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Detecting Exchange Rate Bubbles Using Hamilton Filter

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Abstract

This paper develops an adaptive Hamilton-filter framework for detecting speculative bubbles in exchange rate markets. With the reformulation of the traditional bubble-crash model through a binary transformation, the proposed approach expresses bubble dynamics within a nonlinear regime-switching structure and derives recursive estimates of bubble continuation probabilities. Unlike conventional explosive root tests, the method provides real-time, time-varying conditional probabilities of speculative regimes.

The empirical application focuses on the USD - Iranian Rial exchange rate over the period 2000–2020, examining six episodes of heightened volatility. Bubble detection results are compared with ADF, SADF, and GSADF tests, showing that the proposed filter effectively identifies multiple rational bubble episodes consistent with macroeconomic and policy developments. Additional analysis of tradable and non-tradable goods prices suggests that external-sector imbalances significantly contribute to exchange rate explosiveness. Overall, the findings demonstrate that the adaptive Hamilton-filter approach offers a robust and economically interpretable tool for real-time bubble monitoring and exchange rate risk assessment.

Keywords: exchange rate bubbles; Hamilton filter; GSADF; regime-switching; speculative dynamics.

JEL Classification: C32; F31; G01.

Introduction

Asset price bubbles arise when market prices deviate persistently from their fundamental values, often driven by speculative behaviour, herd dynamics, and investor overreaction. Such episodes generate excessive volatility, distort price signals, and weaken the allocative efficiency of financial markets (Astill et al., 2018). When bubbles burst, the consequences may extend beyond financial markets, affecting macroeconomic stability, resource allocation, and long-term growth (Whitehouse et al., 2023). These distortions are frequently associated with information asymmetry, behavioural biases, and mispricing relative to economic fundamentals (Aloosh et al., 2022; Yang et al., 2020).

While bubbles have been documented in equity, real estate, and cryptocurrency markets, exchange rate markets are equally vulnerable. Episodes such as the Plaza Accord adjustment (1985), the Asian financial crisis (1997), the euro introduction (1999–2000), and more recent currency crises illustrate how speculative pressures can generate explosive exchange rate dynamics (Phillips & Shi, 2018).

Detecting such episodes in real time remains a central challenge in applied econometrics and monetary policy analysis. Traditional explosive root tests, including ADF, SADF, and GSADF, provide valuable ex post identification of bubble periods but offer limited adaptability for dynamic probability assessment.

Against this background, this study contributes to the applied econometrics and exchange rate literature in three main ways. First, it reformulates the rational bubble-crash model within a nonlinear regime-switching framework using a binary transformation, enabling the Hamilton filter to generate adaptive, real-time estimates of bubble continuation probabilities. Unlike conventional explosive root tests, the proposed framework produces time-varying conditional probabilities that allow continuous monitoring of speculative regimes. Second, the methodological extension is applied to the USD - Iranian Rial exchange rate over the period 2000–2020, identifying multiple rational bubble episodes aligned with major macroeconomic and policy shocks. Third, by comparing the results with GSADF and Markov-switching approaches, the study demonstrates the robustness and operational advantages of the adaptive Hamilton-filter framework for detecting exchange rate misalignments. Overall, the proposed approach provides a policy-relevant tool for real-time bubble monitoring and exchange rate risk assessment.

The remainder of the paper is structured as follows. Section 1 reviews the related literature. Section 2 presents the methodological framework and derivation of the Hamilton filter representation. Section 3 discusses parameter estimation, adaptive recursive filtering, and convergence properties. Section 4 reports the empirical case study results. Section 5 concludes.

1. Literature Review

The empirical detection of speculative bubbles has long posed methodological challenges in applied econometrics. Early contributions emphasized rational bubble testing within present-value frameworks, arguing that explosive behaviour in asset prices may signal deviations from fundamentals (Diba & Grossman, 1988). However, subsequent work demonstrated that periodically collapsing bubbles may evade standard unit root tests, complicating empirical identification (Evans, 1991). These limitations stimulated the development of more refined econometric approaches capable of detecting explosive dynamics under structural instability.

A major advance in this literature emerged through right-tailed recursive unit root testing. Phillips, Wu, and Yu (2011) introduced the Supremum ADF (SADF) procedure to detect the origination of explosive episodes, while Phillips, Shi, and Yu (2015) generalized the method via the GSADF test to allow for multiple bubble episodes. These approaches have become standard tools for date-stamping exuberance across equity, housing, and currency markets. Recent extensions emphasize real-time monitoring frameworks (Phillips & Shi, 2020; Whitehouse et al., 2023), allowing policymakers to identify bubble escalation as data accumulate. Despite their statistical power, these methods primarily provide ex post identification of explosive behavior and do not directly generate time-varying probabilistic measures of bubble continuation.

Parallel to explosive-root testing, another strand of research models speculative regimes through nonlinear and regime-switching structures. Markov-switching and state-space models treat bubbles as latent states that evolve probabilistically over time (Hamilton, 1989; Nael & Wilfling, 2011). Such frameworks are particularly useful when financial markets exhibit structural breaks, regime persistence, and asymmetric transitions. Recent applications to exchange rate dynamics highlight the importance of nonlinear adjustment mechanisms and speculative shifts in currency markets (Alola et al., 2021, Tang & Xie, 2025). However, standard regime-switching models often require strong parametric assumptions and may lack a direct link to rational bubble-crash structures derived from theoretical models.

In the context of exchange rates, speculative misalignments have received renewed attention following episodes of currency crises and external imbalances. Empirical evidence suggests that exchange rate explosiveness may be associated with external-sector pressures, capital flow volatility, and policy uncertainty (Baxa & Pascual, 2020). At the same time, recursive explosive-root methods have been applied to detect exuberance in foreign exchange and cryptocurrency markets (Corbet et al., 2025; Enoksen et al., 2020), reinforcing the relevance of real-time monitoring tools in increasingly volatile environments. Nevertheless, the existing literature largely separates three methodological streams: (i) explosive-root detection, (ii) regime-switching estimation, and (iii) adaptive Bayesian learning approaches.

Asako and Liu (2013) bridge part of this gap by proposing a stochastic bubble model with adaptive recursive estimation of continuation probabilities. Their Bayesian learning framework allows parameter updating but does not explicitly exploit the state-space representation inherent in regime-switching processes. This leaves open an important applied question: can the rational bubble-crash model be reformulated in a way that combines the probabilistic interpretation of regime-switching models with the adaptive updating logic of recursive filtering?

The present study addresses this gap by reformulating the rational bubble model within a nonlinear regime-switching structure and deriving an equivalent Hamilton-filter representation. Unlike conventional explosive-root tests that detect bubbles retrospectively, the proposed framework generates time-varying conditional probabilities of bubble continuation, enabling dynamic monitoring of speculative regimes. By integrating recursive filtering with explicit benchmarking against GSADF and Markov-switching results, the paper contributes to a unified perspective on bubble detection that is both statistically rigorous and operationally relevant for exchange rate risk assessment.

Recent developments in the econometric detection of asset and exchange rate bubbles emphasize the need for real-time, adaptive monitoring rather than solely ex post identification. Traditional explosive-root tests such as SADF and GSADF have become standard tools for dating bubble episodes (Phillips, Wu, & Yu, 2011; Phillips, Shi, & Yu, 2015), and recent extensions have improved their power and interpretability in ongoing data settings (Phillips & Shi, 2020; Whitehouse, Harvey, & Leybourne, 2023). Parallel work has explored state-space and regime-switching models that treat speculative dynamics as latent regimes with probabilistic transitions (Hamilton, 1989; Nael & Wilfling, 2011; Alola, 2021). In exchange rates specifically, nonlinear dynamics and misalignment have been documented across multiple currency episodes (Kyrtsou & Terraza, 2020). Despite these advances, existing methods either focus on retrospective bubble detection or rely on strong parametric assumptions. This motivates an approach that preserves real-time adaptive updating, formal regime interpretation, and policy relevance, objectives that underlie the methodology developed in this study.

2. Methodology: Derivation of Filter

As follows, first, we review Asako & Liu (2013) model. Following their notations, at time $t = 1,2, \ldots$, let x_t be the sequence of price deviation from its fundamental value. In the current paper, we focus on exchange rate bubbles. As bubble is alive, then x_t is given by divergent AR(1):

$$x_t = \beta_t x_{t-1} + \varepsilon_t.$$

Time varying coefficient β_t, exceeding unity, follows a random walk as:

$$\beta_t = \beta_{t-1} + \zeta_t.$$

Innovations ζ_t and ε_t are independent and contain independent random variables with normal $N(0, \sigma_\varepsilon^2)$ and $N(0, \sigma_\zeta^2)$, respectively. Let J_t be a binary random variable where $J_t = 1$ stands for the bubble is alive and as soon as $J_t = 0$, then the bubble is crashed and then

xt=εt$x_t = \varepsilon_t$.

In this way, the two rules model of Asako & Liu (2013) is given by:

$$x_t = J_t \beta_t x_{t-1} + \varepsilon_t.$$

Let $\mathcal{F}_t, \mathcal{G}_t$ be the information sets (σ-fields) made by $\{x_1, \ldots, x_t\}$ and $\{\beta_1, \ldots, \beta_t\}$, respectively. To derive the Hamilton filter recursive equations, this model is viewed as a regime switching model. In the current paper, using the odds radios, an equivalent form for the Hamilton filter is proposed, see Kole (2019). To this end, let u_t and v_t be odds of smoothing and predictive probabilities:

$$P(J_t = 1 | \mathcal{F}_t, \mathcal{G}_t),\ P(J_t = 1 | \mathcal{F}_{t-1}, \mathcal{G}_t),$$

respectively, i.e.,

$$u_t = \frac{P(J_t=1|\mathcal{F}_t, \mathcal{G}_t)}{P(J_t=0|\mathcal{F}_t, \mathcal{G}_t)},$$

$$v_t = \frac{P(J_t=1|\mathcal{F}_{t-1}, \mathcal{G}_t)}{P(J_t=0|\mathcal{F}_{t-1}, \mathcal{G}_t)}.$$

Before going future, the following lemma gives two equivalent versions of Bayesian theorem.

Lemma 1. Suppose that A and B are two non-empty arbitrary events of a specified sample space. Let:

$$\frac{P(B|A)}{P(B|A')},$$

be the likelihood ratio, and:

$$\frac{P(A|B)}{P(A'|B)}, \frac{P(A)}{P(A')}$$

be the odds of posterior and prior, respectively. Then,

(a) $\text{Odds(posterior)} = \text{Odds(Prior)} \times \text{Likelihood ratio}$, i.e., $\frac{P(B|A)}{P(B|A')} = \frac{P(A|B)}{P(A'|B)} \times \frac{P(A)}{P(A')}$.

(b) By taking logarithm of both sides, it is seen that:

$\mathrm{logit(Posterior)} = \mathrm{logit(Prior)} + \log(\text{ Likelihood ratio})$,

where $\mathrm{logit(a)} = \log\left(\frac{a}{1-a}\right), 0 < a < 1$.

Part b of lemma shows the additive effects of Likelihood ratio and Prior on Posterior. In the speculative bubbles problem, the Bayesian rule implies that

$$P(J_t = 1|F_t, G_t) = \frac{f(x_t|J_t = 1, x_{t-1}, G_t)P(J_t = 1|F_{t-1}, G_t)}{f(x_t|J_t = 1, x_{t-1}, G_t)P(J_t = 1|F_{t-1}, G_t) + f(x_t|J_t = 0, x_{t-1}, G_t)P(J_t = 0|F_{t-1}, G_t)}.$$

where:

$f(x_t|J_t = 1, x_{t-1}, G_t) \qquad f(x_t|J_t = 0, x_{t-1}, G_t)$

are densities of x_t under two regimes $J_t = 1$ and $J_t = 0$. Let δ_t be the likelihood ratio, that is

$$\delta_t = \frac{f(x_t|J_t=1, x_{t-1}, G_t)}{f(x_t|J_t=0, x_{t-1}, G_t)}.$$

Using the above lemma, it is easy to see that:

$$u_t = \frac{f(x_t|J_t=1, x_{t-1}, G_t)P(J_t=1|F_{t-1}, G_t)}{f(x_t|J_t=0, x_{t-1}, G_t)P(J_t=0|F_{t-1}, G_t)},$$

equivalently, it is:

$$u_t = v_t\delta_t.$$

As extension of Asako & Liu (2013), suppose that J_t is a Markov chain with time varying transition probabilities:

$$P_{ij}^t = P(J_t = j|J_{t-1} = i).$$

The second equation of Hamilton filter implies that:

$$P(J_t = 1|F_{t-1}, G_t) = P_{11}^t P(J_{t-1} = 1|F_{t-1}, G_{t-1}) + P_{01}^t P(J_{t-1} = 0|F_{t-1}, G_{t-1})$$

$$P(J_t = 0|F_{t-1}, G_t) = P_{00}^t P(J_{t-1} = 0|F_{t-1}, G_{t-1}) + P_{10}^t P(J_{t-1} = 1|F_{t-1}, G_{t-1}).$$

It is easy to see that:

$$v_t = \frac{P_{11}^t u_{t-1} + P_{01}^t}{P_{10}^t u_{t-1} + P_{00}^t}.$$

Define function $g_t(u) = \frac{u+1}{P_{10}^t u + P_{00}^t} - 1$ and notice that:

$$v_t + 1 = \frac{u_{t-1} + 1}{P_{10}^t u_{t-1} + P_{00}^t}.$$

Thus,

$v_t = g_t(u_{t-1})$ and $u_t = g_t(u_{t-1})\delta_t$.

For the likelihood ratio δ_t, notice that, under $J_t = 1$ and G_t, then:

$$x_t = \beta_t x_{t-1} + \varepsilon_t.$$

Thus, the x_t is normally distributed with mean and variance:

$$E(x_t|x_{t-1}, G_t) = \beta_t x_{t-1},$$

$\mathrm{Var}(x_t|x_{t-1}, \mathcal{G}_t) = \sigma_\varepsilon^2.$

As soon as $J_t = 0$, then $x_t = \varepsilon_t$ is distributed with zero mean and variance σ_ζ^2.

3. Case Studies: Estimations

In this section, based on the proposed filter, estimation of components and parameters of model of Asako & Liu (2013) is proposed.

Bubble Probabilities

An important component of Asako and Liu's model is π_t, the probability of bubble continuation given $\mathcal{G}_t$. Asako & Liu (2013) assumed that:

$$\pi_t = \exp\{-\alpha - \gamma|x_{t-1}|\}, \alpha, \gamma > 0.$$

Another candidate for π_t is the logit regression model given by:

$$\log\left(\frac{1-\pi_t}{\pi_t}\right) = \alpha - \gamma|x_{t-1}|.$$

As special case, when J_t is independent of J_{t-1}, then:

$$P_{11}^t = P_{01}^t = \pi_t,$$

In this case,

$$v_t = \frac{\pi_t}{1-\pi_t}.$$

which implies that:

$$P(J_t = 1|\mathcal{F}_{t-1}, \mathcal{G}_t) = P(J_t = 1|\mathcal{G}_t).$$

As δ_t is close to 1, then:

$$P(J_t = 1|\mathcal{F}_t, \mathcal{G}_t) = P(J_t = 1|\mathcal{G}_t).$$

In general case, when J_t forms a Markov chain, there is no guarantee for closeness of smooth probability $P(J_t = 1|\mathcal{F}_t)$ and predictive probability:

$$P(J_t = 1|\mathcal{F}_{t-1}, \mathcal{G}_t),$$

to actual probability:

$$P(J_t = 1|\mathcal{G}_t).$$

Here, transition probabilities P_{ij}^t, $i, j = 0,1$ are estimated such that smooth and predictive probabilities are close to actual probability, under different scenarios. To this end, let:

$$l_t = \frac{\pi_t}{1-\pi_t}.$$

Notice that:

$$\pi_t = P_{11}^t\pi_{t-1} + P_{01}^t(1 - \pi_{t-1}),$$

$$1 - \pi_t = P_{10}^t\pi_{t-1} + P_{00}^t(1 - \pi_{t-1}).$$

Therefore, $l_t = g_t(l_{t-1})$. Before going ahead, the following lemma is necessary. Let:

$$\gamma_t = \frac{|P_{00}^t - P_{10}^t|}{(P_{00}^t)^2}.$$

Lemma 2.

For every $0 < a < b$, then: $|g_t(b) - g_t(a)| \leq \gamma_t |b - a|$.

Proof. The first derivative of $g_t(u)$ with respect u is given by:

$$g_t'(u) = \frac{P_{00}^t - P_{10}^t}{(P_{10}^t u + P_{00}^t)^2}.$$

Notice that:

$$P_{10}^t u + P_{00}^t \geq P_{00}^t.$$

Thus,

$$|g_t'(u)| < \gamma_t.$$

The mean value theorem implies that there is a c between a and b, such that:

$$|g_t(b) - g_t(a)| = |g_t'(c)||b - a| \leq \gamma_t |b - a|.$$

This completes the proof of lemma. Notice that:

$$|u_t - l_t| = |\delta_t g_t(u_{t-1}) - g_t(l_{t-1})|.$$

Without loss of generality, suppose that $\delta_t < 1$, then,

$$g_t(u_{t-1}) > \delta_t g_t(u_{t-1}) > g_t(l_{t-1}).$$

Then,

$$|u_t - l_t| < |g_t(u_{t-1}) - g_t(l_{t-1})| < \gamma_t |u_{t-1} - l_{t-1}|.$$

Therefore,

$$|u_t - l_t| < \gamma_t |u_{t-1} - l_{t-1}|.$$

By solving the above inequality, sequentially, and assuming γ_t tends to zero, it is seen that u_t converges to l_t. For the case of $\delta_t > 1$, then,

$$|u_t - l_t| = \delta_t \left|g_t(u_{t-1}) - \frac{1}{\delta_t} g_t(l_{t-1})\right| \leq |g_t(u_{t-1}) - g_t(l_{t-1})|.$$

It is also seen that:

$$|u_t - l_t| < \gamma_t |u_{t-1} - l_{t-1}|.$$

This shows the convergence of u_t to l_t. One can see that the above arguments work well. To this end, notice that:

$$|v_t - l_t| < \gamma_t |u_{t-1} - l_{t-1}|.$$

Therefore, v_t converges to l_t if γ_t is close to zero.

β_t Time Series

The second important element of Asako and Liu's model is the time series of β_t. When $P(J_t = 1|F_t)$ is computed, then it is known that whether $J_t = 1$ or not? Thus, the least square estimate of β_t is given by:

$$\hat{\beta}_t = \frac{\sum_{k=1}^t J_k x_k x_{k-1}}{\sum_{k=1}^t J_k x_{k-1}^2}.$$

One can see that $\hat{\beta}_t$ forms adaptive filter expression as follows:

$$\hat{\beta}_t = (1 - \lambda_t)\hat{\beta}_{t-1} + \lambda_t \frac{x_t}{x_{t-1}},$$

where the forgetting factor is given by:

$$\lambda_t = \frac{J_t x_{t-1}^2}{\sum_{k=1}^{t} J_k x_{k-1}^2} \in (0,1).$$

In practice, it is possible to substitute the J_t with π_t. The above discussions are summarized in the following algorithm.

Computational Algorithm

The computational procedure operationalizes the nonlinear bubble-detection framework within a recursive state-space setting. The algorithm is designed to identify structural shifts in the deviation process, estimate model hyperparameters under both stable and explosive regimes, and generate real-time adaptive probabilities of bubble continuation through Hamilton filtering. The procedure consists of three main stages: diagnosis, parameter estimation, and recursive updating.

- *Diagnosis*. Take a segment of initial values of deviation time series and find the sequential beta estimates. Determine whether there has been a change in betas based on difference of successive betas. Change point analysis test are useful in this case.
- *Parameter estimation*. Estimate the hyper-parameters σ_ε and σ_ζ from a homogenous set where there is no significant change (bubble) in betas. Again, from a set with the presence of change in betas, estimate parameters of α, γ of logit regression.
- *Recursive updating*. Fix t. Compute π_t, δ_t, u_t, v_t and β_t. Then, update π_{t+1} and β_{t+1} from their posterior recursive relations.

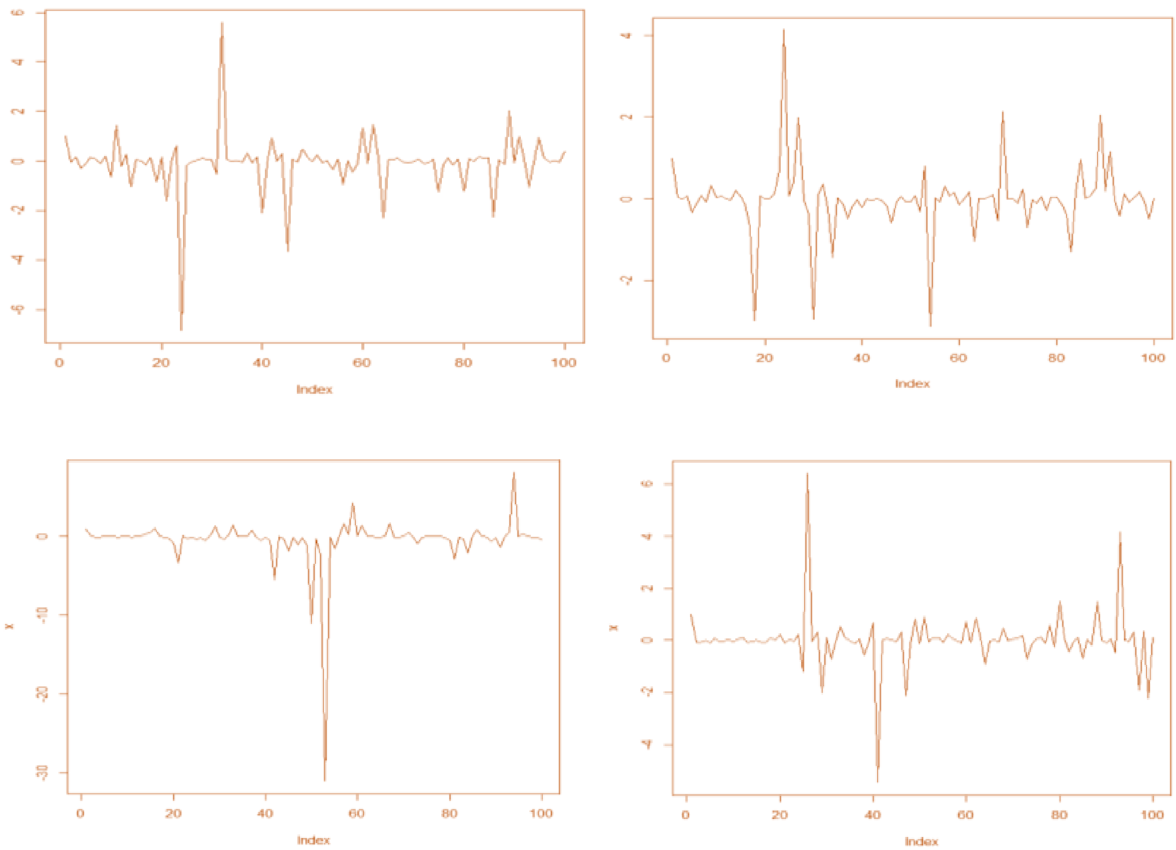
The proposed recursive filtering procedure is computationally efficient and scales linearly with the sample size. Each iteration involves a fixed number of state-space updating operations, yielding an overall computational complexity. In contrast, the GSADF procedure requires repeated estimation over multiple expanding and rolling subsamples, leading to substantially higher computational burden, typically of order depending on window specifications. While GSADF is powerful for retrospective bubble date-stamping, its recursive re-estimation structure can be computationally intensive in large samples or high-frequency settings. Relying solely on forward recursion, the Hamilton-filter implementation avoids repeated re-estimation and matrix recompilation, making it particularly suitable for real-time monitoring applications and continuous updating environments.

Example. Consider 100 deviations with parameters

$n = 100$	$x_0 = 0.3$
$\sigma_\varepsilon = 0.1$	$\sigma_\zeta = 1$
$\alpha = 0.5$	$\gamma = 2$

The following graphs from Figure 1 give some realizations of time series:

Figure 1: Some realization of above time series



Rate of Convergences

In this section, following Nascimento & Sayed (2000) learning curves of filters, the rate of convergences is studied. The relation of Remark 1 states that:

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \lambda_t \frac{e_t}{x_{t-1}},$$

where: $e_t = x_t - \hat{\beta}_{t-1} x_{t-1}$.

One can see that:

$$e_t = \beta_{t-1} x_{t-1} - \hat{\beta}_{t-1} x_{t-1} = \tilde{\beta}_{t-1} x_{t-1},$$

where $\tilde{\beta}_t = \beta_t - \hat{\beta}_t$.

Thus,

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \lambda_t \tilde{\beta}_{t-1}.$$

Equivalently, it is seen that:

$$\tilde{\beta}_t = (1 - \lambda_t)\tilde{\beta}_{t-1} + \zeta_t.$$

It is obvious that $E(\tilde{\beta}_t) = 0$ and the above relation forms a stationary AR(1)-type process for $\tilde{\beta}_t$. Before going ahead, the following lemma is useful.

Lemma 3: For every finite variance random variable X, Y, then:

$$\mathrm{Var}(X+Y) \le 2(\mathrm{Var}(X)+\mathrm{Var}(Y)).$$

For proof, using Cauchy Schwartz inequality, it is seen that:

$$\mathrm{Var}(X+Y) \le \mathrm{var}(X)+\mathrm{Var}(Y)+2\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)} = (\sqrt{\mathrm{Var}(X)}+\sqrt{\mathrm{Var}(Y)})^2 \le 2\big(\mathrm{Var}(X)+\mathrm{Var}(Y)\big).$$

Therefore,

$$\mathrm{Var}(\tilde{\beta}_t) \le 2\mathrm{Var}\left((1-\lambda_t)\tilde{\beta}_{t-1}\right)+2\mathrm{Var}(\zeta_t).$$

Assuming $(1-\lambda_t) \le \frac{1}{\sqrt{2}}$, then:

$$\mathrm{Var}(\tilde{\beta}_t) \le \mathrm{Var}(\tilde{\beta}_{t-1})+2\mathrm{Var}(\zeta_t).$$

Hence,

$$\mathrm{Var}(\tilde{\beta}_t) \le 2\sum_{k=1}^{t}\mathrm{Var}(\zeta_t) \coloneqq 2\varphi_t.$$

Hence,

$$\mathrm{Var}(\tilde{\beta}_t) = O(\varphi_t).$$

Some Remarks

Remark 1. The above relation is useful for estimating hyper-parameters. Let T_1 be the set of times t's, at which for all $t \in T_1$, then $J_t = 1$. Then,

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \lambda_t\left(\frac{x_t}{x_{t-1}} - \hat{\beta}_{t-1}\right).$$

Therefore, the sample variance of:

$$\lambda_t\left(\frac{x_t}{x_{t-1}} - \hat{\beta}_{t-1}\right)$$

estimates the σ_ζ^2. The sample mean of initial $\hat{\beta}_t$'s, estimates the β_0. Again, let for all $t \in T_0$, then $J_t = 0$. Then, $x_t = \varepsilon_t$. Thus, the sample variance of x_t's $t \in T_0$ estimates the σ_ε^2.

Remark 2. In practice, the probability π_t is a good proxy for J_t. Here, again the above-mentioned recursive relation of $\hat{\beta}_t$ works, however, λ_t is given by:

$$\lambda_t = \frac{\pi_t x_{t-1}^2}{\sum_{k=1}^{t}\pi_k x_{k-1}^2}.$$

This estimation is achievable by minimizing the weighted recursive least square criterion:

$$\sum_{k=1}^{t}\pi_k(x_k - \beta_k x_{k-1})^2.$$

Remark 3. These are recursive least square (adaptive) filter for $\hat{\beta}_t$. In practice, various types of Kalman filters and particle filters are applicable which is omitted, here. Again, the rate of convergence is $O(\varphi_t)$.

4. Empirical Evidence from the USD–Iranian Rial Exchange Rate

To provide an applied economic validation of the proposed Hamilton-filter framework, this section examines the existence of multiple rational bubbles in the USD–Iranian Rial exchange rate over the period 2000–2020. Monthly data are used to analyse six sub-periods characterized by heightened exchange rate volatility: (Mar 2000–Nov 2000), (Nov 2005–Nov 2006), (Mar 2009–Jun 2009), (Nov 2014–Feb 2015), (Apr 2015–Sept 2015), and (Jan 2017–Jan 2020).

In addition to the nominal exchange rate, the relative prices of tradable and non-tradable goods are examined to assess whether explosive exchange rate behaviour is driven by structural price imbalances. Bubble detection is conducted using ADF, SADF, and GSADF tests following Phillips and Shi (2018). The results are reported in Table 1.

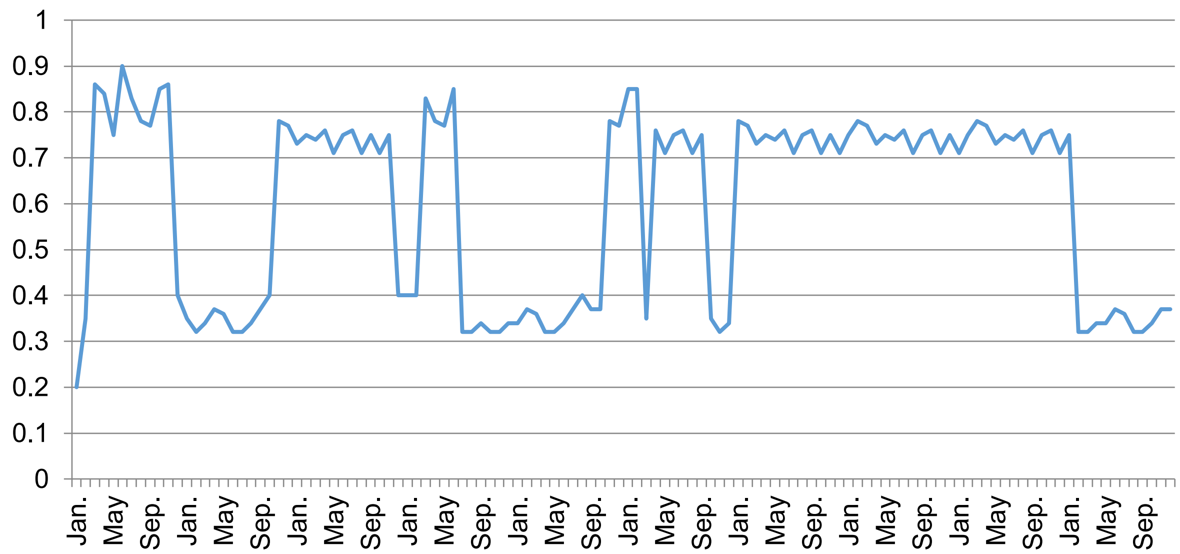
Table 1: ADF, SADF, and GSADF Test statistics

Variable	ADF	SADF	GSADF
Exchange Rate	-1.31	2.70	10.60
	-3.42	0.49	1.37
Tradable Prices	-1.36	1.90	5.80
	-3,22	0.49	1.37
Non-Tradable Prices	-2.38	0.94	1.41
	-3.42	0.49	1.55

The null hypothesis of no speculative bubble is rejected in favour of the alternative hypothesis of multiple explosive episodes, particularly under the GSADF test, which exhibits greater power in detecting episodic bubbles. The results indicate that explosive behaviour is more pronounced in the exchange rate and tradable goods prices compared to non-tradable goods. Markov-switching models yield qualitatively similar results, reinforcing the robustness of the findings.

Figure 2 presents the time-varying conditional probabilities of bubble existence obtained from the Hamilton filter. These probabilities allow precise identification of bubble episodes and provide a real-time monitoring perspective.

Figure 2: Plots of conditional probabilities of $J_t = 1$



The detected bubble periods correspond to significant macroeconomic and policy events in Iran. For example, exchange rate unification policies and macroeconomic adjustment programs in the early 2000s generated temporary misalignments. Subsequent episodes (2008–2012 and 2017–2020) are associated with currency depreciation pressures, sanctions, capital flight, and increased public demand for foreign currency as a store of value.

Importantly, the stronger explosive signals observed in tradable goods prices suggest that external-sector imbalances and import price pressures play a larger role in exchange rate instability than domestic non-tradable price dynamics.

Overall, the empirical findings demonstrate that the proposed Hamilton-filter approach successfully captures multiple rational bubble episodes and provides economically interpretable signals aligned with macroeconomic developments.

Conclusion

This paper has explored the application of adaptive filters in the detection and analysis of speculative bubbles within financial markets. By employing adaptive filtering techniques, we were able to dynamically estimate the underlying value of assets amidst the inherent volatility and noise characteristic of financial data. Our findings indicate that adaptive filters not only improve the accuracy of bubble detection compared to traditional methods but also enhance the responsiveness of the models to rapidly changing market conditions. The results underscore the effectiveness of adaptive filtering in distinguishing between fundamental price movements and speculative mispricing. This capability is particularly valuable in environments characterized by significant behavioural biases and irrational investor sentiments, where traditional valuation approaches may fall short.

The empirical applications presented herein illustrate the potential for adaptive filters to serve as integral tools for policymakers, investors, and risk managers in mitigating the adverse effects of speculative bubbles. However, this study is not without limitations. The performance of adaptive filters can vary depending on the specific design choices, such as the selection of filter parameters and the underlying model assumptions. Additionally, while we focused on certain asset classes, the general-ability of our findings across different markets warrants further investigation.

Future research could also explore the integration of adaptive filters with machine learning techniques, aiming for even greater predictive power and robustness in bubble detection. In conclusion, the application of adaptive filters represents a promising avenue for improving our understanding of speculative bubbles and enhancing decision-making in financial markets. As the financial landscape continues to evolve, ongoing exploration and refinement of these methods will be crucial in effectively managing the risks associated with asset price volatility and irrational behaviour.

Credit Authorship Contribution Statement:

Habibi, R. is only author of paper and is corresponding author and he is responsible for ensuring that the descriptions are accurate.

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Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Data Availability Statement

Simulated openly available data: The simulated data that support the findings of this study are openly available and generated in random number generators of R software.

Ethical Approval Statement

The research utilises publicly accessible secondary financial and macroeconomic data and does not involve human subjects, individual-level confidential information, or experimental interventions. Consequently, in accordance with standard academic research guidelines, formal ethical approval was not required.

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