Journal of Applied Economic Sciences
Volume XX, Fall, Issue 3(89), 2025

Seller-Buyer Bargaining Explained by Fixed Bargaining Costs, Risk Preferences and Value Discovery

Joongsan HWANG
https://orcid.org/0009-0000-9423-2192
University of Colorado Boulder, Boulder, Colorado, USA
johw7401@colorado.edu

Article's history:

Received 25th of June, 2025; *Received* 29th of July, 2025; *Accepted* 12th of August, 2025; *Available online*: 30th of September, 2025. *Published* as article in the Volume XX, Fall, Issue 3(89), 2025.

Suggested citation:

Hwang, J. (2025). Seller-Buyer Bargaining Explained by Fixed Bargaining Costs, Risk Preferences and Value Discovery. *Journal of Applied Economic Sciences*, Volume XX, Fall, 3(89), 333 – 362. https://doi.org/10.57017/jaes.v20.3(89).01

Abstract:

This paper solves for equilibria of bargaining games with a seller and a buyer where there is no discounting between periods but players pay fixed bargaining costs for each period they bargain. In this setting, for the seller to cut prices gradually and effectively, the buyer needs to be risk averse. If players are not allowed to terminate bargaining in a finite game, the seller will raise the equilibrium prices. Allowing players to terminate bargaining causes the players to never make a deal with each other. Allowing the buyer to discover the value of the good along with bargaining termination enables the buyer to stop the seller from offering a high price and the seller to engage in price skimming by gradually lowering the price in equilibrium.

Keywords: bargaining costs, risk preference, risk aversion, bargaining termination, option to exit, price skimming.

JEL Classification: C78; C73; D82.

Introduction

DANIEL	Is he here?
AL ROSE	No, he'd like you to come visit with him.
DANIEL	He's boosting his price.
AL ROSE	He said he'd like to speak with whoever is doing the buying.

— Paul Thomas Anderson, *There Will Be Blood*

This paper is on seller-buyer bargaining games with discrete time periods and incomplete information about the buyer. Only the seller makes the offers in the models and the seller may not know what price the buyer is willing to pay. The seller offers the bargaining price and the buyer has the option to accept it or reject it. Rejecting it might mean that the buyer can bargain in the future for a better offer from the seller.

The four key factors that are involved in the game are fixed bargaining costs, buyer's risk preference, bargaining termination and value discovery. In the literature on bargaining, a key issue is in providing the two players with incentives to come to terms in a timely manner. If the parties do not have incentives to make a deal quickly, bargaining may take arbitrarily long. To provide this incentive, this paper uses fixed bargaining costs. Existing literature usually uses discounting for this purpose[1]. However, the analysis excludes discounting and models phenomena for which fixed bargaining costs are more realistic. Fixed bargaining costs imply that, in each

[1] Fudenberg & Tirole (1983), Cramton (1984), Sobel & Takahashi (1983)

period a party engages in bargaining, the party incurs a fixed cost for that period. The weak Perfect Bayesian Equilibriums (wPBEs) and Perfect Bayesian Equilibriums (PBEs)[2] are derived accordingly.

For deals that take years such as the AB InBev and Modelo merger where it took AB InBev almost five years to buy Modelo, discounting may be the right tool[3]. However, for negotiations that take seconds or minutes in total, fixed bargaining costs are a better tool. When people are bargaining for 10 minutes over the price of a painting, the value of the painting or the money from the sale will not change by getting it 10 minutes later unless you need the painting urgently. Instead, what would matter is the opportunity cost of that 10 minutes. That 10 minutes could be spent on productive or enjoyable activities. Such costs are better represented by fixed bargaining costs.

An even stronger case for fixed bargaining costs instead of discounting is in bargaining where the time that the deal is made does not affect when the deal takes effect. One could be bargaining for a purchase of a sofa that can only be delivered months later. A firm and the labor union could be bargaining on October over a labor contract that will take place on January. In such cases, even a few days' delay in reaching a deal will not change when the parties will get the utility from the deal. Instead, a party may consider the cost of arranging a meeting, transportation costs and opportunity cost of time spent in the furtherance of bargaining.

During bargaining, a seller often engages in price skimming. According to Chang & Lee (2022, 1), price skimming refers to price discrimination where the seller first offers a high price and gradually lowers the price as time goes on. In employing such a strategy when the buyer's value is unknown to the seller, the seller's goal is to get the buyers who value the good more to buy at a higher price and to get the buyers who value the good less to buy at a lower price. With discounting, a buyer with a higher value for the good will see a greater utility loss from the delay and has a greater incentive to make an early deal. Therefore, for some series of prices, the buyer with high value will buy early at a high price and the buyer with low value will buy late at a low price.

However, when the buyer has risk-neutral or risk-seeking preferences and fixed bargaining costs but no discounting, a buyer with a higher value for the good no longer has a greater incentive for a quicker deal. This means that the seller loses the ability to make price skimming reliably work as intended. However, when a buyer has decreasing marginal utility because of risk-averse utility, a buyer with a higher value for the good sees greater utility. Thus, the buyer with a higher value has a smaller utility gain from a lower price and hence a smaller incentive to wait for a lower price. For fixed bargaining costs, reliable and effective price skimming is only possible when the buyer is risk averse.

Schweighofer-Kodritsch (2022) and Kambe (2025a, 2025b) are theoretical papers on bargaining games with fixed bargaining costs and alternating offers. None of these papers allow players to quit bargaining before any papers have been made. Schweighofer-Kodritsch (2022) demonstrates that in equilibrium, bargaining can become a trap where a player gets a worse outcome compared to that of no deal. Allowing players to terminate bargaining removes the trap. Kambe (2025a) shows that players use take-it-or-leave-it strategies in the unique equilibrium in the game with bargaining termination. Kambe (2025b) finds that the option to terminate improves the outcome for the player with the higher cost when this player makes an offer. Experiments with bilateral offers have shown that when people are not given the option to terminate bargaining, a winning strategy is to make an opening offer favorable to its proposer (this is the seller in this paper)[4].

[2] wPBEs are defined in Mas-Colell et al. (1995, pp. 283–285), while PBEs are defined in Fudenberg and Tirole (2005, pp. 331–333). Unlike wPBEs, PBEs impose restrictions on beliefs off the equilibrium path. In contrast to Fudenberg & Tirole's definition, the PBEs in this paper assign beliefs to players only at the information sets where the player takes action. Except for the initial belief, updates occur based on the most recent information set where the player acted.

[3] DePamphilis (2015, 50–51); Sorkin (2008), Sanburn (2012).

[4] Chertkoff & Conley (1967), Liebert et al. (1968), Galinsky & Mussweiler (2001), Yukl (1974).

In games of this paper with fixed bargaining costs, only the seller makes the offer. Furthermore, when the option to terminate bargaining exists, it includes the option to quit bargaining before any offers. To see how this changes the results, the analysis derives bargaining outcomes for models in which players are either not allowed to terminate bargaining (as in sections 2 and 3) or are allowed to do so (as in sections 4 and 5).

Sections 2 and 3 show that a model with fixed bargaining costs that does not let the players terminate bargaining is problematic regardless of the buyer's risk preference. Fixed bargaining costs without discounting mean that players' bargaining cost can increase boundlessly if a deal continues to be not made. In a game with finite periods, this amplifies the seller's first-mover advantage and enables the seller to make high offers and secure greater payoffs in weak Perfect Bayesian Equilibriums (wPBEs). This is similar to how bargaining can become a trap in Schweighofer-Kodritsch (2022). However, in a game with infinite periods, the buyer can punish the seller for infinite periods if the seller keeps insisting on a price higher than the equilibrium price. These phenomena mean that some equilibria have extreme prices for both finite and infinite period games without discounting.

In reality, bargaining costs cannot be increased without limit because if total bargaining costs are too high, players would terminate bargaining. To account for this, in section 4, both players have the ability to terminate bargaining. This removes the previous equilibria and leads to a new equilibrium. When the buyer is able to terminate bargaining but doesn't, the seller finds out that the buyer's value for the good is high. This gives the seller an incentive to raise the price. The seller's unyielding attempt to raise the price leads the players to terminate bargaining without a single offer made. Therefore, unlike Schweighofer-Kodritsch (2022) and Kambe (2025a, 2025b) none of which allow bargaining termination without a single offer made, having bargaining termination in this section does not get any player a better transaction nor any transaction.

In section 5, a disincentive for the seller to raise prices is introduced by modeling value discovery. The buyer can find out the value of the good through bargaining that involves asking questions about the good and getting answers. In bargaining, lower values may be harder to discover for the buyer because the seller would want to hide information on lower values. If the buyer finds the value to be low, the buyer may terminate bargaining without buying the good. This possibility provides the disincentive to keep the seller from raising the price and allows bargaining over multiple periods and price skimming in PBEs to exist. A continuum of PBEs may exist due to differences in which situations have players terminate bargaining.

1. Literature Review

This paper is related to four groups of papers. The first group has papers on bargaining with fixed bargaining costs or bargaining termination. Rubinstein (1982) solves for equilibria of the bargaining games where, unlike my paper, bargainers know each other's preference. These games include games with fixed bargaining costs and games with discounting. For the games with fixed bargaining costs, Rubinstein (1982) finds that the player with lower fixed bargaining cost gets a dominating share of the pie. Shaked (1994) and Ponsatí & Sákovics (1998) find a continuum of equilibria for bargaining games with discounting and bargaining termination. This is because variations in when a player terminates bargaining can lead to different equilibria. In the introduction, I discussed Schweighofer-Kodritsch (2022) and Kambe (2025a, 2025b) which are on bargaining with fixed bargaining costs and alternating offers.

Porter & Rosenthal (1989) solve for bargaining under the split-the-difference mechanism with fixed bargaining costs and finds problematic equilibria. However, unlike my paper's extensions, Porter and Rosenthal (1989) does not allow for bargaining termination. Perry (1986) finds that for its bargaining game with bilateral offers and bargaining termination, with infinite periods, there is at most one offer made. While this paper is the paper similar to my paper in this group, unlike my paper, Perry (1986) solves for the case where the bargaining cost is only paid by a bargainer when this bargainer makes an offer. Karagözoglu & Rachmilevitch (2021) finds a symmetric equilibrium for bargaining with fixed participation costs. However, participation costs are different from my bargaining costs as bargainers who do not the participation cost do not immediately drop out of bargaining.

In Özyurt (2023), a player can terminate bargaining and this gives the player bargaining power. However, unlike my paper, only one player has the power to terminate bargaining. Similarly, in Abreu & Manea (2024), a seller can exclude buyers from bargaining and this lets the seller extract profit from buyers. Only the seller has the exclusion power.

The second group has papers that deal with bargaining using risk preferences. Roth (1985) showed that for Rubinstein (1982) 's model with discounting, a risk-averse bargainer has a disadvantage. Dickinson (2003) shows that when bargaining payoffs in case of a dispute are represented by a lottery, risk aversion causes the bargainer to get less in negotiations but risk love makes negotiation failure more likely.

The third group is for papers that have bargaining with incomplete information. Rubinstein (1985) and Harris (1985) have models where one player's discount factor is private information and random. Harris (1985) also has a model where one player's bargaining cost is private and random. Rubinstein (1985) finds that the belief about this private information has a clear connection to the equilibrium. Harris (1985) finds pooling equilibrium and separating equilibrium depending on the distribution of the discount factor or the distribution of the fixed bargaining costs. In Chang & Lee (2022)'s bargaining, both the buyer's valuation and the outside option are private information. Chang & Lee (2022) show how the outside option is related to the seller's profit. Excluding Harris (1985), papers in the second and third group are not on bargaining costs. Harris (1985) has random bargaining costs and fixed value of the good. I have fixed bargaining costs but the value of the good may be random.

The final group has to do with papers on the Coase Conjecture. Coase (1972) presents the Coase Conjecture that when a monopolist sells a durable good to patient consumers, the monopolist will sell at the market price. Dilmé (2025) shows that the Coase Conjecture holds for a bargaining model with infinite periods where the value of the good for the buyer is private and only the seller makes offers. Yoshida (2025) shows that in a game with finite periods, Coase Conjecture results are partially restored by a probabilistic threat to exit the market by the buyers. However, Groseclose (2024) shows that when the monopolist's discount rate approaches 1 and the consumers' does not, the Coase Conjecture fails.

Zhang & Chiang (2020) states price skimming can be optimal for a durable goods seller in the presence of consumers' reference price effects which mean that purchase decisions are based on the reference price formed by past prices. In Ausubel & Deneckere (1989)'s model of a durable goods monopolist, when the time between offers is close to 0, punishments for deviation become effective and a Folk Theorem establishes that seller's payoff can take on a wide range of values. This is contrary to the Coase Conjecture. My buyers are different from those in the Coase Conjecture because my buyers have to pay fixed bargaining costs for delaying purchase and are not infinitely patient.

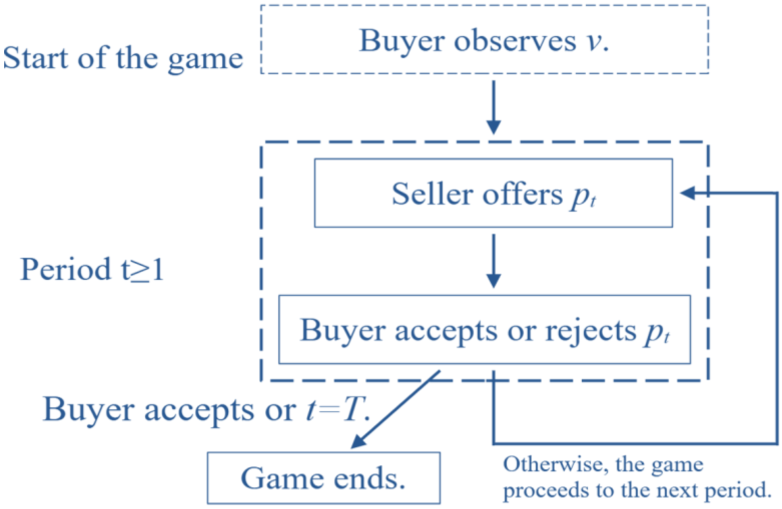
2. Basic Model

2.1 Specification

In the basic model, there is a seller (he) and a buyer (she). Seller's production cost of the good is γ and public information. Buyer's value of the good is v. This is a random variable and buyer's private information. The image of v is $V \in R^1$. V is bounded. At the start of the game, nature decides v.

The game has $T \geq 1$ discrete periods, starting with period 1. T can be finite or infinite. When the game proceeds to period t, the seller first offers a price, $p_t \in R^1$ to the buyer. If the buyer accepts the price. the sale is made and the game ends. If the buyer rejects the price, the game continues to the next period unless $t = T$ in which case, the game ends. The game tree is in Figure 1.

Figure 1. Game tree for the basic model



If the game ends with a sale, let τ be the period when the sale is made. For each period in which the players bargained, the seller paid a fixed bargaining cost of $c_S > 0$ and the buyer paid a fixed bargaining cost of $c_B > 0$. If the sale is made, seller's payoff from the game is:

$$U_S(p_\tau, \tau) = u_S(p_\tau - \gamma) - \tau c_S.$$

where: $u_S(p_\tau - \gamma)$ is the seller's utility from the sale. τc_S is his total bargaining cost accumulated over τ period of bargaining. If the game ends in period T without a sale and T is finite, seller's payoff is $-Tc_S$. If the game never ends, seller's payoff is $-\infty$.[5]

If the purchase is made, buyer's payoff from the game is:

$$U_B(p_\tau, \tau) = u_B(v - p_\tau) - \tau c_B.$$

where: similarly, to the seller, $u_B(v - p_\tau)$ is the buyer's utility from the purchase. τc_B is her total bargaining cost. If the game ends after finite periods without a sale, buyer's payoff is $-Tc_B$. If the game never ends, buyer's payoff is $-\infty$[6]; u_S and u_B are weakly increasing utility functions on R^1. They are concave or convex. This means they can be risk averse, risk-neutral or risk-seeking functions. By changing these functions, I can change the risk preferences of the players.

2.2 Price Skimming and the Buyer's Risk Preference

Price skimming refers to a seller's price discrimination strategy of offering lower prices as time goes on. If the seller is the one who makes the offers, price skimming decides whether bargaining can take multiple offers. If the seller does not lower prices as time goes on and the buyer knows that this is the seller's strategy, the buyer would either not buy or take the first offer. Then, the seller's first offer can be thought of as a take-it-or-leave-it offer. The buyer can ignore subsequent offers and just consider whether buying at the first offer price is optimal.

Therefore, in bargaining where the buyer and the seller bargain over different prices for some time, price skimming happens. The seller's goal in price skimming is to make buyers who value the product highly buy at a high price and buyers who value the product less buy at a low price. Whether this is possible depends on the buyer's utility function.

[5] This means that the seller's payoff and expected payoff can be real numbers or $-\infty$.

[6] This means that the seller's payoff and expected payoff can be real numbers or $-\infty$.

Suppose that the seller's strategy is to offer p_t and upon rejection, to offer $p_{t+1} < p_t$. Then in period *t*, the buyer can buy now or alternatively delay and buy in period *t+1.* The delayed purchase gives her a payoff of $u_B(v - p_{t+1}) - (t+1)c_S$. Buying now gives her a payoff of $u_B(v - p_t) - tc_S$. The difference is:

$$u_B(v - p_{t+1}) - u_B(v - p_t) - c_S. \quad (1)$$

The benefit of delayed purchase is $u_B(v - p_{t+1}) - u_B(v - p_t)$ and the cost of delayed purchase is c_S. If the buyer's utility function is risk neutral, take the case of $u_B(v - p_\tau) = v - p_\tau$. Then, equation (1) becomes: $p_t - p_{t+1} - c_S$. In this case, for a buyer of any type, the benefit of delayed purchase is $p_t - p_{t+1} > 0$. Because the benefit and the cost are unchanging for buyers of all types, if the benefit, $p_t - p_{t+1}$, is greater than the cost, c_S, all buyers want the delayed purchase. If the cost, c_S, is greater, all sellers want to buy now. If the benefit and cost are equal, all buyers are indifferent between the two options. In other words, the seller cannot reliably make buyers with high value buy at a high price and buyers with low value buy at a low price using price skimming.

The situation is worse for the seller when the buyer's utility function is strictly convex so the buyer is risk seeking. The benefit of delayed purchase, $u_B(v - p_{t+1}) - u_B(v - p_t)$, is increasing in v. This is easy to see for the case where u_B is differentiable since then, the marginal utility is increasing.[7] The benefit of the delayed purchase is greater when the buyer values the good more. This means that the buyer with a higher value for the good is more inclined to wait for a lower price. Therefore, if price skimming works to price discriminate buyers of different types, the buyers with high value buy at a low price and the buyers with low value buy at a high price. This is the opposite of what the seller wants. If this happens in reality, the seller should just announce that he will sell at the high price the buyers with low value will buy for and that he will not bargain. This way, all buyers will buy at this high price.

Price skimming can only work reliably the way the seller intends it to when the buyer's utility function is strictly concave i.e., when the buyer is risk averse. Then, the benefit of delayed purchase, $u_B(v - p_{t+1}) - u_B(v - p_t)$, is decreasing in v. Again, this is easy to see when u_B is differentiable since then, the marginal utility is decreasing.[8] The benefit of delayed purchase is now greater for the buyers who value the good less. Therefore, for decreasing prices, the seller may get the buyers with high value to buy at a high price and the buyers with low value to buy at a low price. When the seller does this, the buyer with high value for the good makes the deal early because further bargaining will decrease her payoff.

3. Equilibria

First, key characteristics of the weak Perfect Bayesian Equilibriums (wPBEs) are identified for the basic model.

Proposition 1.

Fix u_S, u_B, c_B and c_S. If u_B is continuous, u_S is increasing and u_B and u_S are unbounded from above and below, for all t and p, there exists some T' for which if $T' < T < \infty$, $P(p_t > p) = 1$ and $E(U_S) > p$ and $E(U_B) < -p$ in any wPBE.

Roughly speaking, the above proposition means that for a finite game with $T < \infty$, as $T \to \infty$, the price for any period in a wPBE goes to ∞, the seller's expected payoff in a wPBE goes to ∞ and the buyer's expected payoff goes to $-\infty$. The logic can be explained by a simplified example. In period T, the last period, a buyer whose willingness to pay is $1 will accept a price of 1. Now apply backward induction. If the buyer's bargaining cost is 1, in period T −1, the buyer will accept a price of 2 since delaying the purchase will not increase her payoff. In period T −2, she will accept 3 and so on.

[7] For the general case, this result is proven in lemma 1's (1).
[8] For the general case, this result is proven in lemma 1's (1).

So, for any period t, the more periods left in the game, the higher the price for the current period. Furthermore, as the number of periods left goes to infinity, the price for the current period will do so as well. Roughly speaking, when prices for all periods, especially the early periods, go to $-\infty$, the buyer faces a choice of whether to wait an extremely large number of periods and incur an extremely large bargaining cost or to buy earlier for an extremely high price. Therefore, the buyer's expected payoff will go to $-\infty$.

An intuitive explanation is provided for why this increase in prices leads to an infinitely high expected payoff for the seller. Take the following alternative strategy by the seller. If the early prices can be increased to infinity, in period 1, the seller can force the buyer to wait a large number of periods till the price gets lower or buy now for a high price. This is because if the buyer concludes that he prefers to not pay the bargaining cost of waiting many periods, the period 1 offer that is better than paying the bargaining cost will be accepted. With infinitely many periods, the total bargaining cost it takes to get to a period with low price can be increased to infinity. With enough periods, the seller can get any high offer accepted in period 1. Since the seller can always choose to deviate to this strategy in a wPBE, the equilibrium strategy payoff is weakly greater.

Next, the following proposition finds pure PBEs when there are infinite periods. Note that instead of wPBEs, the equilibria are specified to be PBEs which is a stronger condition.

Proposition 2.

If $\mathrm{T} = \infty$ and $\mathrm{u_B}$ has no upper bound, for any $\mathrm{p} \in \mathrm{R}^1$, there exists a pure strategy PBE where the sale always happens in period 1 for $\mathrm{p_1} = \mathrm{p}$.

The above proposition states that, under mild conditions, for any price including negative prices, there is a PBE where the buyer always buys and does so in the first period. So, if the proposition holds, there are infinitely many PBEs and any price can be the PBE sale price. The reason can be explicated using a simplified example in which the buyer's bargaining cost is 1. The buyer's strategy involves cutoff prices for which the buyer accepts any offer at this price or below and rejects all offers above this price. Suppose in the first period, the cutoff price is 1. If the seller offers a price greater than 1, the buyer rejects this offer and punishes the seller by lowering the cutoff price to 0 in the second period. If the seller offers a price greater than 0 in the second period, the buyer again rejects this offer and punishes the seller by lowering the cutoff price to −1 in the third period and so on. On the other hand, the seller's strategy is to always make the offer at the current cutoff price and to repeat the last offer if the last offer was at or below that period's cutoff price.

The seller's strategy is optimal because there is no way for the seller to get the buyer to accept a price above the current cutoff price. The buyer's strategy of never accepting an offer above the cutoff price is optimal because by rejecting such offers and punishing the seller in the next period, the buyer can get the seller to succumb to punishment in the next period. Since the seller will make a better offer the next period, the buyer will be sufficiently compensated for rejecting the current offer. The buyer gets no gain from rejecting offers on or below the current cutoff price because when these offers are rejected, the seller offers the same price in the next period. Rejecting those offers will not lower the price. When the game had finite periods, there was no equilibrium like this because in the last period of the game, there was no way for the buyer to punish the seller for the seller's offer in the period.

This proposition is akin to the folk theorem. The similarity is that when there is little or no discounting and infinite periods, punishments become very effective and using punishments, a wide variety of expected payoffs can be supported in equilibria as long as the players' expected payoffs are weakly greater than their expected payoffs from continued punishment.

For the basic model, the cause of equilibria with extreme prices and extremely effective punishments is that players have no ability to terminate bargaining. In reality, for bargaining with finite periods, if a seller asked $1,000 for a pencil and truthfully stated that for the next 1,000 offers, he will insist on unreasonably high prices before making reasonably low offers, the buyer would just leave without buying. However, without this option, the seller has a large bargaining power. This explains why making an opening offer favorable to its proposer is an effective strategy in bargaining experiments without termination. The proposer has a large bargaining power.

For bargaining with infinite periods, if the price offered by the seller is too high and rejecting the offer does not decrease the price, the buyer would also just leave without buying. For this type of bargaining, if the price that the buyer would accept is too low and every time the seller tries to make a higher offer, the buyer responds by punishing the seller, the seller would stop bargaining without a sale. Punishments are limited in their effectiveness in reality.

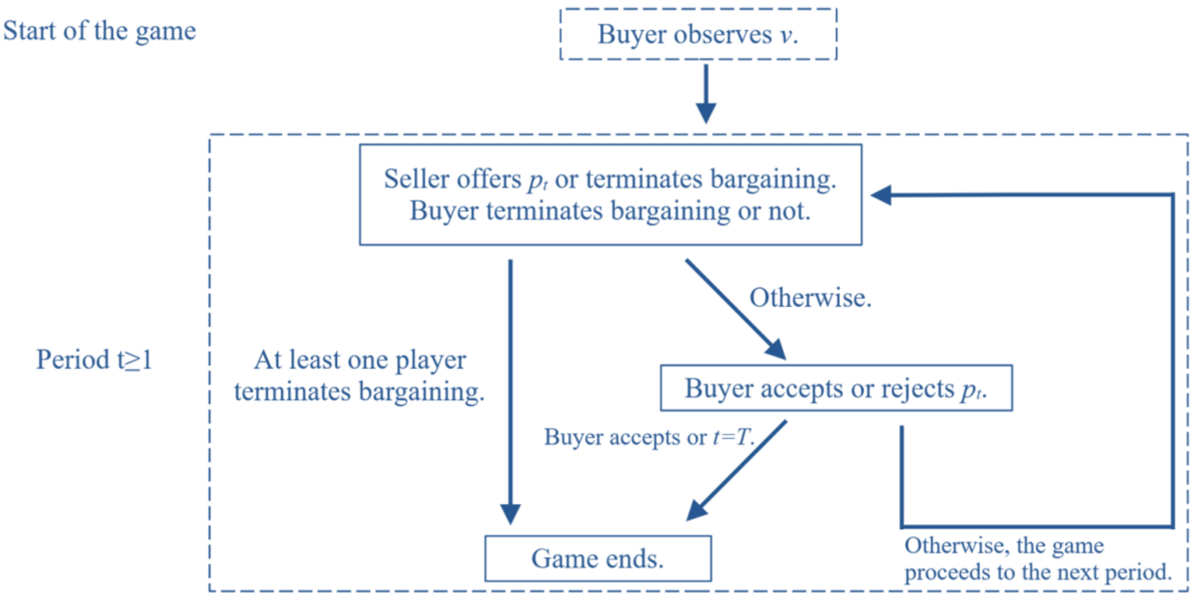
4. Extended Model with Termination

4.1 Specification

For this section's model, the players' decisions are different. At the beginning of a period, the seller does not just offer p_t. Instead, he either offers p_t or terminates bargaining. Simultaneously with this action by the seller, the buyer decides to terminate bargaining or not. Allowing both players to terminate bargaining in any period deals with the issue with the previous section's model. If a player does not terminate bargaining for the period, this is described by saying that the player bargains in the period. If the player does not terminate bargaining in period *1*, the description is that the player bargains.

If at least one player terminates bargaining in period *t*, the game ends in period t. If both players bargain in the period, the buyer sees the price and then decides whether to accept or reject it. As before, if the buyer accepts the offer or $t = T$, the game ends. Otherwise, the game continues to the next period. Figure 2 shows the game tree for this model.

Figure 2: Game tree for the extended model with termination



The players' payoffs for when the game ends with a sale or never ends are the same as before. If the game ends without a sale, the seller's payoff is $-\tau_S c_S$ and the buyer's payoff is $-\tau_B c_B$. τ_S and τ_B are, respectively, the number of periods the buyer and the seller bargained in.

If one player terminates bargaining but the other player bargains for the period, the game still ends without a sale. However, the player who did not terminate still pays the bargaining cost for the period. In other words, if in a period, a player chooses to bargain and pays the associated costs with it but the other player quits, the player bargains in the period in vain.

However, in the period, the seller cannot offer a price without bargaining and the buyer cannot see the price without bargaining. In reality, a party might not know whether the other side has quit bargaining. When a party states that it is unwilling to bargain any further, that statement might not be believed.[9] Thus, a party may attempt to bargain and pay the bargaining cost without knowing that the other side quit. For instance, if a corporate seller exerts effort to make a detailed bargaining proposal but the buyer has already quit bargaining, the seller's effort is futile. Similarly, if the buyer keeps in contact with the seller or stays at the meeting place hoping the seller will make another offer but the seller has quit, the buyer's effort is wasted.

4.2 Lack of Offers in Weak Perfect Bayesian Equilibria

Proposition 3. If u_S and u_B have no lower bound, u_S is increasing and u_B is continuous, in any pure strategy wPBE, players never bargain.

To see why players never bargain, for the finite periods case, consider the following simplified example. Assume that the buyer's bargaining cost is 1. In the last period of the game or a period where both players are set to terminate bargaining in the next period, if the seller's strategy is to offer a price of 2 only buyers whose willingness to pay is 3 or greater will bargain. Others will terminate bargaining. The seller knows this. Therefore, if the buyers' willingness to pay is 3 or greater, the seller knows he can get the buyers to pay 3. So, the seller has an incentive to raise the price to 3. However, if the seller's strategy is to offer a price of 3, only buyers whose willingness to pay is 4 or greater will bargain and the seller can get these buyers to pay 4 and so on. This demonstrates that for any price, the seller has an incentive to raise the price. Therefore, in a wPBE, no price is optimal for the seller. Instead, the seller does not offer a price and the players do not bargain in this period.

Go to the period before. For this earlier period, players will terminate bargaining in the next period. Then, by the same logic, the players will not bargain in this earlier period either. Using backward induction, the players do not bargain in any period.

The proof for the infinite periods case can be described roughly using what prices the seller will offer. Even in a game with infinite periods, there is some lower bound for the seller's price that applies to all wPBE. This is because if the price is too low, the seller would rather terminate bargaining than offer it. Given this lower bound, in a wPBE where the players bargain, the game eventually reaches a period in which the seller will not make any meaningful price cuts. Then in this period, the buyer either bargains or terminates bargaining. If the buyer views and accepts this period's offer, that means the buyer's willingness to pay is equal to or greater than the sum of the bargaining cost for the period and the price. Then, as in the finite periods case, the seller should raise the price right after the buyer decides to bargain for this period. This price raise makes use of the fact that the buyer cannot observe the offer without paying the bargaining cost. The seller's price before the raise is not optimal for him, which is a contradiction. Thus, in a wPBE, the players do not bargain.

In both the finite period model and the infinite period model, the result that both players do not bargain has a fundamental cause. The cause is that the seller recognizes that if the buyer does not terminate, the buyer's value is high. The seller recognizes that the buyer finds the price and the bargaining cost acceptable. Once the buyer enters bargaining for the period, the bargaining cost is sunk cost. The seller can take advantage of this by raising the price by the bargaining cost. The seller uses the bargaining cost to get the buyer to take a deal that the buyer would possibly not have taken if she knew about it from the beginning. The seller's unwavering attempt to raise the price causes the buyer to not bargain[10].

[9] See Ma et al., 2019; Chuang, 2025.

[10] If the number of periods is finite, I believe simply changing the game so that both buyers and sellers make offers would not change the result that both players do not bargain. This is because even in such a model, a similar logic to the one above explaining proposition 3 would still hold.

5. Extended Model with Termination and Value Discovery

5.1 Specification

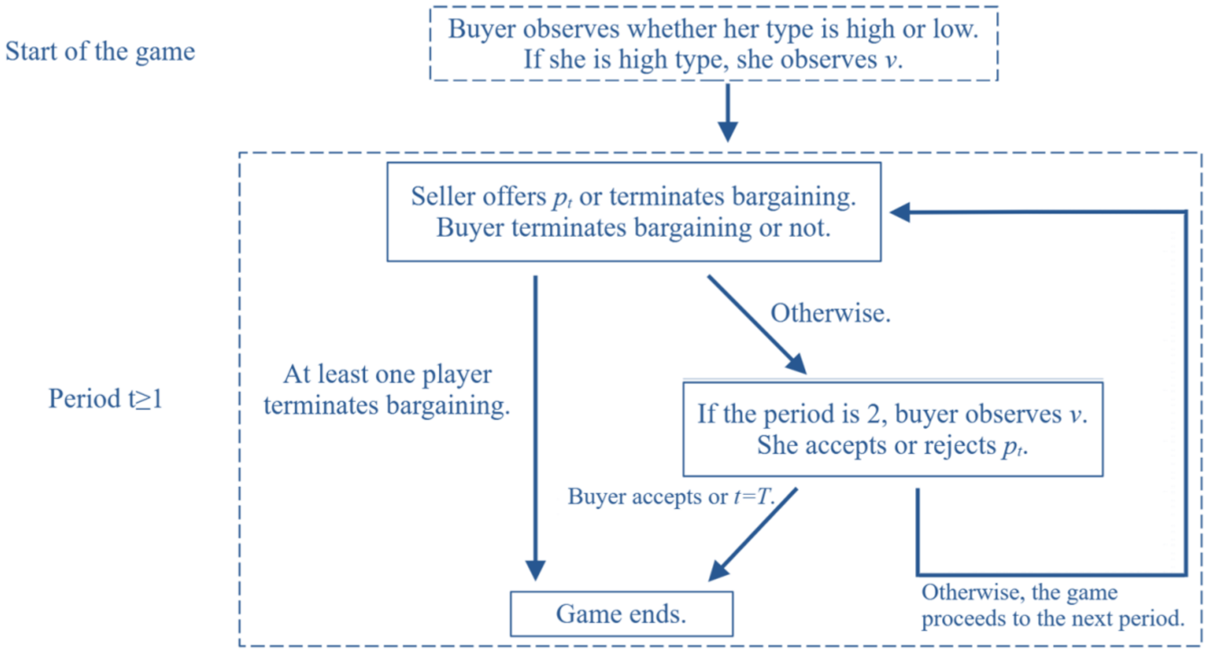
In this section, on top of the previous section's model with bargaining termination, I model the phenomenon where a buyer discovers the value of a good. In the new model, initially, the buyer may not know the value of the good for her when this value is relatively low. In reality, a buyer might be unsure whether the good is worth the cost. When she is unsure, she may try to obtain more information to ascertain the value of the good in bargaining. Then, bargaining would consist of more than just offering prices and would also involve asking questions about the good and getting answers. Through this process, the buyer may find that the good has certain aspects that the buyer does not like.

Obtaining such information would be arduous because the seller would hide it. This argument justifies why the relatively low values are harder to discover in the model. Also, even if the seller gives the buyer information about the good, that does not necessarily mean that the seller knows the buyer's value for the good. For instance, a seller might be reluctant to reveal why the clothing he is selling has been repaired but not know whether that matters to the buyer.

For complicated deals such as labor contract deals, the value discovery may actually involve narrowing down the details of the deal. For instance, instead of just negotiating the wage, the firm and the union may also negotiate overtime pay, sick leave, etc. As these auxiliary conditions are agreed upon through the process of negotiation, the buyer of labor, the firm, also discovers how much surplus it will see from the deal.

The first difference between this section's model and the previous section's model has to do with the buyer's value, v. For this section's model, v is no longer private information because while the seller does not observe v as before, the buyer may not observe it either. The buyer is either low or high type. The buyer is low type with probability P_L in which case, v is chosen from a uniform distribution on [0,1]. The buyer is high type with probability $1 - P_L$ in which case, v is chosen from a uniform distribution on [1,2] (The rest of this paper uses $P_L = \frac{2}{3}$). The buyer observes her type at the start of the game. The high type observes v at the start of the game but the low type does not. In period 2, if both players bargain, the buyer observes v regardless of his type. The new game tree is in Figure 3.

Figure 3. Game tree for the extended model with termination and value discovery



The seller's utility function is $u_S(p_\tau - \gamma) = p_\tau - \gamma$. This is linear. The buyer's utility function is the following:

$$u_B(x) = \begin{cases} \ln(x+1), & \text{if } x \geq 0 \\ -x^2 + x, & \text{otherwise} \end{cases}$$

This function is specifically chosen because it has nice properties that allow me to express the equilibrium prices with simple formulas. (The details on how I use this function to prove the formulas are in appendix 3. While this utility function and the distribution of v are chosen to get the simple formulas, similar equilibria might exist when the buyer has a substantial probability of finding out later that her utility will be meaningfully negative.) This function has a different form for when $x < 0$ because $\lim_{x \to -1} \ln(x+1) = -\infty$. Appendix 3 proves that this function is an increasing, differentiable, strictly concave function. Recall that subsection 2.2 discussed why the buyer's utility had to be strictly concave for reliable and effective price skimming. If $p_\tau = v$, $u_B(p_\tau - v) = 0$. In other words, when the value of the good equals the price of purchase, the buyer's utility is 0.

The game has more than 1 period. It can have infinitely many periods. p_t^* is the price that the seller offers in period t on the equilibrium path. p_t^{**} is the price that the seller offers in period t if the buyer deviated in period t−1 by not buying and this was the only deviation before period t.

When the buyer's value of the good is relatively low, the buyer does not know exactly how low it is. The value could be so low that the buyer does not want to buy the good. By bargaining and talking to the seller, the buyer can extract information about the good from the seller that gives the exact value of the good. However, this process is costly. Note that if the buyer plays a pure strategy, any low type buyer who has not observed v always plays the same action in the same situation.

5.2 Results

The equilibria of the model for this section can differ in the maximum number of periods bargaining takes. Furthermore, as will be shown below, even for the same set of parameters, there can be multiple PBEs with different prices. Therefore, in this subsection, instead of solutions for every PBE of the model, examples 1 and 2 which have representative PBEs are presented. Example 1 demonstrates price skimming and how the buyer stops the seller from raising the prices with her ability to terminate bargaining. Example 2 demonstrates how the seller can lower the period 1 price to have the buyer buy in period 1 and prevent the low type buyer from not buying after discovering her low value.

Definition 1. The following are two settings.

1. $c_B = 0.01$ and $p_1 \in [0.7, 0.71]$.
2. $c_B \in [0.01, 0.02]$ and $p_1 = 0.7$.

The following example looks at PBEs with price skimming under settings 1 and 2.

Example 1. In both of the settings 1 and 2, if $\gamma = 0.01$ and $c_S = 0.005$, there exists a pure strategy PBE where the following holds:

1. $p_1^* > p_2^* = \frac{e^{c_B} p_1^* + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3}$
2. $2 < \frac{e^{c_B} p_1^* - p_2^*}{e^{c_B} - 1} < 3$
3. High type buys in period 1 if $v \geq \frac{e^{c_B} p_1^* - p_2^*}{e^{c_B} - 1} - 1$ and buys in period 2 if $v < \frac{e^{c_B} p_1^* - p_2^*}{e^{c_B} - 1} - 1$.
4. Low type bargains in period 2.
5. Probability that lows type buys in period 2 is positive.
6. If period 3 exists, both players' strategies in period 3 is to terminate bargaining regardless of previous history.

In the above example, (1) means that the equilibrium price decreases in period 2. The high type buys in period 1 if $v \geq \frac{e^{c_B} p_1^* - p_2^*}{e^{c_B} - 1} - 1$ and buys in period 2 if $v < \frac{e^{c_B} p_1^* - p_2^*}{e^{c_B} - 1} - 1$. The low type buys in period 2 or does not buy. This means that the seller engages in price skimming advantageously by lowering the price in the second period and getting the buyers with high value buy early and the buyers with low value buy late. (4) and (5) mean that the low type buyer discovers her value of the good through bargaining in period 2 and based on that, may decide to buy. In (6), I limit bargaining to two periods on the equilibrium path for simplicity. (1) indicates that once p_1^*, the period 1 price on the equilibrium path, is given, p_2^* is known from it. However, as will be explained in detail, the value of p_1^* can vary for different PBEs even for the same parameters. Figure 4 depicts PBEs from example 1 and setting 1[11]. For the parameters here, for any $p_1 \in [0.7, 0.71]$, there exists some PBE where $p_1^* > p_2^* = \frac{e^{c_B} p_1^* + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3}$.

Figure 4. p_1^* and PBEs

The high type buyer's strategy is optimal because of her risk aversion and value. In period 1, she knows that the price will be lower in the next period. However, whether she will wait for the lower price depends on her value. Since she is risk averse, if her value is high, her utility from purchase is already high and she sees little benefit from waiting for a lower price. Therefore, she buys in period 1. She only buys in period 2 when her value and her utility from period 1 purchase are low.

[11] Lemma 8 in appendix 3 proves the characteristics of these PBEs.

If a low type buyer buys in period 1, she does so without knowing the value of the good. This is because it takes two periods of bargaining for a low type buyer to find her exact value of the good. Buying in period 1 means that she might get negative utility from the good if her value is actually low. Therefore, she does not want to purchase in period 1. In period 2, after she decides to bargain, she sees her value of the good. Then, she buys the good if the value is weakly greater than the price. She does not buy if the value is less than the price.

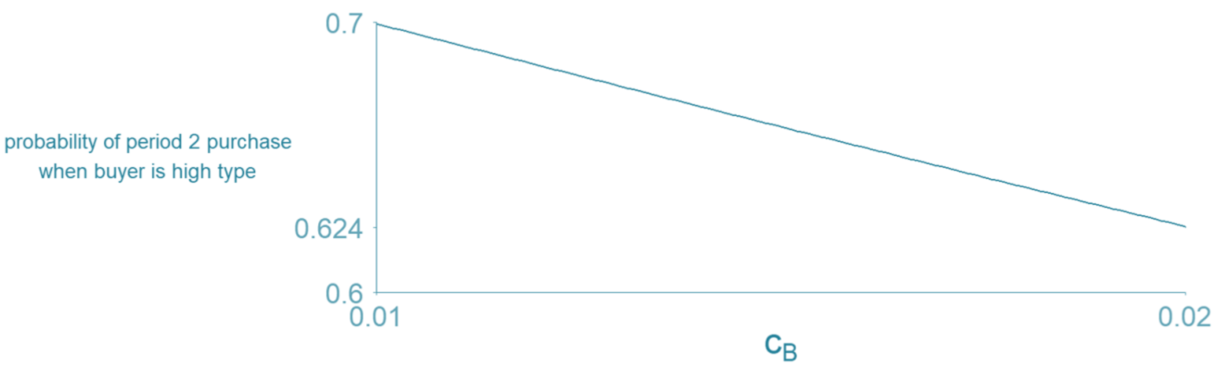
In period 2, the seller does not want to increase the price further because increasing the price means losing more buyers. The low type buyer might not buy in period 2 if after bargaining in the period, she discovers that her value is unexpectedly low and her utility is negative. The seller finds the optimal price by weighing the risk of losing the buyer with low value against the benefit of a higher price. This is different from section 4's model where the seller had no cost of raising the price by a small amount from the expected price both in the last period and the period after which both players terminate bargaining. The difference explains why players bargain in the PBEs of example 1.

The seller finds it optimal to have a higher price in period 1 compared to period 2. This is because in period 1, the high type is the only type that might buy. In period 2, the low type might buy as well and the low type requires a lower price for purchase. As Figure 4's first graph shows, if p_1*is greater, the high type buyer is more likely to delay purchase. Then, in period 2, the seller expects the buyer to have a higher value of the good. Therefore, as p_1*increases, p_2*increases. This is shown in Figure 4's second graph.

Figure 4's third graph shows that the seller's payoff is higher when $p_1^* = 0.71$ compared to when $p_1^* = 0.7$. This raises the question of why he does not always offer $p_1 = 0.71$. The reason is that in the different PBEs with different values of $p_1^* \in [0.7, 0.71]$, the buyer's strategy is different. In the PBE with $p_1^* = 0.7$, if the seller raises the price to $p_1 > 0.7$, the buyer interprets this deviation to mean that the seller will terminate bargaining in the next period and the low type buyer terminates in the next period. In the PBE with $p_1^* = 0.71$, the same thing happens for $p_1 > 0.71$. In other words, for the PBEs with $p_1^* = 0.7$ and $p_1^* = 0.71$, there is a difference in how high p_1 has to be for the low type buyer to terminate bargaining. This demonstrates that the buyer's ability and willingness to terminate bargaining gives her bargaining power. If the seller knows that a too high p_1 will drive away the buyer, the seller does not offer such a high p_1. Low type buyer's strategy to give up on bargaining when p_1 is too high is optimal if the seller will do so as well.

Figure 5 shows PBEs from example 1 and setting 2. As the buyer's bargaining cost, c_B, increases, the high type buyer becomes more likely to buy in period 1 instead of period 2. The low type always bargains in period 2[12].

Figure 5. Buyer's cost of bargaining and purchase delay in PBEs



Now, an example PBE where there is no price skimming. Example 2, is showed.

$\gamma = 0;$ $c_S = 0.25;$ $c_B = 0.01.$

[12] Lemma 8 in appendix 3 proves the characteristics of these PBEs.

In this setting, there exists a PBE where $p_1^* \approx 0.35$, $p_2^{**} = 0.5$ and the buyer always buys in period 1. p_1^* is the period 1 price on the equilibrium path. p_2^{**} is the price the seller offers in period 2 if the buyer deviated by rejecting p_1^* and there was no other deviation in period 1.

For the PBE above, the sale is always made and it is made in period 1. The seller knows that if he bargains with the low type buyer in period 2, the low type buyer might find out that her value for the good is near 0. Then, she might decide to never buy or ask for a low price. Given the seller's high bargaining cost, he chooses to avoid this by offering a low price in period 1. This is how he has the sale always happen in period 1. Note that in the PBE of this example, the buyer never actually discovers her value because the bargaining ends before that. However, unlike section 4's model, the example's PBE still have the player's bargain. This proves that the probability of the value discovery does not have to be positive for a PBE where players bargain. The potential for the discovery can be enough. This is because the potential may provide enough incentive for the seller to not raise the price.

5.3 Effective Bargaining Strategies

In this section's model, the buyer's ability to keep the seller from raising the price can come from two possibilities. The first is that if the price is too high, the buyer might think that no acceptable deal is possible and terminate bargaining. The second is that through bargaining, the buyer might find out that her value is low and reject the seller's offer.

To raise the first possibility, the buyer should create the impression that if the offer is too high, she will think that there is no hope of getting an acceptable deal and she will never buy the good. For instance, the buyer could state that she is "just looking around" and ready to leave if she cannot get a low price. To raise the second possibility, the buyer should be inquisitive. She should find out as much as she can about the good through bargaining and then stress that because of the downsides of the good that she found, she cannot buy at a high price. However, this strategy should not be employed to the extent that the seller gives up trying to sell the good.

The seller can counter these strategies to increase his expected payoff. This happens in the equilibria shown in section 5. To counter the first strategy, in making offers, the seller should not just consider the possibility that the buyer will reject the offer and the seller has to make a new offer. He should also refrain from making offers that cause the buyer to give up on bargaining and never buy. A common strategy for negotiations is to make an opening offer extremely favorable for the person making the offer.[13] Some experiments have found that in bargaining, an opening offer favorable to the proposer leads to a better deal for the proposer.[14] However, in many of these studies, subjects were not given the option to terminate the bargaining and the experiment. Even if subjects were given the option to terminate, they may not have wanted to because they were interested in the experiment or they believed it was not what the experimenters wanted. In reality, an extreme opening offer may lead the other side to believe that there is no chance of a good deal and terminate bargaining.

To counter the second strategy, the seller can use the following strategy. If the probability that the buyer will find out something she doesn't like about the good through bargaining and reject the buyer's offer is too high, the seller can reduce this possibility and bargaining costs with a limited-time discount. In other words, in such situations, the seller should say "If you buy this now, I will give you a discount." This is consistent with the seller's low equilibrium price in example 2. This is similar to how, in reality, limited-time discounts such as Black Friday deals may encourage consumers to buy on impulse without carefully considering the benefits and the costs of a good.

---

[13] See Volkema, 1999.

[14] See Chertkoff & Conley, 1967; Liebert et al., 1968; Yukl, 1974; Galinsky & Mussweiler, 2001. Galinsky & Mussweiler (2001, 665) states that "the first offer, once made, appears to function as an anchor toward which final agreements are assimilated."

6. Discussion

6.1. Testing the Predictions of the Models

The models of this paper make four main predictions for a bargaining game where only the seller makes offers. All can be tested by asking subjects to repeatedly bargain with the chance of winning actual money or goods. Subjects should be informed about the random distribution of the buyer's value or break-even point. Bargaining costs can be implemented by reducing the research participation payment for longer bargaining. Since subjects may take time to figure out how to play the game well, PBE behaviors should be compared against subjects' behaviors after they played the game many times.

The first two predictions are from the basic model. Following subsection 2.2, the first prediction is that reliable and useful price skimming requires risk aversion of the buyer. This can be tested by bargaining games where the seller has the option to just post a price instead of bargaining. The researcher can measure subjects' risk preferences and if there is a lack of risk-seeking subjects, risk-seeking behavior can be induced by telling buyers that if they "win" bargaining games, they will receive an extra payment at the end of the test. The sellers should be told about the risk preferences of the buyers. In equilibrium, the sellers will post prices for risk-seeking buyers. Risk-avoiding buyers may buy in later periods. If price skimming is mandated for sellers and the buyer is risk-seeking, the buyer will buy early when her value is low and buy later when her value is high.

Secondly, similar to proposition 1, when bargaining termination is not allowed, if the number of bargaining periods increases, the sale price should increase. Furthermore, since this increase in the price is driven by the buyer's bargaining cost, increasing this bargaining cost should increase the sale price as well. This can be tested with games with different numbers of periods and different bargaining costs for the buyer. Subjects should be told that they cannot quit bargaining in the middle.

The extended models predict that when people can terminate bargaining, without value discovery, buyers and sellers would choose to not bargain. However, if value discovery can also be made, deals can be secured. These are the third and fourth predictions. For testing these, subjects should be told that while they cannot quit testing till the allotted time is over, whenever they are matched with a bargaining partner, they can refuse to bargain with the partner or terminate bargaining with the partner in the middle. (To avoid demand characteristics where subjects feel that they should bargain because they received a research participation payment, the participants can be told that they should bargain with at least three subjects but afterwards, they can refuse all bargains.)

In proposition 3, the equilibrium strategy is to not bargain and this is because of the seller's incentive to raise the price. In reality, subjects may take time to figure out this equilibrium. The proof of this result relies on the fact that in the last period of bargaining, the subjects would choose to terminate bargaining because for the buyer, playing in the last period is not worth the buyer's bargaining cost.

If the game subjects play has only one period, it may be easy to see that in the last period, the subjects should not bargain. However, with a larger number of periods, the subjects may find it more difficult to reach the same conclusion. By varying the number of periods, the researcher can test under which conditions the subjects reach the conclusion that they should not bargain. Furthermore, if the bargaining cost is high, the buyer may feel more strongly that trying to get a deal is not worth the bargaining cost. Therefore, subjects should be tested with different levels of bargaining costs. The researcher should also see if the sellers gradually come to the conclusion that they can charge more than the buyer's value in the last period as the proof of proposition 3 in Appendix 2 states.

Finally, to test the prediction that when the buyer can discover the value, players may reach a deal, there should be a condition where the seller is given some information that might lower the value of the good for the buyer. The buyer is told that the seller knows this but not the information itself and the players are allowed to communicate on this information. The researcher should analyze how this condition changes the probability of a deal and bargaining termination.

6.2 Behavioral Economics Explanations

Section 5's model cannot explain bargaining for goods whose values are not revealed through bargaining. If there are real cases where people bargain for such goods, alternative explanations have to be found. Four possible explanations that use behavior economics and no longer assume that players maximize their payoffs are considered. Behavior economics incorporates insights from psychology to explain irrational economic behavior.

Spite, Fairness Concerns and The Sunk Cost Fallacy

The first two behavioral economic explanations of bargaining involve rejection of unfavorable deals by the buyer, which creates disincentives that apply to the seller for raising the price. Recall that in section 4, the reason that players did not bargain was that given the bargaining cost already paid by the buyer, the seller could raise the price by a small amount and still have the buyer buy.

However, players may have spite or fairness concerns, which is the first explanation[15]. Spite and fairness concerns are consequential for games of the basic model and section 4's model. If the buyer rejects the offers unfavorable to her for spite at the offer or being unfair, unlike proposition 1 for the basic model, the seller cannot raise prices without limit as the number of periods increases. In section 4's model, if the buyer will reject the offer because of the same reasons when the seller increases the price, the seller would not raise the price too much. Then, a sale might be made. However, what causes spite or what people consider unfair in reality for seller-buyer bargaining where the total surplus is uncertain is a complex issue.

The second explanation involves the sunk cost fallacy. The seller can raise the price for buyers who paid the bargaining cost because the buyer does not consider the sunk bargaining cost when she decides whether to accept a deal. If the buyer exhibits the sunk cost fallacy, the buyer would reject such price increases and the seller would not raise prices in this manner. However, introducing the sunk cost fallacy creates a new problem for bargaining. If both players consider the total sunk cost they paid for bargaining, both players may refuse to accept a deal that does not compensate them for their total sunk costs. In this case, price skimming may become impossible because after bargaining costs have accumulated, there will be no deal that both sides accept.

Present Bias and Myopia

The last two explanations apply to bargaining with multiple offers over time. For most circumstances, the benefit from a good or money would not differ by a nontrivial amount depending on whether one gets the good now or a few minutes later. However, people may exhibit present bias. They might want to get the good or the money now and irrationally discount future gains even in such circumstances[16]. In such cases, models with rational discounting may better explain bargaining because people's mental calculations are more similar to rational discounting compared to accounting for fixed bargaining costs.

However, present bias cannot explain bargaining for which the time that the deal takes effect is not affected by when the agreement is made. In the introduction, there were two such examples. One was a case where the buyer could only receive the good after months. The other was a labor contract negotiation that would only take effect in a few months. For these cases, even if present-biased people want the benefit of the deal now, they should not discount a later deal because it would not change when they actually get the benefit.

If people also discount later deals in such situations, this can be explained by myopia. Myopic people fail to accurately judge what will happen in the future.[17] In this case, people would care more about the deal they can get now and irrationally discount the deal they can get in the future even if for both deals, the time at which they see

[15] Radulović & Mojasevic (2021), Forsythe et al. (1994).

[16] Chakraborty (2021).

[17] Gabaix & Laibson (2017, 4) states "But myopia also means a "lack of foresight or discernment". Such forecasting limitations matter when agents need to judge the value of events that will occur at a temporal distance".

the benefit from the deal is the same. However, such myopia would be harder to justify and verify compared to present bias.

Conclusion

This paper explains bargaining with incomplete information about the buyer when there is no discounting but fixed bargaining costs exist. For such bargaining games, the seller can only advantageously and reliably employ price skimming when the buyer is risk averse. The paper has three models for such games. For the basic model with finite periods, as the number of periods goes to infinity, in wPBEs, prices and the seller's expected payoff go to infinity as well and the buyer's expected payoff goes to negative infinity. This explains why bargaining experiments without bargaining termination find that an opening offer favorable to its proposer is advantageous. If the game has infinite periods, for any price, there exists a PBE with that as the sale price.

For the extended model with termination, in the pure strategy wPBE, players terminate bargaining as soon as possible without a deal. In the extended model with termination and value discovery, I find PBEs with price skimming and a PBE without it. In the PBEs with price skimming, the first price can differ depending on what price causes the low type buyer to not bargain in the future. The seller does not offer a price so high that this happens. In the PBE without price skimming, the seller offers a low price in the first period so that the sale is always made and it is made in the first period. This strategy works to preclude the possibility that the buyer discovers the exact value of the good and consequently does not buy the good.

Authorship Contribution Statement

Hwang, J. is the sole author of the paper. The author was responsible for the conception of the research idea, development of the theoretical model, formal analysis, derivation of results, and preparation of the manuscript. All work, including literature review, mathematical proofs, and writing, was completed independently by the author.

Acknowledgments/Funding

Joongsan Hwang's advisor, Professor Jin-Hyuk Kim at the University of Colorado Boulder, provided valuable advice for this paper.

Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Data Availability Statement

This study does not analyse data. Data sharing is not applicable to this article.

References

Abreu, D., & Manea, M. (2024). Bargaining and exclusion with multiple buyers. *Econometrica*, 92(2), 429–465. https://doi.org/10.3982/ecta19675

Apostol, T. M. (1985). *Mathematical Analysis.* Narosa Publishing House. ISBN: 978-8185015668

Ausubel, L. M., & Deneckere, R. J. (1989). Reputation in bargaining and durable goods monopoly. *Econometrica*, 57(3), 511–531. https://doi.org/10.2307/1911050

Avriel, M., Diewert, W. E., Schaible, S., & Zang, I. (2010). Generalized concavity. Society for Industrial and Applied Mathematics. ISBN: 978-0-898718-96-6

Chakraborty, A. (2021). Present bias. *Econometrica*, 89(4), 1921–1961. https://doi.org/10.3982/ecta16467

Chang, D., & Lee, J. J. (2022). Price skimming: Commitment and delay in bargaining with outside option. *Journal of Economic Theory*, 205, 105528. https://doi.org/10.1016/j.jet.2022.105528

Chertkoff, J. M., & Conley, M. (1967). Opening offer and frequency of concession as bargaining strategies. *Journal of Personality and Social Psychology,* 7(2), 181–185. https://doi.org/10.1037/h0024997

Chuang, T. (2025). King Soopers and grocery workers end strike for 100 days as bargaining resumes. Retrieved March 22, 2025, from https://coloradosun.com/2025/02/18/king-soopers-grocery-workers-end-strike/

Coase, R. H. (1972). Durability and monopoly. *The Journal of Law and Economics*, 15(1), 143–149. https://doi.org/10.1086/466731

Cramton, P. C. (1984). Bargaining with incomplete information: An infinite-horizon model with two-sided uncertainty. *The Review of Economic Studies,* 51(4), 579–593. https://doi.org/10.2307/2297780

DePamphilis, D. (2015). Mergers, acquisitions, and other restructuring activities. Academic Press. ISBN: 978-0128016091

Dickinson, D. L. (2003). Illustrated examples of the effects of risk preferences and expectations on bargaining outcomes. *The Journal of Economic Education,* 34(2), 169–180. https://doi.org/10.1080/00220480309595210

Dilmé, F. (2025). Bargaining with binary private information. *Games and Economic Behavior*, 152, 423–442. https://doi.org/10.1016/j.geb.2025.05.008

Forsythe, R., Horowitz, J. L., Savin, N. E., & Sefton, M. (1994). Fairness in simple bargaining experiments. *Games and Economic Behavior,* 6(3), 347–369. https://doi.org/10.1006/game.1994.1021

Fudenberg, D., & Tirole, J. (1983). Sequential bargaining with incomplete information. *The Review of Economic Studies*, 50(2), 221–247. https://doi.org/10.2307/2297414

Fudenberg, D., & Tirole, J. (2005). Game theory. Ane Books Pvt. Ltd. ISBN: 978-8180520822

Gabaix, X., & Laibson, D. (2017). Myopia and discounting. Working paper. National Bureau of Economic Research. https://www.nber.org/system/files/working_papers/w23254/w23254.pdf (Accessed: 21 July 2025).

Galinsky, A. D., & Mussweiler, T. (2001). First offers as anchors: The role of perspective-taking and negotiator focus. *Journal of Personality and Social Psychology,* 81(4), 657–669. https://doi.org/10.1037//0022-3514.81.4.657

Groseclose, T. (2024). The coase conjecture when the monopolist and customers have different discount rates. *Review of Industrial Organization,* 66, 349–365. https://doi.org/10.1007/s11151-024-09990-w

Harris, C. (1985). An alternative solution to Rubinstein's model of sequential bargaining under incomplete information. *The Economic Journal,* 95, 102–112. https://doi.org/10.2307/2232874

Kambe, S. (2025a). The prevalence of take-it-or-leave-it offers. *Games and Economic Behavior*, 151, 42–58. https://doi.org/10.1016/j.geb.2025.02.010

Kambe, S. (2025b). The weaker player's option to exit as a source of bargaining power in bilateral bargaining with fixed costs. *Economics Letters,* 247, 112090. https://doi.org/10.1016/j.econlet.2024.112090

Karagözoğlu, E., & Rachmilevitch, S. (2021). Costly preparations in bargaining. *The Scandinavian Journal of Economics*, 123(2), 532-557. https://doi.org/10.2307/1913153

Liebert, R. M., Smith, W. P., Hill, J. H., & Keiffer, M. (1968). The effects of information and magnitude of initial offer on interpersonal negotiation. *Journal of Experimental Social Psychology*, 4(4), 431–441. https://doi.org/10.1016/0022-1031(68)90068-1

Ma, A., Yang, Y., & Savani, K. (2019). "Take it or leave it!" A choice mindset leads to greater persistence and better outcomes in negotiations. *Organizational Behavior and Human Decision Processes*, 153, 1–12. https://doi.org/10.1016/j.obhdp.2019.05.003

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). Microeconomic theory. Oxford University Press. ISBN: 978-0195073409

Özyurt, S. (2023). Take-it-or-leave-it offers in negotiations: Behavioral types and endogenous deadlines. *Journal of Economic Psychology*, 95, 102588. https://doi.org/10.1016/j.joep.2022.102588

Perry, M. (1986). An example of price formation in bilateral situations: A bargaining model with incomplete information. *Econometrica*, 54(2), 313–321. https://doi.org/10.2307/1913153

Ponsatí, C., & Sákovics, J. (1998). Rubinstein bargaining with two-sided outside options. *Economic Theory*, 11(3), 667–672. https://doi.org/10.1007/s001990050208

Porter, D., & Rosenthal, J.-L. (1989). The scope of bargaining failures with complete information. UCLA economics working papers 564. UCLA Department of Economics. https://core.ac.uk/download/pdf/7283136.pdf (Accessed: 21 July 2025).

Radulović, B., & Mojasevic, A. (2021). Spite and strong reciprocity in the bargaining game: An experimental study. *Teme*, 45(3), 1041–1056. https://doi.org/10.22190/TEME200819061M

Roth, A. E. (1985). A note on risk aversion in a perfect equilibrium model of bargaining. *Econometrica*, 53(1), 207–211. https://doi.org/10.2307/1911733

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50(1), 97–109. https://doi.org/10.2307/1912531

Rubinstein, A. (1985). A bargaining model with incomplete information about time preferences. *Econometrica*, 53(5), 1151–1172. https://doi.org/10.2307/1911016

Sanburn, J. (2012, July 3). One company will soon control half of the US Beer market. https://business.time.com/2012/07/03/one-company-will-soon-control-half-of-the-u-s-beer-market/

Schweighofer-Kodritsch, S. (2022). The bargaining trap. Games and Economic *Behavior*, 136, 249–254. https://doi.org/10.1016/j.geb.2022.09.006

Shaked, A. (1994). Opting out: Bazaars versus 'Hi Tech'markets. *Investigaciones Economicas*, 18(3), 421–432.

Sobel, J., & Takahashi, I. (1983). A multistage model of bargaining. *The Review of Economic Studies*, 50(3), 411–426. https://doi.org/10.2307/2297673

Sorkin, A. R. (2008, June 26). Anheuser to reject InBev offer. Retrieved February 15, 2025, from https://www.nytimes.com/2008/06/26/business/26busch.html

Volkema, R. J. (1999). *The negotiation toolkit: How to get exactly what you want in any business or personal situation.* Amacom Books. ISBN: 978-0814480083

Yoshida, M. (2025). Using a soft deadline to counter monopoly. *The Journal of Industrial Economics*, 73(3), 446–457. https://doi.org/10.1111/joie.12416

Yukl, G. (1974). Effects of the opponent's initial offer, concession magnitude and concession frequency on bargaining behavior. *Journal of Personality and Social Psychology*, 30(3), 323–335. https://doi.org/10.1037/h0036895

Zhang, J., & Chiang, W. Y. K. (2020). Durable goods pricing with reference price effects. Omega, 91, 102018. https://doi.org/10.1016/j.omega.2018.12.007

APPENDIX

1. Lemmas and Proofs for Section 3

Lemma 1 is used to prove lemma 2 and propositions 1 and 3. Lemma 2 is used to prove proposition 1.

Lemma 1. Suppose $\psi' > \psi$ and $p' > p$.

1. If u_B is strictly concave (strictly convex), $u_B(\psi' - p) - u_B(\psi' - p') < (>) u_B(\psi - p) - u_B(\psi - p')$.
2. If u_B is concave (convex), $u_B(\psi' - p) - u_B(\psi' - p') \leq (\geq) u_B(\psi - p) - u_B(\psi - p')$.

Proof. Let $x = \psi - p'$, $y = \psi' - p$ and $z = p' - p$.

$y - x = \psi' - \psi + p' - p > z = p' - p > 0$.

$y - x > z > 0$ means that if u_B is (strictly) concave,

$$\frac{y-x-z}{y-x} u_B(x) + \frac{z}{y-x} u_B(y) \leq (<) u_B\left(x\frac{y-x-z}{y-x} + y\frac{z}{y-x}\right) = u_B(x+z)$$

and,

$$\frac{z}{y-x} u_B(x) + \frac{y-x-z}{y-x} u_B(y) \leq (<) u_B\left(x\frac{z}{y-x} + y\frac{y-x-z}{y-x}\right) = u_B(y-z).$$

Therefore, $u_B(x) + u_B(y) \leq (<) u_B(y-z) + u_B(x+z)$. Similarly, if u_B is (strictly) convex, $u_B(x) + u_B(y) \geq (>) u_B(y-z) + u_B(x+z)$.

$u_B(y-z) + u_B(x+z) \gtreqless u_B(x) + u_B(y) \leftrightarrow u_B(\psi' - p') + u_B(\psi - p) \gtreqless u_B(\psi - p') + u_B(\psi' - p) \leftrightarrow u_B(\psi - p) - u_B(\psi - p') \gtreqless u_B(\psi' - p) - u_B(\psi' - p')$.

Definition 2.

$\underline{v} \equiv \inf V$;

$\bar{v} \equiv \sup V$.

$\overline{V}$ is the closure of V.

Lemma 2. If $t < t'$, u_B is continuous and $\forall v' \in \overline{V}, u_B(v' - p_t) - tc_B > u_B(v' - p) - t'c_B$, then $\exists \epsilon > 0, \forall v' \in \overline{V}, u_B(v' - p_t - \epsilon) - tc_B > u_B(v' - p) - t'c_B$.

Proof.

$0 > tc_B - t'c_B$.
If $p_t < p$,
$\forall v' \in \overline{V}, u_B(v' - p_t) - u_B(v' - p) \geq 0$.
If $p_t + \epsilon = p$,
$\forall v' \in \overline{V}, u_B(v' - p_t - \epsilon) - u_B(v' - p) = 0$.
The $p_t < p$ case is proven.

Suppose $p_t \geq p$. If u_B is concave, $\check{v} = \underline{v}$. If u_B is convex, $\check{v} = \bar{v}$. By lemma 1's (2), when $\epsilon > 0$,
$\forall v' \in \overline{V}, u_B(\check{v} - p) - u_B(\check{v} - p_t - \epsilon) \geq u_B(v' - p) - u_B(v' - p_t - \epsilon)$

and,

$\forall v' \in \overline{V}, u_B(v' - p_t - \epsilon) - u_B(v' - p) \geq u_B(\check{v} - p_t - \epsilon) - u_B(\check{v} - p)$.
$u_B(\check{v} - p_t) - tc_B > u_B(\check{v} - p) - t'c_B \leftrightarrow u_B(\check{v} - p_t) - u_B(\check{v} - p) > (t - t')c_B \leftrightarrow \exists \epsilon > 0, u_B(\check{v} - p_t - \epsilon) - u_B(\check{v} - p) > (t - t')c_B$.

For this ϵ,

$\forall v' \in \overline{V}, u_B(v' - p_t - \epsilon) - u_B(v' - p) > (t - t')c_B$.

Proof of Proposition 1.

The first part of this proof establishes that for any t, p and history of the game before period t, there exists some T' for which if $T' < T$ and $T < \infty$, $P(p_t > p) = 1$. In period T, if $u_B(\underline{v} - p_T) > 0$ and $u_S(p_T - \gamma) < 0$, any buyer buys and seller prefers to offer p'_T for which $u_S(p'_T - \gamma) \geq 0$. In an wPBE, for any history before period T, in period T, $P(u_B(\underline{v} - p_T) > 0 \text{ and } u_S(p_T - \gamma) < 0) = 0$.

Pick p_L and $p'_L > p_L$ such that $u_B(\underline{v} - p'_L) > 0$ and $u_S(p'_L - \gamma) < 0$. $\epsilon = p'_L - p_L$. In period t, $P(p_t > p_L + 0.5^{T-t+1}\epsilon) = 1$. Then, in an wPBE, in any period t, for any history before period t, $P(p_t > p_L) = 1$.

Pick $\underline{T} \geq 1$ and $T \geq \underline{T}$ such that $u_B(\underline{v} - p_L) - \underline{T}c_B \succ -Tc_B$. Assume a wPBE with T periods and consider constants, $\underline{p_1}, \cdots, \underline{p_T}$.

Let $\underline{p_{\underline{T}}} = p_L$. In this wPBE, for any history before period t, $P\left(p_{\underline{T}} \geq \underline{p_{\underline{T}}}\right) = 1$.

Pick $t < \underline{T}$. Suppose that for all $t' \in (t, \underline{T}]$, $P\left(p_t \geq \underline{p_t}\right) = 1$ for any history before period t. If, by the seller's action,

$\forall v' \in \overline{V}$ and $t' \in (t, \underline{T}]$, $u_B(v' - p_t) - u_B\left(v' - \underline{p_{t'}}\right) > (t - t')c_B$, then,

$\forall v' \in \overline{V}$ and $t' \in (t, \underline{T}]$, $u_B(v' - p_t) - tc_B > u_B\left(v' - \underline{p_{t'}}\right) - t'c_B$.

Then, by lemma 2, seller's action is suboptimal because there exists some $\epsilon' > 0$ by which seller can raise p_t and have the buyer buy in period t. Therefore, if p_t is optimal, for some $t' \in (t, \underline{T}]$,

$$\exists v' \in \overline{V}, u_B(v' - p_t) - u_B\left(v' - \underline{p_{t'}}\right) \leq (t - t')c_B.$$

$$S^* \equiv \{p_t | \exists v' \in \overline{V}, u_B(v' - p_t) \leq u_B\left(v' - \underline{p_{t'}}\right) + (t - t')c_B\}$$

By the least-upper-bound property, there exists some $\check{p}$ such that if $p_t < \check{p}$, $p_t \notin S^*$ and if $p_t > \check{p}$, $p_t \in S^*$. $P(p_t \geq \check{p}) = 1$ for any history before period t. By the continuity of u_B,

$$\exists v' \in \overline{V}, u_B(v' - \check{p}) - u_B\left(v' - \underline{p_{t'}}\right) \leq (t - t')c_B.$$

Set $\underline{p_t} = \check{p}$. This way I set $\underline{p_t}$ for any period $t < \underline{T}$. For any $t \leq \underline{T}$ and any history before t, $P\left(p_t \geq \underline{p_t}\right) = 1$.

For any period $t < \underline{T}$, there exists some $t' \in (t, \underline{T}]$ such that:

$$\exists v' \in \overline{V}, u_B\left(v' - \underline{p_t}\right) - u_B\left(v' - \underline{p_{t'}}\right) \leq (t - t')c_B,$$

and $\underline{p_t} > \underline{p_{t'}}$.

With this intermediate result, I will use proof by induction. Suppose u_B is concave. $\check{v}$ is a function whose codomain is $\overline{V}$. $\check{v}(\underline{T}, \underline{T}) = \bar{v} \in \overline{V}$.

$$\forall v' \leq \check{v}(\underline{T}, \underline{T}), u_B\left(v' - \underline{p_{\underline{T}}}\right) - u_B\left(v' - \underline{p_{\underline{T}}}\right) \leq (\underline{T} - \underline{T})c_B.$$

Now, will prove for $T'' < \underline{T}$. Suppose that for all $T' \in (T'', \underline{T}]$,

$$\check{v}(T', \underline{T}) \in \overline{V}$$

and,

$$\forall v' \leq \check{v}(T', \underline{T}), u_B\left(v' - \underline{p_{T'}}\right) - u_B\left(v' - \underline{p_{\underline{T}}}\right) \leq (T' - \underline{T})c_B.$$

For some $T' \in (T'', \underline{T}]$,

$$\exists v' \in \overline{V}, u_B\left(v' - \underline{p_{T''}}\right) - u_B\left(v' - \underline{p_{T'}}\right) \leq (T'' - T')c_B.$$

Fix this v' as $\dot{v}$. By lemma 1's (2),

$\forall v'' \leq \dot{v}, u_B\left(v'' - \underline{p}_{T''}\right) - u_B\left(v'' - \underline{p}_{T'}\right) \leq (T'' - T')c_B$.

Set $\check{v}(T'', \underline{T}) = \min\{\dot{v}, \check{v}(T', \underline{T})\}$.

$\forall v'' \leq \check{v}(T'', \underline{T}), u_B\left(v'' - \underline{p}_{T''}\right) - u_B\left(v'' - \underline{p}_{\underline{T}}\right) \leq (T'' - \underline{T})c_B$.

Suppose u_B is convex, by a similar logic, for $T'' < T$, I can set $\check{v}(T'', \underline{T}) \in \overline{V}$ such that:

$\forall v'' \geq \check{v}(T'', \underline{T}), u_B\left(v'' - \underline{p}_{T''}\right) - u_B\left(v'' - \underline{p}_{\underline{T}}\right) \leq (T'' - \underline{T})c_B$.

Since $\overline{V}$ is bounded and u_B is monotonic and has bounded variation on any closed and bounded interval in R^1, the first part of this proof that I stated in the beginning of the proof is proven[18].

For the second part, pick an arbitrary p_1'. Since V is bounded, $u_B(v - p_L)$ is bounded from above and $u_B(v - p_1') - c_B$ is bounded from below. There exists some $\bar{t}$ for which if $t > \bar{t}$,

$\forall v \in V, u_B(v - p_L) - tc_B < u_B(v - p_1') - c_B$.

If seller deviates to p_1' and the game has $\bar{t} + 1$ or more periods, buyer prefers to buy in period 1 compared to buying in period $t > \bar{t}$.

For a sufficiently large T, $P(p_t > p_1') = 1$ with any $t \leq \bar{t}$ in any wPBE. Furthermore, $\forall v \in V, -Tc_B < u_B(v - p_1') - c_B$. Therefore, if seller deviates to p_1' and T is sufficiently large, buyer buys in period 1 with probability 1. In this case, the seller gets a payoff of $u_S(p_1' - \gamma) - c_S$. Since the seller gets this payoff from the deviation, the expected payoff from the equilibrium strategy is weakly greater.

Pick an arbitrary p, given p_L, there exists some period $\underline{t} \geq 1$ such that in any wPBE, if $t > \underline{t}$, $u_B(\bar{v} - p_L) - tc_B < -p$. Pick p' such that $u_B(\bar{v} - p') < -p$. For a sufficiently large $T < \infty$, in any wPBE, if $t \leq \underline{t}$, $P(p_t > p') = 1$. Furthermore, $-Tc_B < -p$. Thus, for a sufficiently large $T < \infty$, $E(U_B) < -p$ in any PBE.

Proof of Proposition 2.

Pick $p \in R^1$. Since u_B is weakly increasing, u_B is bounded on $[\underline{v} - p, \bar{v} - p]$. There exists some $\bar{u}$ for which $\forall v \in [\underline{v}, \bar{v}]$ and $v' \in [\underline{v}, \bar{v}], u_B(v - p) - u_B(v' - p) \leq \bar{u}$. There exists some $p' \in R^1$ such that $c_B + \bar{u} \leq u_B(\underline{v} - p') - u_B(\bar{v} - p)$.

$$\forall v \in V, c_B + \bar{u} \leq u_B(v - p') - u_B(\bar{v} - p) \tag{2}$$

$$\forall v \in V, -\bar{u} \leq u_B(\bar{v} - p) - u_B(v - p) \tag{3}$$

Combine formulas (2) and (3).

$$\forall v \in V, c_B \leq u_B(v - p') - u_B(v - p) \tag{4}$$

For any $p \in R^1$, there exists some $p' \in R^1$ satisfying formula (4).

Start again from an arbitrary p. For the rest of this proof, I will use $p_0 = p$ and the function $\acute{p}(p_i)$ for $i \geq 1$ where:

$\forall v \in V, c_B \leq u_B\left(v - \acute{p}(p_i)\right) - u_B(v - p_i)$.

Sequence π_i is defined here to denote the state of the game. $\pi_1 = 0$. Consider $i \geq 1$. If $p_i > p_{i-1}$, $\pi_{i+1} = 1$. If $p_i \leq \min\{\acute{p}(p_j) | 0 \leq j \leq i - 1\}$, $\pi_{i+1} = 0$. Otherwise, $\pi_{i+1} = \pi_i$.

Suppose the seller's strategy is the following. For $i \geq 1$, if $\pi_1 = 0$, $p_i = p_{i-1}$ and if $\pi_i = 1$, $p_i = \min\{\acute{p}(p_j) | 0 \leq j \leq i - 1\}$. This means $p_1 = p$.

The buyer's strategy is defined using ζ_t. $\zeta_t = p''$ means that the buyer's cutoff price for period t is p''. In other words, in period t, buyer accepts $p_t \leq p''$ and rejects $p_t > p''$.

For $i \geq 1$, if $\pi_i = 0$, $\zeta_i = p_{i-1}$ and if $\pi_i = 1$, $\zeta_i = \min\{\acute{p}(p_j) | 0 \leq j \leq i - 1\}$. This means $\zeta_1 = p$. By this strategy, buyer always chooses $\zeta_i \leq p_{i-1}$. It is trivial to check that buyer always chooses $\zeta_{i+1} \leq \zeta_i$.

[18] Apostol (1985, 94, 128).

The strategies are optimal by the following arguments. Consider the periods when it is the seller's turn to act. In period $i \geq 1$, if $\pi_i = 0$, seller knows that the buyer will never accept any price above p_{i-1}. In period $i \geq 1$, if $\pi_i = 1$, seller knows that buyer will never accept any price above $\min\{\acute{p}(p_j)|0 \leq j \leq i-1\}$.

Consider the periods when it is the buyer's turn to act. If in period i, $\pi_i = 0$ and $p_i \leq p_{i-1}$, buyer knows that seller will never offer a price lower than p_i. If $p_i > p_{i-1}$ instead, buyer knows that if she rejects p_i, she can weakly increase her payoff by accepting p_{i+1}.

If in period i, $\pi_i = 1$ and $p_i \leq \min\{\acute{p}(p_j)|0 \leq j \leq i-1\}$, buyer knows that seller will never offer a price lower than p_i. If $\pi_i = 1$ and $p_i > \min\{\acute{p}(p_j)|0 \leq j \leq i-1\}$ instead, buyer knows that if she rejects p_i, she can weakly increase her payoff by accepting p_{i+1}.

2. Lemma and Proofs for Subsection 4.2

The following lemma is used to prove proposition 3.

Lemma 3. Suppose that u_S has no lower bound and is given. The set of all prices for which it is the seller's strategy to offer the price for some history, in some pure strategy wPBE that satisfies the following 2 conditions has a lower bound.

- $T = \infty$;
- The seller's strategy is to always bargain.

Proof. This set is referred to as S. For a pure strategy wPBE, suppose that the game has infinite periods and seller always bargains.

There exists some p such that $u_s(p - \gamma) < 0$. If by the seller's strategy, there is some period t where $p_t < p$ and by the buyer type v's strategy, she buys in a later period, there exists some period $t' > t$ where $p'_t < p_t$ and the buyer type v buys. If $p_t < p$ and the sale is made in period t or later or never made, seller's utility is non-positive or he does not get utility.

This means that if by the players' strategies, there is some period t where $p_t < p$, seller prefers to terminate bargaining in period t. Therefore, S is bounded from below.

Proof of Proposition 3.

Suppose that $t = T < \infty$ or in period t, both player's strategy is to always terminate bargaining in the next period no matter what happened in period t and before. In period t, using price the buyer expects, if $u_B(v - p_t) - c_B < 0$, she terminates bargaining and if $u_B(v - p_t) - c_B > 0$, she buys. If the seller terminates bargaining in this period, the buyer terminates bargaining as well.

To demonstrate that the seller does terminate bargaining, assume the contrary. Consider the distribution of v according to the seller's belief in this period when the seller does not terminate bargaining in this period. If the seller believes $P(u_B(v - p_t) - c_B \geq 0) = 0$, the seller prefers to terminate bargaining in this period.

If the seller believes $P(u_B(v - p_t) - c_B \geq 0) > 0$, by the least-upper-bound property, the infimum exists for $\{v|u\_B(v - p\_t) - c\_B \geq 0\}$. Call this infimum $\underline{v}'$. Since u_B is continuous, $u_B(\underline{v}' - p_t) - c_B < 0$ cannot be true.

$u_B(\underline{v}' - p_t) - c_B \geq 0$

In this period, the buyer does not bargain if $v < \underline{v}'$ because this means $u_B(v - p_t) - c_B < 0$. Since u_B is continuous,

$$\exists \epsilon > 0, \forall v \geq \underline{v}', u_B\big(v - (p_t + \epsilon)\big) > 0. \quad (5)$$

The seller prefers to raise the price to $p_t + \varepsilon$ in which case, by the buyer's original strategy, she will buy if her strategy is to bargain in this period. Therefore, in any pure strategy wPBE with $T < \infty$, both players terminate bargaining in period t. Induction means that the players terminate bargaining in any period no matter what happened before that period.

Next, will be proven for $T = \infty$. In period t+1, if the seller's strategy is to terminate bargaining in period t+1 and regardless of what p_t was played, the buyer also terminates bargaining in period t+1. Induction using the logic above means that in period t and any earlier period, players will terminate bargaining.

Pick $t > 0$. Consider the game in period t. I need only consider cases where the seller does not bargain in this period. If the seller believes that $P(u_B(v - p_t) - c_B \geq 0) = 0$, the seller prefers to terminate bargaining. Consider the case where the seller believes that $P(u_B(v - p_t) - c_B \geq 0) > 0$. For $t' > t$, $h(t')$ is a sequence of prices the seller played starting from period t and ending at $t' - 1$ that satisfy the condition that in period t', the seller's strategy is to bargain. If no such $h(t')$ exists, the case is proven.

If u_B is concave, pick $\check{v}$ as a lower bound of V. If not and u_B is convex, pick $\check{v}$ as an upper bound of V. Let $p_{h(t'),t'}$ be the $p_{t'}$ that the seller plays in period $t' > t$ given $h(t')$.

In period t, suppose that $\forall t' > t$ and $h(t')$, $u_B\left(\check{v} - p_{h(t'),t'}\right) - t'c_B < u_B(\check{v} - p_t) - tc_B - \frac{t'-t}{2}c_B$. If $t' > t$ and $p_{h(t'),t'} \geq p_t$,

$$v \in V: u_B\left(v - p_{h(t'),t'}\right) - t'c_B < u_B(v - p_t) - tc_B - \frac{t' - t}{2}c_B. \tag{6}$$

If $t' > t$ and $p_{h(t'),t'} < p_t$, by lemma 1's (2), formula (6) holds as well.

$$\forall t' > t, h(t') \text{ and } v \in V: u_B\left(v - p_{h(t'),t'}\right) - t'c_B < u_B(v - p_t) - tc_B - \frac{t' - t}{2}c_B \tag{7}$$

Since u_B is continuous, by formula (7), there exists some $\epsilon' > 0$ for which:

$$\forall t' > t, h(t') \text{ and } v \in V: u_B\left(v - p_{h(t'),t'}\right) - t'c_B < u_B\big(v - (p_t + \epsilon')\big) - tc_B.$$

If $u_B(v - p_t) - c_B < 0$, by formula (7), the buyer's strategy in this period is to terminate bargaining. By the least-upper-bound property, $\underline{v}'$ exists. Combine the above formula with formula (5). Then, I have the result that the seller prefers to raise the price to $p_t + \min\{\epsilon, \epsilon'\}$ in which case, the buyer will buy if her strategy is to bargain in this period. Therefore, for some $t' > t$ and $h(t')$, $u_B\left(\check{v} - p_{h(t'),t'}\right) - \frac{t'}{2}c_B \geq u_B(\check{v} - p_t) - \frac{t}{2}c_B$.

If some $h(t')$ can be found as above, I can go to period t' with this $h(t')$ and for some $t'' > t'$, find a sequence of prices from t' to $t'' - 1$ the same way. Using induction, for any $\bar{t} > t$, there exists some $t'' > \bar{t}$ and $h(t'')$ where $u_B\left(\check{v} - p_{h(t''),t''}\right) - \frac{t''}{2}c_B \geq u_B(\check{v} - p_t) - \frac{t}{2}c_B$ and $u_B\left(\check{v} - p_{h(t''),t''}\right) - u_B(\check{v} - p_t) \geq \frac{t''-t}{2}c_B$. However, this violates lemma 3. Therefore, there is no pure strategy wPBE where the seller does not terminate bargaining in any period.

3. Lemmas and Proofs for Section 5

Lemma 4 is used to prove example 1. Lemma 5 is used to prove example 1. Lemma 6 is used to prove lemma 8 and example 1. Lemma 7 is used to prove examples 1 and 2. Lemma 8 is used to prove lemma 9. Lemma 9 is used to prove example 1.

Note that the following proposition is for the u_B defined in section 5.1.

Proposition 4.

u_B is increasing, differentiable and strictly concave.

Proof. Consider $\ln(x + 1)$ for $x \geq 0$.

$$\frac{d\ln(x + 1)}{dx} = \frac{1}{x + 1} > 0 \tag{8}$$

$$\frac{d^2\ln(x + 1)}{dx^2} = -\frac{1}{(x + 1)^2} < 0 \tag{9}$$

Consider $-x^2 + x$ for $x \leq 0$.

$$\frac{d(-x^2 + x)}{dx} = -2x + 1 > 0 \tag{10}$$

$$\frac{d^2(-x^2 + x)}{dx^2} = -2 < 0 \tag{11}$$

When $x = 0$, $\ln(x + 1) = -x^2 + x = 0$. Also, for this case, equations (8) and (10) are equal. u_B is differentiable. Since the derivative is positive, u_B is increasing.

Formulas (8), (9), (10) and (11) show that $\frac{du_B(x)}{dx}$ is decreasing and that $u_B(x)$ is strictly concave[19].

Lemma 4. If $p_1 \in [0.5,1)$, the low type's expected utility from buying period 1 is negative.

Proof. When $p_1 \in [0,1]$, the low type's expected utility from buying in period 1 is the following.

$$\int_{p_1+1}^{2} \ln(v_u - p_1)\, dv_u + \int_{1}^{p_1+1} -(v_u - p_1 - 1)^2 + (v_u - p_1 - 1)\, dv_u \tag{12}$$

$$= (2 - p_1)\ln(2 - p_1) - (2 - p_1) - \big(-(p_1 + 1 - p_1)\big) + (0 + 0) - \left(\frac{p_1^3}{3} + \frac{p_1^2}{2}\right) \tag{13}$$

$$= (2 - p_1)\ln(2 - p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} \tag{14}$$

The derivative of the last line for $p_1 \in [0,1]$ was taken:

$$\frac{d\left((2 - p_1)\ln(2 - p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2}\right)}{dp_1} = -\ln(2 - p_1) - p_1^2 - p_1 \tag{15}$$

By equation (15), when $p_1 \in [0.5,1)$, the low type's expected utility is decreasing in p_1. If $p_1 = 0.5$,
$(2 - p_1)\ln(2 - p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} < 0.$
Therefore, when $p_1 \in [0.5,1)$, the low type's expected utility from buying in period 1 is negative.

Definition 3.

$v_u = v + 1;$ v_u is a random variable like v.

Lemma 5.

If $v_u > p_1, v_u > p_2$, $\ln(v_u - p_1) \gtreqless \ln(v_u - p_2) - c_B$ is equivalent to $\frac{e^{c_B}p_1 - p_2}{e^{c_B} - 1} \lesseqgtr v_u$.

Proof.

$\ln(v_u - p_1) \gtreqless \ln(v_u - p_2) - c_B;$

$\frac{v_u - p_1}{v_u - p_2} \gtreqless e^{-c_B};$ $\frac{v_u - p_2}{v_u - p_1} \lesseqgtr e^{c_B};$ $v_u - p_2 \lesseqgtr e^{c_B}v_u - e^{c_B}p_1;$

$e^{c_B}p_1 - p_2 \lesseqgtr e^{c_B}v_u - v_u;$

$e^{c_B}p_1 - p_2 \lesseqgtr v_u(e^{c_B} - 1);$

$\frac{e^{c_B}p_1 - p_2}{e^{c_B} - 1} \lesseqgtr v_u.$

Lemma 6.

$\frac{e^{c_B}p_1 - p_2}{e^{c_B} - 1} - 4p_2 + 2\gamma = 0 \leftrightarrow p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3};$

Proof.

$\frac{e^{c_B}p_1 - p_2}{e^{c_B} - 1} - 4p_2 + 2\gamma = 0;$

$\left(-\frac{1}{e^{c_B} - 1} - 4\right)p_2 = -\frac{e^{c_B}p_1}{e^{c_B} - 1} - 2\gamma;$

$(-1 - 4e^{c_B} + 4)p_2 = -e^{c_B}p_1 - 2\gamma(e^{c_B} - 1);$

$p_2 = \frac{-e^{c_B}p_1 - 2\gamma(e^{c_B} - 1)}{-4e^{c_B} + 3};$

$p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3}.$

[19] Avriel et al. (2010, 22–23)

Lemma 7.

1. Under $p_1 \in [0,1]$, there exists a unique $p_1 \approx 0.45$ for which: $(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} = 0$.
2. If $c_B \in [0.01,0.02]$ and $\gamma \in [0,0.01]$, under $p_1 \in [0,1]$, there exists a unique $p_1 \leq 0.365$ for which: $(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} = \frac{3-\gamma}{2}\ln\left(\frac{3-\gamma}{2}\right) + \frac{\gamma-1}{2} - c_B$.

This unique $p_1 \leq 0.365$ is increasing in c_B and γ.

Proof. Suppose $\gamma \in [0,0.01]$.

$\frac{d\left(\frac{3-\gamma}{2}\ln\left(\frac{3-\gamma}{2}\right)+\frac{\gamma-1}{2}-c_B\right)}{d\gamma} = -\frac{1}{2}\ln\left(\frac{3-\gamma}{2}\right) - \frac{1}{2}\left(\frac{3-\gamma}{2}\right)\frac{1}{\frac{3-\gamma}{2}} + \frac{1}{2} = -\frac{1}{2}\ln\frac{3-\gamma}{2} < 0$;

$\frac{3-\gamma}{2}\ln\left(\frac{3-\gamma}{2}\right) + \frac{\gamma-1}{2} - c_B$ is decreasing in c_B and γ.

If $p_1 \in [0,1]$, by equation (15), $(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2}$ is decreasing in p_1.

If $p_1 = 0$, $(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} = 2\ln(2) - 1 > 0.38$.

If $p_1 = 1$, $(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} < 0$.

By the intermediate value theorem, the unique p_1 of 7.1 exists. Similarly, since $c_B \in [0.01,0.02]$ means $1.5\ln(1.5) - 0.5 - c_B \in [0.08,0.1]$, the unique p_1 of 7.2 exists.

Equation (15) means that as c_B or γ increases, the unique p_1 of 7.2 increases.

Lemma 8.

If $p_2 = \frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$, $\frac{e^{c_B}p_1-p_2}{e^{c_B}-1}$ and p_2 are increasing in p_1. If $p_1 > \gamma$ as well, $\frac{e^{c_B}p_1-p_2}{e^{c_B}-1}$ and p_2 are decreasing in c_B.

Proof. Lemma 6 means that:

$$\frac{e^{c_B}p_1 - p_2}{e^{c_B} - 1} = 4p_2 - 2\gamma.$$

$p_2 = \frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$ is increasing in p_1.

$p_1 > \gamma$;

$3p_1 > 2\gamma$;

$-3p_1 < -2\gamma$;

$4\left(2\gamma(e^{c_B}-1)\right) = 2\gamma(4e^{c_B}-3) - 2\gamma$;

$4\left(e^{c_B}p_1 + 2\gamma(e^{c_B}-1)\right) = p_1 \times 4e^{c_B} + 2\gamma(4e^{c_B}-3) - 2\gamma$;

$$(p_1+2\gamma)(4e^{c_B}-3) = p_1 \times 4e^{c_B} - 3p_1 + 2\gamma(4e^{c_B}-3) < p_1 \times 4e^{c_B} + 2\gamma(4e^{c_B}-3) - 2\gamma = 4\left(e^{c_B}p_1 + 2\gamma(e^{c_B}-1)\right)$$

$\frac{dp_2}{de^{c_B}} = \frac{(p_1+2\gamma)(4e^{c_B}-3)-4\left(e^{c_B}p_1+2\gamma(e^{c_B}-1)\right)}{(4e^{c_B}-3)^2} < 0$.

p_2 is decreasing in c_B.

Lemma 9.

In settings 1 and 2, let $p_2 = \frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$ and $\gamma = 0.01$. p_2 is increasing in p_1 and decreasing in c_B. $p_1 > p_2 > 0.6$.

Proof. By lemma 8, p_2 is increasing in p_1 and decreasing in c_B.

$$p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3};$$

$2\gamma < 3p_1$;

$2\gamma(e^{c_B}-1) < (3e^{c_B}-3)p_1$;

$$\frac{2\gamma(e^{c_B}-1)}{4e^{c_B}-3} < \frac{3e^{c_B}-3}{4e^{c_B}-3}p_1;$$

$$p_2 = \frac{e^{c_B} p_1 + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3} < p_1; \tag{16}$$

$p_2 = \frac{e^{c_B} p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3} \geq \frac{0.7}{4e^{c_B}-3} > 0.6.$

Proof of Example 1.

First, the equilibrium strategies are described. Both players' strategies are to bargain. For period 3 and any period after, if the period exists, the players' strategies are to terminate bargaining regardless of previous history.

In period 1, seller offers the equilibrium p_1. If the buyer rejects this, in period 2, seller offers the equilibrium p_2. Suppose the seller deviates to a different price in period 1 and the game continues to period 2. Let p be a function of γ and c_B. The output of this function is given by lemma 7.2. Let p' be given by 7.1. If $p_1 \in [p(\gamma, c_B), p']$, the seller believes that the buyer is low type and offers $p_2 = \frac{\gamma+1}{2}$. If $p_1 \notin [p(\gamma, c_B), p']$, the seller terminates bargaining in period 2.

The buyer's strategy for period 1 is the following. If p_1 is not a deviation, high type buyer buys if $v_u \geq \frac{e^{c_B} p_1^* - p_2^*}{e^{c_B}-1}$. If p_1 is a deviation, then, the high type buyer buys for this offer if and only if $v \geq p_1$. In period 1, the low type buyer buys if and only if $p_1 < p(\gamma, c_B)$.

In period 2, the buyer bargains if p_1 is not a deviation or $p_1 \in [p(\gamma, c_B), p']$. In period 2, when the buyer sees the price, the buyer buys if $v \geq p_2$.

Since both players terminate bargaining in period 3 and any period after if the period exists, the players' strategies to do so are optimal. If p_1 was a deviation and $p_1 \notin [p(\gamma, c_B), p']$, both players terminate bargaining in period 2. Therefore, the strategies to do so are optimal.

The optimality of the seller's strategy will be proven. If the seller deviates to $p_1 = 1$, his expected utility is $\frac{1}{3}(1-\gamma) \geq 0.3$. The seller prefers to bargain. If p_1 was the equilibrium p_1 or $p_1 \in [p(\gamma, c_B), p']$, in period 2, his expected utility from $p_2 = 0.5$ is at least $0.5(0.5-\gamma)$. Therefore, in these cases, the seller prefers to bargain in period 2.

Consider the optimality of p_2. By lemma 6, for the equilibrium p_1 and p_2,

$$\frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} = 4p_2 - 2\gamma.$$

By lemma 9,

$$2 < \frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} < 2.9.$$

$$1 < \frac{e^{c_B} p_1 - p_2}{e^{c_B}-1} - 1 < 1.9.$$

Let $\check{v}$ be this $\frac{e^{c_B} p_1 - p_2}{e^{c_B}-1} - 1$. If p_1 is not a deviation, $p_2 < \gamma$ and $p_2 > \check{v}$ are not optimal. If the seller sets $p_2 \in [1, \check{v}]$, seller's expected utility in period 2 is $\frac{1}{3}(p_2-\gamma)(\check{v}-p_2)$.

$$\frac{1}{3}(p_2-\gamma)(\check{v}-p_2) = \frac{1}{3}(-p_2^2 + (\gamma+\check{v})p_2 - \gamma\check{v}) = \frac{1}{3}\left(-\left(p_2 - \frac{\gamma+\check{v}}{2}\right)^2 + \left(\frac{\gamma+\check{v}}{2}\right)^2 - \gamma\check{v}\right).$$

From $p_2 \in [1, \check{v}]$, $p_2 = 1$ is optimal.

If the seller sets $p_2 \in [\gamma, 1]$, his expected utility in period 2 is $(p_2-\gamma)\left(\frac{1}{3}(\check{v}-1) + \frac{2}{3}(1-p_2)\right)$. The first order condition is:

$$\frac{1}{3}(\check{v}-1) + \frac{2}{3}(1-p_2) + (p_2-\gamma) \times \left(-\frac{2}{3}\right) = 0. \tag{17}$$

If $p_2 = 1$, $\frac{1}{3}(\check{v}-1) - \frac{2}{3}(1-\gamma) < 0$.

The second order condition is: $-\frac{2}{3} - \frac{2}{3} < 0$. When $p_2 = \gamma$, the expected utility is 0. The seller prefers $p_2 = 0.5$. $p_2 \in \{\gamma, 1\}$ is not optimal.

Equation (17) is equivalent to:

$$(\check{v}-1) - 4p_2 + 2\gamma + 2 = 0.$$

When both expected and chosen p_2 are $\frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$, Lemma 6 proves that $(\check{v}-1)-4p_2+2\gamma+2=0$. $p_2=\frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$ is optimal.

Consider the case where $p_1\in[p(\gamma,c_B),p']$. Then, in period 2, seller believes that the buyer is low type. When the buyer is low type, the seller prefers $p_2=0.5$ to $p_2<0$ or $p_2>1$. Furthermore, when $p_2\in[0,1]$, the seller's expected utility in this period is $\frac{2}{3}(p_2-\gamma)(1-p_2)$.

$$(p_2-\gamma)(1-p_2)=-p_2^2+(\gamma+1)p_2-\gamma=-\left(p_2-\frac{\gamma+1}{2}\right)^2+\left(\frac{\gamma+1}{2}\right)^2-\gamma$$

Therefore, in this case, $p_2=\frac{\gamma+1}{2}$ is optimal.

Consider the optimality of p_1. If the seller does not deviate in period 1 and plays $p_2=p_1$ in period 2, his expected payoff is:

$$\frac{1}{3}\left(3-\frac{e^{c_B}p_1-p_2^*}{e^{c_B}-1}\right)(p_1-\gamma-c_S)+\frac{1}{3}\left(\frac{e^{c_B}p_1-p_2^*}{e^{c_B}-1}-2\right)(p_2-\gamma-2c_S)+\frac{2}{3}(1-p_2)(p_2-\gamma)-\frac{2}{3}\times 2c_S$$
$$>\frac{1}{3}\left(3-\frac{e^{c_B}p_1-p_2^*}{e^{c_B}-1}\right)(p_1-\gamma-2c_S)+\frac{1}{3}\left(\frac{e^{c_B}p_1-p_2^*}{e^{c_B}-1}-2\right)(p_1-\gamma-2c_S)$$
$$+\frac{2}{3}(1-p_2)(p_2-\gamma)-\frac{2}{3}\times 2c_S=\frac{1}{3}(1)(p_1-\gamma)+\frac{2}{3}(1-p_2)(p_2-\gamma)-2c_S$$
$$=\frac{1}{3}(p_1-\gamma)+\frac{2}{3}(1-p_1)(p_1-\gamma)-2c_S=\frac{3-2p_1}{3}(p_1-\gamma)-2c_S.$$

$$\frac{d(3-2p_1)(p_1-\gamma)}{dp_1}=-2(p_1-\gamma)+(3-2p_1)=3+2\gamma-4p_1>0;$$

$$\frac{3-2p_1}{3}(p_1-\gamma)-2c_S\geq\frac{3-2\times 0.7}{3}(0.7-0.01)-0.005-c_S>0.36-c_S.$$

If the seller deviates to $p_1<p(\gamma,c_B)$ in period 1, his payoff is less than $p(\gamma,c_B)-\gamma-c_S$.

$$p(\gamma,c_B)-\gamma-c_S\leq 0.355-c_S$$

If the seller deviates to $p_1\in[p(\gamma,c_B),p']$ in this period, the price of sale does not exceed $\frac{\gamma+1}{2}$ in periods 1 and 2. The expected payoff is less than $\frac{2}{3}\times\frac{\gamma+1}{2}-c_S$.

$$\frac{2}{3}\times\frac{\gamma+1}{2}-c_S\leq 0.34-c_S.$$

If p_1 is a deviation and $p_1>p'$, in this period, the low type does not buy. After this p_1, if the game continues to the next period, the buyer terminates bargaining in the next period. $p_1<1$ and $p_1>2$ are not optimal. If $p_1\in[1,2]$, seller's expected payoff is $\frac{1}{3}(p_1-\gamma)(2-p_1)-c_S$.

$$\frac{1}{3}(p_1-\gamma)(2-p_1)-c_S=\frac{1}{3}(-p_1^2+(\gamma+2)p_1-2\gamma)-c_S=\frac{1}{3}\left(-\left(p_1-\frac{\gamma+2}{2}\right)^2+\left(\frac{\gamma+2}{2}\right)^2-2\gamma\right)-c_S=$$
$$\frac{1}{3}\left(-\left(p_1-\frac{\gamma+2}{2}\right)^2+\frac{(\gamma-2)^2}{4}\right)-c_S\leq\frac{1}{3}-c_S.$$

Now, that the buyer's strategy is optimal is proved. Suppose p_1 is a deviation, $p_1\notin[p(\gamma,c_B),p']$ and the buyer is deciding whether to accept it. Since the seller will terminate bargaining if the game proceeds to the next period, it is optimal for the buyer to buy when her expected utility is 0 or greater. Therefore, it is optimal for the high type to buy if and only if $v\geq p_1$.

If the buyer is low type, expected utility is positive for $p_1\leq 0$ but negative for $p_1\geq 1$. When $p_1\in[0,1]$, low type's expected utility of purchase is given by formulas (12)~(14) and equation (15) means that the expected utility is decreasing in p_1. By lemma 7, it is optimal for the low type to accept $p_1<p(\gamma,c_B)$ and reject $p'<p_1$.

Suppose that $p_1\in[p(\gamma,c_B),p']$ and the buyer is deciding whether to accept it. If the buyer rejects, she can get $p_2=\frac{\gamma+1}{2}$. However, this is a higher price and the high type prefers to buy now.

Consider the low type buyer's expected utility from bargaining in period 2 when $p_1 \geq p(\gamma, c_B)$ and $p_2 \in [0,1]$ and buying in period 2 when $v \geq p_2$.

$$\int_{p_2+1}^{2} \ln(v_u - p_2)\ dv_u = (2-p_2)\ln(2-p_2) - (2-p_2) - \big(-(p_2+1-p_2)\big)$$
$$= (2-p_2)\ln(2-p_2) - 1 + p_2 \quad (18)$$

If $p_2 = \frac{\gamma+1}{2}$,

$$(2-p_2)\ln(2-p_2) - 1 + p_2 = \frac{3-\gamma}{2}\ln\left(\frac{3-\gamma}{2}\right) + \frac{\gamma-1}{2}$$

If $p_2 < 1$,

$$\frac{d\big((2-p_2)\ln(2-p_2) - 1 + p_2\big)}{dp_2} = -\ln(2-p_2) - (2-p_2)\frac{1}{2-p_2} + 1$$
$$= -\ln(2-p_2) < 0 \quad (19)$$

For the low type, when $p_1 \in [p(\gamma, c_B), p']$ rejecting p_1 and bargaining for p_2 is better than accepting p_1.

Consider p_2 on the equilibrium path. By lemma 9, p_2 is increasing in p_1 and decreasing in c_B. If $p_1 = 0.71$, $\gamma = 0.01$ and $c_B = 0.01$, $p_2 < 0.7$. If $p_2 = 0.7$, by formula (18),

$$\int_{p_2+1}^{2} \ln(v_u - p_2)\ dv_u > 0.04.$$

Therefore, by formula (19), in both settings, low type's expected equilibrium payoff is positive. In both settings, any high type's payoff from buying in period 2 is positive if the seller plays the equilibrium p_2. It is optimal for the low type and the high type to bargain. By lemma 4, it is optimal for the low type to not buy for the equilibrium price in period 1 and to bargain in period 2. In period 2, it is optimal for the high type to bargain. Furthermore, by formula (19), after seeing $p_1 \in [p(\gamma, c_B), p']$, in period 2, it is optimal for the buyer to bargain.

Consider the high type's strategy when she sees that p_1 is not a deviation. Lemmas 5 and 9 mean if $v_u \geq \frac{e^{c_B}p_1 - p_2}{e^{c_B} - 1}$, she weakly prefers buying in period 1 to buying in period 2. In period 2, it is optimal for the buyer who saw the offer to accept it if $v \geq p_2$.

Proof of Example 2.

Both players' strategy is to bargain. They terminate bargaining in period 3 and any subsequent period regardless of the previous history if the period exists. In period 1, the buyer offers p_1 defined by lemma 7.2 which is approximately 0.35. Let p_1 defined by lemma 7.1 be p_1'. In period 2, if $p_1 < p_1'$, seller believes that the buyer is low type. In this period, if $p_1 < p_1'$, seller offers $p_2 = 0.5$ and if $p_1 \geq p_1'$, he terminates bargaining.

In period 1, the high type buyer buys as long as $v \geq p_1$. If $p_1 \leq p_1^*$, the low type buys in period 1. If $p_1 > p_1^*$, the low type does not buy in this period. In period 2, if $p_1 < p_1'$, the buyer bargains and if $p_1 \geq p_1'$, she terminates bargaining. In period 2, the buyer who saw the price buys if $v \geq p_2$.

The players' strategies to terminate bargaining in period 3 if the period exists are optimal because if the period exists, the other player terminates bargaining in the period. The players' strategies to terminate bargaining in period 2 when $p_1 \geq p_1'$ are optimal for a similar reason.

The optimality of the seller's strategy will be proven. Consider period 2 for $p_1 < p_1'$. Here, seller prefers $p_2 = 0$ to $p_2 < 0$ and weakly prefers $p_2 = 1$ to $p_2 > 1$. If $p_2 \in [0,1]$, seller's expected utility in period 2 calculated in period 2 is: $p_2(1-p_2) = -(p_2-0.5)^2 + 0.25$. Since the seller can get an expected utility of 0.25, the seller weakly prefers to bargain in this period. $p_2 = 0.5$ is optimal.

Consider period 1. In the equilibrium, seller sells in this period and gets a utility of approximately 0.35. The seller prefers to bargain. Seller can make the sale with the equilibrium price or a lower one. If the price is lower, the seller's utility is less. If the price is higher, the seller does not sell to the low type in this period. If $p_1 \in (p_1^*, p_1')$, seller's expected utility in period 2 calculated in period 2 is 0.25 or less.

$$\frac{1}{3}p_1' + \frac{2}{3} \times 0.25 < p_1^*$$

Therefore, deviating to $p_1 \in (p_1^*, p_1')$ is not optimal.

The seller's expected utility when $p_1 \in [1,2]$ is: $\frac{1}{3}p_1(2-p_1) = \frac{1}{3}(-(p_2-1)^2+1) \leq \frac{1}{3}$. In the next period, the buyer terminates bargaining. The seller prefers to play the equilibrium strategy compared to deviating to a higher price.

The optimality of the buyer's strategy will be proven. In period 2, after seeing the price, buying when $v \geq p_2$ is optimal for the buyer. Given this, in period 2, when $p_1 < p_1'$, by bargaining the low type buyer can get a utility of $\int_{0.5}^{1} \ln(v-0.5+1)\ dv$ for $p_2 = 0.5$.

$$\int_{0.5}^{1} \ln(v-0.5+1)\ dv = \int_{1}^{1.5} \ln(v)\ dv = 1.5\ln(1.5) - 1.5 - (\ln(1) - 1)$$
$$= 1.5\ln(1.5) - 0.5 \approx 0.11 > 2c_B > 0 \tag{20}$$

In period 2, when $p_1 < p_1'$, it is optimal for the low type buyer to bargain. For $p_2 = 0.5$, the high type's utility from the purchase is at least $\ln(2-0.5)$.

$$\ln(2-0.5) \approx 0.41 > 2c_B \tag{21}$$

In period 2, when $p_1 < p_1'$, it is optimal for the high type to bargain as well.

Consider period 1. For the equilibrium p_1, a high type buyer's utility from purchase is at least $\ln(2-p_1)$. By formula (21), she prefers to bargain. If the seller offers $p_1 < p_1'$, by formula (21), the high type's utility from the purchase is positive. If she buys in the next period instead, her utility will be smaller. If the seller offers $p_1 \geq p_1'$, the seller will not offer a price in period 2. After seeing p_1, buying when $v \geq p_1$ is optimal.

If $p_1 > 1$ and low type buyer accepts p_1, her expected utility is negative. The low type buyer's optimal strategy in this case is to terminate bargaining in period 2. If $p_1 \leq 0$ and the low type buyer accepts p_1, her expected utility is:

$\int_{1}^{2} \ln(v_u - p_1)\ dv_u = \int_{1-p_1}^{2-p_1} \ln(v_u)\ dv_u = (2-p_1)\ln(2-p_1) - (2-p_1) - \big((1-p_1)\ln(1-p_1) - (1-p_1)\big) = (2-p_1)\ln(2-p_1) - (1-p_1)\ln(1-p_1) - 1.$

If $p_1 = 0$,

$(2-p_1)\ln(2-p_1) - (1-p_1)\ln(1-p_1) - 1 \approx 0.39.$

If $p_1 \leq 0$,

$\frac{d(2-p_1)\ln(2-p_1)-(1-p_1)\ln(1-p_1)-1}{dp_1} = -\ln(2-p_1) + (2-p_1) \times -\frac{1}{2-p_1} + \ln(1-p_1) - (1-p_1) \times -\frac{1}{1-p_1} = \ln(1-p_1) - \ln(2-p_1) < 0.$

Therefore, when the low type buyer sees $p_1 \leq 0$, accepting it is optimal.

Suppose $p_1 \in [0,1]$. If the low type buyer accepts p_1, her expected utility is given by formulas (12)~(14). If the low type buyer rejects p_1^* and buys for $p_2 = 0.5$, her utility is given by formula (20). For the equilibrium p_1, lemma 7.2 establishes that in period 1, the low type buyer weakly prefers accepting it to bargaining in period 2. Formula (20) means that the low type prefers to bargain. Formula (15) means that the low type buyer prefers accepting $p_1 < p_1^*$. Suppose that p_1 is higher. If $p_1 < p_1'$, by equation (15), low type prefers to reject p_1 and buy in period 2. If $p_1 \geq p_1'$, the seller terminates bargaining in the next period. Lemma 7.1 establishes that the low type buyer weakly prefers not buying to buying in period 1.