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Demographic Stability of a Degressively Proportional Allocation

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Abstract:

According to the Treaty on European Union, the composition of the European Parliament must be degressively proportional with respect to the population size of the individual Member States of the European Union. The reference point is always the population data from the year preceding the elections for the five-year parliamentary term. During the term, however, population sizes may change, which can lead to a violation of the principle of degressive proportionality. In this paper, the concept of demographic stability of a degressively proportional allocation rule is defined, and based on this definition, a coefficient is constructed whose maximisation leads to the identification of allocations that are stable in the sense defined above. A rule based on this maximization has been empirically verified using data from the 2024 – 2029 parliamentary term and changes in population size that occurred by 2025. The verification confirmed the high effectiveness of this rule in preserving the property of degressive proportionality.

Keywords: apportionment problem, degressive proportionality, demography, European Parliament, fair division.

JEL Classification: C02, D63, D71, J11, K16.

Introduction

In the context of fair distribution of goods and entitlements, a fundamental principle of allocation was shaped as early as antiquity. The principle of proportional allocation, proportional to the value of all agents participating in the distribution, became undisputed. In Aristotle's Nicomachean Ethics, one can find a formulation of this principle, namely the assertion that justice is something proportional, followed by the clarification that everyone agrees that a just distribution should be carried out based on some value, although not everyone agrees on what that value should be.

According to Aristotle, the construction of a fair distribution is therefore simple. First, one must determine the values of the agents that is, numerically define their entitlements to a share of the good and then allocate proportionally to those values. These values are, of course, understood numerically and are almost always non-negative numbers. Aristotle himself pointed out the first obvious problem with this reasoning: there are no objective guidelines on how to determine the values of agents. Egalitarians will point to equal values, electoral system designers to the population sizes of individual electoral districts, and authors of prize distribution rules in sports or

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artistic competitions to parameters describing the merits of the participants. In particular, under the winner-takes-all principle, the only positive value may turn out to be that of the winner.

The second problem is the durability of the principle over time. This issue does not arise in the case of one-off distributions or when the agents' values do not change over time. The former occurs, for instance, in the distribution of a one-time annual financial bonus among organizational units of a company, proportionally to the profits generated by each unit during that year. Annual profits may change year by year, but the distribution concerns only a one-off event and refers to a single moment in time. Unchanging agent values are characteristic, for example, in the case of prize distribution in tennis tournaments. The proportional shares of the total prize pool allocated to players in a given tournament are determined once over many years and depend solely on performance in that tournament most commonly the stage reached, especially when the tournament uses a knockout format.

The durability of the principle, however, becomes relevant when the effects of a given distribution are spread over time. A classic example is the allocation of seats in the European Parliament among EU Member States. The allocation is made based on the population data from the year preceding the start of the term, but it remains in effect, despite demographic changes, for the subsequent five years.

The problem of the durability of an allocation principle, from the perspective of proportionality relative to entitlement values, has a trivial solution: without any additional assumptions, no allocation proportional to changing entitlements is stable. Only a proportional change in all values does not affect the allocation outcome. In all other cases, any change in entitlements leads to changes in the quantity of goods allocated. Therefore, in the context of stability, only allocation rules that permit deviations from strict proportionality lend themselves to meaningful analysis. Such rules include the classical methods applied to the apportionment problem, as well as the degressively proportional rule for allocating seats among Member States in the European Parliament.

The apportionment problem has been known for many years, and many of its aspects have been thoroughly described in the academic literature (Balinski & Young, 2001; Palomares at al., 2024; Pukelsheim, 2014; Young, 1995). Practically an undisputed principle of allocation is its proportionality with respect to the level of support for political parties or the population size of electoral districts. Departures from strict proportionality are necessary in this context due to the need to produce integer allocations. The goods being distributed, seats in representative bodies, are indivisible, and therefore, it is generally not possible to adhere strictly to the proportionality principle. Its numerical realization takes the form of the so-called quota sequence of proportional division, and the actual allocation is an integer approximation of this sequence. The goal is to compute a sequence of natural numbers that approximate the exact proportional shares.

Among the methods used to derive such integer allocations, two main categories stand out: quota methods (Balinski & Young, 1975) and divisor methods (Balinski & Young, 1978; Lyu & Zhao, 2023). In practical applications, divisor methods are more commonly used. These methods have also been the subject of attempts to analyse their demographic stability, since the models assume that the sequence of entitlements determining the allocation corresponds to the population sizes of individual electoral districts. The results obtained identified the range of possible changes in population size that, under three classical rounding methods, rounding up, rounding down, and rounding to the nearest integer, do not necessitate a change in allocation for the following electoral term. Thus, the maximum range of demographic changes was defined, within which the allocation established at the beginning of an elected body's term continues to satisfy the proportionality principle (as modified by the specific divisor method) throughout the entire term. There are no studies in the literature addressing the stability of quota methods.

The principle of degressive proportionality, discussed in more detail in the following section and applied in the allocation of seats in the European Parliament, allows for significant deviations from proportionality. This makes the issue of demographic stability more interesting than in the case of proportional methods from the apportionment problem group. A degressively proportional allocation may be equal, where each Member State receives the same number of seats regardless of population or proportional, subject to rounding, to population size.

This wide range of possibilities means that, for the 2024 – 2029 term, there are over 800 million allocations that comply with the legal provisions. On one hand, this presents a selection problem; on the other, it enables negotiation and the application of various optimization criteria. One such criterion may be demographic stability of the allocation, understood as maintaining the conditions of degressive proportionality for as long as possible despite year-to-year changes in population.

This is particularly relevant because, until the last elections for the 2024 – 2029 term, no repeatable formula for determining the composition of the European Parliament was accepted. Each time, the composition is established through negotiations, and one of the key principles followed by the Committee on Constitutional Affairs when formulating its proposals is to make as few changes as possible to the current composition. Demographic stability as an allocation criterion is therefore consistent with the practice currently in place.

It should be emphasized that the most demographically stable degressively proportional allocation is, of course, the equal allocation. It is independent of population size and thus complies with the degressive proportionality conditions regardless of the scale of demographic changes. However, it cannot be applied in practice when shaping the composition of the European Parliament due to the boundary conditions specified in the Treaty on European Union, which require differentiation between the number of seats allocated to Germany and Malta, that is, to the most and least populous Member States of the Union.

In the present study, the concept of demographic stability of a degressively proportional allocation and allocation rule is first formally defined. Next, a method for measuring such stability is proposed through the introduction of appropriate demographic stability coefficients. In the applied part of the paper, these coefficients are used both as an optimization criterion for the allocation of seats in the European Parliament and as a criterion for adjusting the current composition of the chamber. In the first case, by maximizing the demographic stability index of the degressively proportional rule, an allocation is selected, among all legally permissible solutions, that is most demographically stable according to the defined criterion. In the second case, a sequential correction is applied to the allocation valid for the 2024–2029 term to obtain an allocation with the property that no single-seat transfer between any two countries would increase its demographic stability coefficient.

1. Research Background

The degressively proportional allocation of seats in the European Parliament is a relatively new problem. It emerged with the entry into force of the Treaty on European Union, which introduced degressive proportionality as the guiding principle for structuring this body.

Article 14(2) of the Treaty on European Union states: "The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats".

Additionally, Article 1 of European Council Decision 2023/2061 defines degressive proportionality as follows: "Degressive proportionality is defined as follows: the ratio between the population and the number of seats of each Member State before rounding up or down to the nearest whole number is to vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member of the European Parliament from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats in the European Parliament".

Despite the fact that the Treaty on European Union has been in force for over a decade, no mathematical formula has yet been approved for allocating seats among Member States of the European Parliament. Article 14(2), which governs this issue, is sufficiently general to allow for many allocations that comply with its provisions, but none of the proposed models has been legally adopted as a definitive solution. Recognizing the importance of establishing a reproducible mathematical formula for determining the Parliament's composition, the Committee on Constitutional Affairs has organized academic symposia dedicated to this topic. One special issue of the journal

Mathematical Social Sciences was entirely devoted to the problems associated with degressive proportionality. However, no consensus was reached that could serve as a legally accepted formula, and the proposed composition of the chamber remains a result of negotiations that adjust the previously adopted allocation. As a result, the question of how to systematically determine the composition of the European Parliament remains open.

Among the methods presented in the literature, the largest group consists of adaptations of well-known apportionment rules used for proportional allocation (Cegiełka & Łyko, 2014; Charvát, 2024; Dniestrzański, 2014; Haman, 2017; Martínez-Aroza & Ramírez-González, 2008; Pukelsheim, 2010; Ramírez-González, 2012; Słomczyński & Życzkowski, 2012). This is a natural approach, considering the electoral nature of the problem. The rules belonging to this group, most notably the Cambridge Compromise and the Power Compromise, came closest to being legally adopted and are widely recognized as the most practically relevant proposals. In addition, there have been attempts to address the problem using numerical methods and optimization techniques (Cegiełka et al., 2019, 2021a, 2021b; Cegiełka et al., 2025; Florek, 2012; Łyko et al., 2025; Łyko & Rudek, 2017; Serafini, 2012). Within the latter group, special attention should be given to the paper (Łyko & Rudek, 2013) where the LaRsa algorithm which identifies the entire set of all permissible allocations was presented. This opens the door to any kind of minimization or maximization analysis based on a defined objective function.

The LaRsa algorithm is employed in the applied section of this article to generate all possible allocations for the 2024 – 2029 term. From this set, the allocation that maximizes the demographic stability index of a degressively proportional rule is selected. This construction leads to the proposal of a new rule for shaping the composition of the European Parliament, based on the previously unused criterion of demographic stability. Additionally, in line with the current practice of determining the composition through negotiation, a proposal is also made to adjust the current allocation in a way that ensures the highest possible demographic stability under the given constraints.

2. Research Methodology

An allocation problem is defined as a pair (p, h), where $p = (p_1, p_2, \dots, p_n)$, $p_i > 0$, is a sequence of agents' entitlements, and $h > 0$ is the number of goods to be distributed among them. For a given allocation problem (p, h), an allocation is defined as a positive sequence $s = (s_1, s_2, \dots, s_n)$ such that $\sum_{i=1}^{\infty} s_i = h$. In the case of the allocation of seats in the European Parliament, the sequence p represents the population sizes of the Member States, the sequence s represents the number of seats allocated to them, and h is the total number of seats. The set of all allocations for a problem (p, h) will be denoted by $A_{(p,h)}$.

An allocation rule is understood as any function that assigns to an allocation problem (p, h) a subset $A_{R(p,h)}$ of the set $A_{(p,h)}$ of all allocations for that problem, i.e., a function:

$$R: \mathbb{R}_+^n \times \mathbb{R}_+ \to 2^{\mathbb{R}_+^n}.$$

If the set $A_{R(p,h)}$ is a singleton, the allocation rule is called deterministic. An example of a deterministic allocation rule is the equal division rule R_E, according to which each agent receives the same amount of the good. Another important example of a deterministic rule is the proportional allocation rule R_P, in which the allocation is proportional to the entitlements, i.e., $s_i = \frac{hp_i}{\sum_{i=1}^{n} p_i}$. In full generality, the degressively proportional allocation rule used in the distribution of seats in the European Parliament is not deterministic.

An integer allocation $s = (s_1, s_2, \dots, s_n)$ is degressively proportional with respect to a non-increasing sequence of entitlements $p = (p_1, p_2, \dots, p_n)$ if and only if, for every $i \in \{1,2, \dots, n-1\}$,

$$s_i \geq s_{i+1}, \tag{1}$$

$$\frac{s_i}{p_i} \leq \frac{s_{i+1}}{p_{i+1}}. \tag{2}$$

An allocation rule $R(p,h)$ is said to be degressively proportional with respect to p if every allocation $s \in A_{R(\boldsymbol{p},h)}$ is degressively proportional with respect to p.

The concept of degressively proportional allocation is relative, as it depends on the population sequence and therefore refers to a specific point in time, namely the moment at which population data is obtained. The above conditions (1) and (2) are verified based on that data. However, population sizes may change over time, and as a result, inequalities (1) and (2) may no longer hold for updated values. In particular, an allocation that is degressively proportional with respect to the population sequence measured at the beginning of a parliamentary term may not remain so with respect to the population at the end of that term. From a formal standpoint, during the term of the European Parliament, its composition may thus fail to comply with the principle expressed in Article 14(2) of the Treaty on European Union. Hence the idea of considering demographic stability over time in the context of degressively proportional allocation (DP demographic stability), where stability is understood as the preservation of conditions (1) and (2) throughout the entire period under analysis.

Let $[t_1,t_2]$, with $t_1 < t_2$, be any time interval, and let $t \in [t_1,t_2]$. Let $p^t = (p_1^t, p_2^t, \dots, p_n^t)$ denote a non-increasing sequence representing the population sizes at time t.

Definition 1. An allocation $s = (s_1, s_2, \dots, s_n)$ is DP demographically stable over the period $[t_1,t_2]$ if and only if, for every $t \in [t_1,t_2]$, the allocation s is degressively proportional with respect to the non-increasing population sequence $p^t = (p_1^t, p_2^t, \dots, p_n^t)$.

Definition 2. An allocation rule $R(p,h)$ is DP demographically stable over the period $[t_1,t_2]$ if and only if there exists an allocation $s \in A_{R(p,h)}$ such that for every $t \in [t_1,t_2]$, s is degressively proportional with respect to the non-increasing population sequence $p^t = (p_1^t, p_2^t, \dots, p_n^t)$.

In the following sections, only *DP* demographic stability will be considered and will be referred to simply as demographic stability or stability.

From a practical perspective, the most relevant case is the demographic stability of an allocation during a single parliamentary term, that is, when t_1 and t_2 mark the beginning and end of the term, respectively. In the practice of determining the composition of the European Parliament, especially given the frequently emphasized need to preserve the *status quo*, meaning to minimize seat changes between successive terms, demographic stability also acquires an intertemporal dimension. It is important to note that the empirical verification of demographic stability is significantly constrained by the lack of continuous population measurement. Since such data is recorded at discrete intervals, the practical feasibility of analysing demographic stability depends on the frequency of updates to the sequence p^t.

For the first time, the issue of demographic stability of a degressively proportional allocation was considered in the work by (Dniestrzański et al., 2013). Starting from condition (2) in Definition 1, *i.e.*, the inequality $\frac{s_i}{p_i} \le \frac{s_{i+1}}{p_{i+1}}$, the authors justified that for all degressively proportional allocations and for every $i \in \{1,2,\dots,n-1\}$, the following inequalities hold $0 \le \frac{p_{i+1}s_i}{p_i s_{i+1}} \le 1$. Demographic stability of an allocation means these inequalities are satisfied for all $n-1$ pairs of agents i and $i+1$. If $s_i = s_{i+1}$, then inequalities (1) and (2) are fulfilled for any values such that $p_i \le p_{i+1}$, which means that such pairs do not affect the demographic stability of the allocation. Consequently, only those pairs *where* $s_i \ge s_{i+1}$ are relevant. Since the stability of an allocation $\boldsymbol{s}$ implies stability for all such agent pairs, one can modify the idea from the cited work and first define for any allocation $s = (s_1, s_2, \dots, s_n)$, which is degressively proportional with respect to the non-increasing sequence of populations $p = (p_1, p_2, \dots, p_n)$ a sequence $r = (r_1, r_2, \dots, r_{n-1})$ that represents the degree of fulfilment of inequalities (1) and (2) for each consecutive pair of agents i and $i+1$:

$$r_i = \begin{cases} 1, \text{for } s_i = s_{i+1} \\ 1 - \frac{p_{i+1}s_i}{p_i s_{i+1}}, \text{for } s_i \neq s_{i+1} \end{cases}. \quad (3)$$

Then, the demographic stability coefficient of allocation s is defined as:

$$STD_{\boldsymbol{s}} = \min_{i \in \{1,2,\dots,n-1\}} r_i. \quad (4)$$

Between two allocations, the one with the higher $STD_{\boldsymbol{s}}$ is considered more demographically stable.

Looking at the problem more broadly, one can also evaluate the demographic stability of an allocation rule $R(p,h)$, defining its measure as:

$$STD_{R(\boldsymbol{p},h)} = \max_{s \in A_{R(\boldsymbol{p},h)}} STD_{\boldsymbol{s}}. \quad (5)$$

If $R_P(p,h)$ is the proportional allocation rule with respect to sequence p, then $\frac{p_{i+1}s_i}{p_i s_{i+1}} = 1$ and hence $STD_{R_P(\boldsymbol{p},h)} = 0$. In the case of the equal allocation rule $R_E(p,h)$, we have $STD_{R_E(\boldsymbol{p},h)} = 1$. From the definition of degressively proportional allocation, it follows that $0 \leq \frac{p_{i+1}s_i}{p_i s_{i+1}} \leq 1$, which means that for any degressively proportional allocation rule $R(p,h)$, the following inequalities hold:

$$0 = STD_{R_P(\boldsymbol{p},h)} \leq STD_{R(\boldsymbol{p},h)} \leq STD_{R_E(\boldsymbol{p},h)} = 1. \quad (6)$$

Thus, it is evident that the equal allocation rule is the most demographically stable. No change in population can make it lose its degressive proportionality. On the opposite end, the proportional allocation rule is the least stable, even the slightest change in population renders the allocation no longer degressively proportional. If one aims to ensure long-term robustness of the allocation, then between two candidate rules, the one with the higher $STD_{R(\boldsymbol{p},h)}$ should be chosen.

3. Case Studies

To illustrate the issue, two practical tasks related to shaping the composition of the European Parliament were considered:

Maximizing Demographic Stability

In the first task, based on the population figures used by the Committee on Constitutional Affairs to determine the 2024 – 2029 composition, a degressively proportional allocation was found that maximized the demographic stability coefficient $STD_{\boldsymbol{s}}$ in lexicographic order. To do this, the LaRsa algorithm was implemented, which identified all 874,025,775 degressively proportional allocations of 720 seats under the constraints $s_1 = 96$ and $s_{27} = 6$. These constraints match the total number of seats and the boundary conditions used for the official allocation of the current term.

From this set, 42 allocations were selected with the highest demographic stability coefficient $STD_{\boldsymbol{s}} = 0.04540513703469362$. In all these 42 allocations, 15 countries received the same number of seats across all solutions, while the number of seats for the remaining countries varied between 2 and 4 mandates. These values are presented in the column "Max $STD_{\boldsymbol{s}}$ Allocations" of Table 1.

In the next steps, this set of 42 allocations was narrowed down by choosing those allocations where the next value in the sequence $\boldsymbol{r}$ was the largest. After 9 iterations, a final allocation was selected, which is presented in column "Lex Max $STD_{\boldsymbol{s}}$ Allocation" of Table 1.

Improving Allocation Through Seat Transfers

In the second task, the starting point was the current allocation s^{2024}, for which the demographic stability coefficient was computed as $STD_{s^{2024}} = 0.00244339$. Next, one-seat transfers between countries were considered, with the goal of producing a degressively proportional allocation (respecting the constraints $s_1 = 96$, $s_{27} = 6$) with a higher demographic stability coefficient. This one-seat transfer procedure was continued as long as a transfer existed that improved the value of STD_s. After three transfers, a final allocation was reached, presented in column "Seat Transfer Allocation", with the property that no further one-seat shift between any two countries would increase the allocation's demographic stability.

Table 1. Demographic stability of the allocation in the 10th term of the European Parliament

Countries	Population 2023	Seats 2024-2029	Max $\mathrm{STD_s}$ Allocations	Lex Max $\mathrm{STD_s}$ Allocation	Seat Transfer Allocation	Population 2025
Germany	83,203,320	96	96	96	96	83,577,140
France	67,842,582	81	82	82	82	68,635,943
Italy	59,607,184	76	76	76	75	58,934,177
Spain	47,432,805	61	64	64	62	49,077,984
Poland	37,654,247	53	54	54	53	36,497,495
Romania	19,038,098	33	29	29	32	19,036,031
Netherlands	17,734,036	31	29	29	31	18,044,027
Belgium	11,631,136	22	20	20	22	11,900,123
Greece	10,603,810	21	20	20	21	10,409,547
Czech Republic	10,545,457	21	20	20	21	10,909,500
Sweden	10,440,000	21	20	20	21	10,587,710
Portugal	10,352,042	21	20	20	21	10,749,635
Hungary	9,689,010	21	20	20	21	9,539,502
Austria	8,967,500	20	20	20	20	9,197,213
Bulgaria	6,838,937	17	16,17	17	16	6,437,360
Denmark	5,864,667	15	15,16	16	15	5,992,734
Finland	5,541,241	15	15,16	16	15	5,635,971
Slovakia	5,434,712	15	15,16	16	15	5,419,451
Ireland	5,060,004	14	15,16	16	15	5,439,898
Croatia	3,862,305	12	12,13,14,15	14	12	3,874,350
Lithuania	2,805,998	11	10,11,12	11	11	2,890,664
Slovenia	2,107,180	9	9,10,11	9	9	2,130,850
Latvia	1,875,757	9	9,10,11	9	9	1,856,932
Estonia	1,331,796	7	7,8,9,10	8	7	1,369,995
Cyprus	904,700	6	6,7,8,9	6	6	979,865
Luxembourg	643,648	6	6,7	6	6	681,973

Countries	Population 2023	Seats 2024-2029	Max STD_S Allocations	Lex Max STD_S Allocation	Seat Transfer Allocation	Population 2025
Malta	520,971	6	6	6	6	574,250

Source: EUR-Lex, Eurostat

Conclusion

The composition of the European Parliament is determined based on population data from the year preceding the start of a new term. According to the Treaty on European Union, this allocation must be degressively proportional with respect to the population figures of the Member States. Since each parliamentary term lasts five years, population changes during the term may cause the condition of degressive proportionality to no longer hold. This raises the question of how to design the allocation so that the condition remains satisfied for as long as possible.

In the absence of a clear, legally approved rule for determining the composition of the European Parliament, the demographic stability coefficient presented in this study may serve as a criterion for shaping the chamber. This paper illustrates its application using the case of the 2024–2029 term, comparing the resulting allocations with updated population data from the year 2025.

The current allocation is no longer degressively proportional with respect to the 2025 population figures. This condition is violated for five pairs of countries, highlighted in bold in the relevant table. The correction proposed in the previous section, which involved just three seat transfers, is insufficient to fix the violations for all five pairs. However, it is worth noting that after the correction, the degressive proportionality condition is violated for only two pairs, indicating the effectiveness of each individual transfer.

The conclusions are significantly different when the maximization of the demographic stability coefficient is used as the primary criterion from the outset. All 42 allocations identified using this method remained degressively proportional also with respect to the 2025 population data. This is a very strong result, especially considering that, out of the initial 874,025,775 admissible allocations at the start of the term, only 16,956,362 i.e., about 1.94% maintained degressive proportionality by 2025. This outcome confirms the effectiveness of the proposed criterion and justifies further research in this area.

Credit Authorship Contribution Statement

All work, including conceptualization, methodology, analysis, and writing, was carried out solely by Janusz Łyko.

Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Data Availability Statement

The data that support the findings of this study are openly available in EUR-Lex, providing access to the legal framework on degressively proportional allocation, at https://eur-lex.europa.eu/eli/dec/2022/2518/oj, and in Eurostat, offering demographic statistics on population by country, at https://ec.europa.eu/eurostat/databrowser/view/tps00001/default/table?lang= en&category=t_demo.t_demo_pop.

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