that the productivity v^{-1} is positively correlated with $\hat{p}_A(FI)$. Hence, by equation (21), for a given value of η, the productivity v^{-1} can be modeled most simply as:

$$v^{-1} = \bar{\gamma}_0 + \bar{\gamma}_1(x-1). \quad (22)$$

3.1.6. Relation between v and η

Inequality (16) and equation (17) indicate that as η increases, x decreases and thereby, by equation (22), v^{-1} decreases. That is, there is a negative correlation between v^{-1} and η, (i.e., a positive correlation between v and η) such that

$$v = f_v(\eta), \quad (23)$$

and

$$\frac{df_v(\eta)}{d\eta} > 0. \quad (24)$$

3.1.7. The Case of Very Small z

By equation (9),

$$\lim_{z\to 0} N_{R,FI} = 0,$$

and therefore, if *z* of an economy is very small (*i.e.,* there are a limited number of workers with high levels of fluid intelligence), few workers can be researchers in an economy. In this case, equations (13) and (17) cannot be satisfied *(i.e.,* ηT is not fully filled), and therefore, not enough innovations are produced to fully sustain economic growth. If z is close to zero, few innovations are produced in each period, and the economy will experience little endogenous growth.

3.2. Arbitrage between Investments in Technologies and Capital

The productivity v^{-1} is not determined only by the distribution of worker fluid intelligence because, as equation (17) indicates, *x* depends not only on *z* but also on *η*. The value of η indicates how many innovations have to be produced in a period and is determined by arbitrage between investments in capital and technology, as indicated by equation (2). If the return on investments in technologies exceeds that in capital, investments in technologies increase up to the point that equation (2) is satisfied and vice versa. If v^{-1} increases, the return on investments in technologies increases, and therefore, *η* increases. Hence, there is a positive correlation between v^{-1} and *η* (*i.e.,* a negative correlation between v and η) such that:

$$\eta = f_\eta(v), \tag{25}$$

and

$$\frac{df_\eta(v)}{dv} < 0\,. \tag{26}$$

3.3. Equilibrium v and η, and the Upper Bound of Economic Growth

Equations (23) and (25) can be seen as the reduced form of equations with regard to the determination of *v* and *η*. Because equation (23) is monotonously and continuously increasing and equation (25) is monotonously and continuously decreasing, an equilibrium combination of *v* and *η* exists except for corner solutions, and this equilibrium determines the values of v and η in the economy.

With this equilibrium value of *v,* the constant growth rate on a balanced growth path is determined by equations (5) and (7). In other words, this equilibrium binds the long-run economic growth rate *(i.e.,* it generates the upper bound).

4. Heterogeneous Economies

Next, I examine the production of innovations when there are many heterogeneous economies. To start, let ω be the average productivity of all workers in an economy. As indicated in ection 2.2, ω is constant and the average productivity of usual workers is almost equal to ω.

Suppose for simplicity that there are two economies (Economy 1 and Economy 2) that are identical except for the values of z and ω. Let z_1 and z_2 be z of Economies 1 and 2, and ω_1 and ω_2 be ω of them, respectively; in addition, $z_1 > z_2$ and $\omega_1 > \omega_2$. Because the values of z are heterogeneous, η, $N_{R,FI}$, and x are also heterogeneous. Finally, let η_j, $N_{R,FI,j}$, and x_j be η, $N_{R,FI}$, and x of Economy j (= 1, 2), respectively.

4.1. Closed Economies

Suppose first that the two economies are isolated and closed to each other. In this case, if $\eta_1 = \eta_2$, $x_1 > x_2$ by inequality (16) because $z_1 > z_2$, which indicates that $\hat{p}_A(FI)$ (the productivity v^{-1}) in Economy 1 is higher than that in Economy 2 by equation (22).

However, as v^{-1} increases, η also increases, as inequality (26) indicates. At the same time, an increase in η makes v^{-1} decrease, as inequality (24) indicates. As a result, $N_{R,FI,1}$ and $N_{R,FI,2}$ can take various values depending on the shapes of equations (23) and (25) in the two economies. Hence, in general, it is difficult to say whether $N_{R,FI,1}$ or $N_{R,FI,2}$ is larger.

4.2. Open Economies

If the two economies are fully open and capital moves perfectly elastically between them (labor does not move between the economies), new innovations generated by either economy are accumulated as the common knowledge (A) for both economies. This case can be interpreted as the two economies being combined to form Economy 1+2.

Harashima (2010, 2017) showed that, even if households are heterogeneous in RTP (θ), DRA (ε), and ω, the state in which all optimality conditions of all heterogeneous households are satisfied exists; that is, "sustainable heterogeneity" (SH) can be achieved. Heterogeneous z will naturally accompany heterogeneous ω as assumed above, but it necessitates only heterogeneity in ω and not in the other sources of heterogeneity (RTP and DRA). Harashima (2010, 2017) showed that, if only ω is heterogeneous, SH is naturally achieved without government interventions. That is, SH is naturally achieved in Economy 1+2.

By equation (9), in Economy 1+2, the number of researchers from Economy 1 is:

NR,I,1 = z1exp$FIN_{R,FI,1} = \frac{z_1}{\exp(FI)}$

and that from Economy 2 is

$$N_{R,FI,2} = \frac{z_2}{\exp(FI)}.$$

Because the two economies are fully open and capital moves perfectly elastically, the wages of workers with the same level of productivity are kept identical between the two economies, regardless of whether they are usual workers or researchers. Therefore, the value of *x* is kept identical between the two economies *(i.e.,* a unique common value of x exists) through arbitrage in markets, and thus,

$$\eta_{1+2}T = \int_x^\infty N_{R,FI,1}\hat{p}(FI)\,\mathrm{d}FI_i + \int_x^\infty N_{R,FI,2}\hat{p}(FI)\,\mathrm{d}FI = \frac{z_1[\bar{\gamma}_0 + \bar{\gamma}_1(x-1)]}{\exp(x)} + \frac{z_2[\bar{\gamma}_0 + \bar{\gamma}_1(x-1)]}{\exp(x)}$$

and thereby,

$$T = \left(\frac{z_1 + z_2}{\eta_{1+2}}\right)\frac{\bar{\gamma}_0 + \bar{\gamma}_1(x-1)}{\exp(x)}\ , \tag{27}$$

where η_{1+2} is η of Economy 1+2.

As indicated by equation (25) and inequality (26), η increases if the productivity v^{-1} increases. Nevertheless, η also increases if the worker's average productivity (ω_i) increases because the production per capita (y_t) increases as ω_i increases by equation (8), which indicates that more new innovations have to be produced for balanced growth in each period, and thereby η_i increases (see Harashima 2010, 2017). That is, η depends not only on v^{-1} but also on ω, and if $\omega_2 < \omega_1$, then $\eta_2 < \eta_1$ (*i.e.,* η_i is an increasing function of ω_i).

Here, it is highly likely that, if $z_2 < z_1$, then $\omega_2 < \omega_1$, and thereby $\eta_2 < \eta_1$ by the above reasoning. Furthermore, because the scale effect is small, as noted in Section 1.1, the number of necessary innovations for balanced growth does not largely change even if the population doubles (Harashima, 2013, 2019b). Therefore,

$$\eta_2 < \eta_{1+2} < \eta_1 \tag{28}$$

because $\omega_2 < \omega_{1+2} < \omega_1$. Hence,

$$\frac{z_1}{\eta_1} < \frac{z_1+z_2}{\eta_1} < \frac{z_1+z_2}{\eta_{1+2}}\ . \tag{29}$$

By equations (17) and (27) and inequality (29), x in Economy 1+2 is greater than *x* in Economy 1, and thereby some researchers in the case of isolated Economy 1 cannot be researchers anymore in Economy 1+2 because their levels of fluid intelligence are not sufficiently high.

On the other hand, for Economy 2, there are two possibilities:

$$\frac{z_2}{\eta_2} < \frac{z_1+z_2}{\eta_{1+2}} \tag{30}$$

or

$$\frac{z_1+z_2}{\eta_{1+2}} < \frac{z_2}{\eta_2}\ . \tag{31}$$

If inequality (30) holds, some researchers in the case of isolated Economy 2 cannot be researchers anymore in Economy 1+2 because their levels of fluid intelligence are not sufficiently high in Economy 1+2. Conversely, if inequality (31) holds, some workers who cannot be researchers in the isolated Economy 2 because of their relatively low levels of fluid intelligence can be researchers in Economy 1+2.

Nevertheless, because

$$\omega_{1+2} = \frac{\omega_1 + \omega_2}{2} ,$$

as shown in Harashima (2010, 2017), then in general, approximately

$$\eta_{1+2} \cong \frac{\eta_1+\eta_2}{2} . \tag{32}$$

Hence, it seems more likely that inequality (30) will hold, not inequality (31). Therefore, it is highly likely that some workers who can be researchers in the isolated Economy 2 can no longer be researchers in Economy 1+2.

As noted above and in Harashima (2013, 2019b), because the scale effect is small, even if the population doubles, the number of innovations needed for balanced growth does not largely change, which means that the necessary number of researchers does not necessarily increase as the number of high fluid intelligence workers does with an expanding population. Therefore, as the population increases, some of the relatively lower fluid intelligence researchers have to quit their jobs as researchers.

4.3. The Case of Very Small z2

Because equation (32) approximately holds,

$$\frac{z_1+z_2}{\eta_{1+2}} \cong \frac{2(z_1+z_2)}{\eta_1+\eta_2}. \tag{33}$$

Therefore, if $z_2 \cong 0$, by inequality (28) and equation (33),

$$\frac{z_1+z_2}{\eta_{1+2}} \cong \frac{2z_1}{\eta_1+\eta_2} > \frac{z_1}{\eta_1}. \tag{34}$$

Hence, by equations (16) and (17), if $z_2 \cong 0$, the value of x in Economy 1+2 is higher than that in the isolated Economy 1. There are few high fluid intelligence workers in Economy 2 because $z_2 \cong 0$. Even so, some researchers in the case of isolated Economy 1 cannot continue to be researchers in Economy 1+2 because, as inequality (28) indicates, $\eta_2 < \eta_1$, and thereby, fewer innovations need be produced in Economy 1+2 than in the isolated Economy 1.

Conversely, even though high fluid intelligence workers exist mostly in Economy 1, Economy 1+2 can "normally" grow endogenously. In this case, most researchers are from Economy 1. This nature originates in the non-rivalry of technologies; that is, accumulated technologies (knowledge) A can be used equally in both Economies 1 and 2. Hence, the endogenous growth of Economy 1+2 is not affected even if almost all of the researchers are from Economy 1.

4.4. The Upper Bound of Long-run Growth Rate

The result in Section 4.3 indicates that, even if Economies 1 and 2 are identical and high fluid intelligence workers sufficiently exist equally in both Economies 1 and 2, the long-run growth rate of Economy 1+2 will be almost the same as that of Economy 1+2 in the case where high fluid intelligence workers sufficiently exist almost exclusively in Economy 1 because $\eta_1 = \eta_2$ and furthermore $\eta_1 = \eta_2 \cong \eta_{1+2}$, as implied by inequality (34). Thereby, x and v^{-1} are almost the same in both cases.

This means that increasing the number of high fluid intelligence workers will not be an effective way of raising the upper bound of the long-run economic growth rate (see Section 3.3) if a sufficiently high number of these workers already exist. A better way to do so may be to increase the level of fluid intelligence in the existing workers, although that may not be an easy task.

Concluding Remarks

The average growth rate in the long run has historically had an upper bound, which means that the number of innovations produced is constrained by some factors. In the literature of innovation-based endogenous (or Schumpeterian) growth theory, the growth rate is basically constrained by the finite nature of the labor supply. However, innovations are the fruits of intellectual activities, and it seems highly likely that the constraint originates not in a finite labor supply but in the finiteness of workers' innovative talents or intelligence.

The decreasing rate of marginal utility has to be constant on a balanced growth path, most likely because the number of innovations produced in each period is constrained by some factors. In this paper, I showed that the number is constrained because (1) the amount of fluid intelligence of researchers is limited in an economy, and (2) the return on investments in technologies and capital are kept equal through arbitrage in markets, which means that the amount of innovation is also substantially constrained by activities not only in the labor market but also in other markets, particularly financial markets. With these constraints, equilibrium values of the number of researchers and their average productivity exist in an economy, and innovations are produced at a level that is consistent with the equilibrium in each period. This equilibrium average productivity of researchers determines the number of innovations produced in each period.

Distributions of fluid intelligence levels among researchers are most likely heterogeneous across economies, and therefore, the decreasing rates of marginal utility and the growth rates of technologies (innovations) will be also heterogeneous if these heterogeneous economies are closed to each other. However, if they are open, the decreasing rate of marginal utility and the growth rates of technologies are equalized across economies, and therefore, even an economy with a smaller number of researchers with a high level of fluid intelligence can grow at the same rate as an economy with more researchers.

References

[1] Philippe, A. and Peter Howitt, P. 1992. A Model of Growth through Creative Destruction, *Econometrica*, Volume 60, pp. 323–351. DOI: https://doi.org/10.2307/2951599

[2] Philippe, A. and Peter Howitt, P. 1998. Endogenous Growth Theory, Cambridge, MA, MIT Press.

[3] Philippe, A., Akcigit, U. and Peter Howitt, P. 2014. What Do We Learn from Schumpeterian Growth Theory? in: Philippe Aghion and Steven Durlauf (ed.), *Handbook of Economic Growth*, Volume 2, pp. 515-563.

[4] Cattell, R.B.1963. Theory of Fluid and Crystallized Intelligence: A Critical Experiment, *Journal of Educational Psychology,* Volume 54, pp. 1-22. DOI: https://doi.org/10.1037/h0046743

[5] Cattell, R.B. 1971. *Abilities: Their Structure, Growth, and Action*, Houghton Mifflin, Boston.

[6] Dinopoulos, E. and Thompson, P. 1998. Schumpeterian Growth without Scale Effects*, Journal of Economic Growth*, Volume 3, pp. 313–335.

[7] Eicher, T.S. and Turnovsky, S.J. 1999. Non-Scale Models of Economic Growth, *The Economic Journal*, Volume 109, pp. 394–415. DOI: https://doi.org/10.1111/1468-0297.00454

[8] Grossman, G.M. and Helpman, E. 1991. *Innovation and Growth in the Global Economy,* Cambridge, MA, MIT Press.

[13] Harashima, T. 2016. A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers' Innovations as an Essential Element, *Journal of Kanazawa Seiryo University*, 50(1): 55-80, *in Japanese*.

[14] Harashima, T. 2017. Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population, *Journal of Kanazawa Seiryo University,* 51(1): 31-80, *in Japanese*.

[16] Harashima, T. 2018b. Wage Inequality and Innovative Intelligence-Biased Technological Change, *Theoretical and Practical Research in Economic Fields*, Volume IX, Summer, 1(17): 17-24.

[17] Harashima, T. 2019a. Do Households Actually Generate Rational Expectations? "Invisible Hand" for Steady State, *Journal of Kanazawa Seiryo University*, 52(2): 49-70, *in Japanese*.

[18] Harashima, T. 2019b. An Asymptotically Non-Scale Endogenous Growth Model, *Journal of Kanazawa Seiryo University,* 52(2): 71-86, *in Japanese*.

[19] Harashima, T. 2020a. A Theory of Intelligence and Total Factor Productivity: Value Added Reflects the Fruits of Fluid Intelligence, *Journal of Kanazawa Seiryo University*, 53(2): 65-82, *in Japanese*.

[20] Harashima, T. 2020b. Why Is Risk Aversion Essentially Important for Endogenous Economic Growth? *Journal of Applied Economic Sciences*, Volume XV, 3(69): 556-569.

[21] Harashima, T. 2020c. The Correlation between Time Preference and Incomes Is Spurious: They Are Bridged by Fluid Intelligence, *Journal of Applied Economic Sciences*, Volume XV, Spring, 1(67): 107-123.

[22] Jones, Charles I. 1995a. Time Series Tests of Endogenous Growth Models, *Quarterly Journal of Economics*, 110(2): 495–525. DOI: https://doi.org/10.2307/2118448

[23] Jones, C.I. 1995b. R&D-Based Models of Economic Growth, *Journal of Political Economy,* 103(4): 759–84. DOI: https://doi.org/10.1086/262002

[24] Jones, C.I. 1999. Growth: With or without Scale Effects? *The American Economic Review*, 89(2): 139-144. DOI: 10.1257/aer.89.2.139

[25] Kortum, S.S. 1997. Research, Patenting, and Technological Change, *Econometrica*, 65: 1389–1419. DOI: https://doi.org/10.2307/2171741

[26] Lord, F.M. and Novick, M.R. 1968 Statistical Theories of Mental Test Scores, Addison-Wesley, Reading, MA.

[27] Peretto, P. 1998. Technological Change and Population Growth, *Journal of Economic Growth*, 3: 283–311. DOI:10.1023/A:1009799405456

[28] Peretto, P. and Smulders, S. 2002. Technological Distance, Growth and Scale Effects, *The Economic Journal*, 112: 603–624. DOI: https://doi.org/10.1111/1468-0297.00732

[29] Romer, P. M. 1990. Endogenous Technological Change, *Journal of Political Economy*, 98(5): 71–102.

[30] Segerstrom, P. 1998. Endogenous Growth without Scale Effects, *American Economic Review*, 88: 1290–1310.

[31] Young, A. 1998. Growth without Scale Effects, *Journal of Political Economy*, Volume 106: 41–63. DOI: https://doi.org/10.1086/250002

[32] van der Linden, Wim J. and Hambleton, R.K. (Eds.). 1997. *Handbook of Modern Item Response Theory*, Springer-Verlag, New York.