A Data Envelopment Analysis or Goal Programming Model with Fuzzy Preferences of Decision Makers

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Abstract

Due to the complexity and uncertainty involved in real world decision problems, the determination and interpretation of Decision Makers' preference relations remain a challenging task for them. This paper develops a novel procedure for incorporating preference information in the efficiency analysis of Decision Making Units. The efficiency of Decision Making Units is defined in the spirit of Data Envelopment Analysis, complemented with Decision Maker's preference information. Our procedure begins by aiding the different decision making group members to express their incomplete fuzzy preferences by using a multiple objective linear programming approach for generating a common set of weights in the DEA framework.

Keywords: goal programming; group decision making; data envelopment analysis; modeling; fuzzy.

JEL Codes: E63; 011; Q18; R15.

Introduction

The research area of multiple criteria decision analysis (MCDA) is developed to provide decision aids for complex decision situations. MCDA aims to furnish a set of decision analysis techniques to help decision makers (DMs) that logically identify, compare, and evaluate alternatives according to diversity, usually conflicting, criteria arising from social, economic, and environmental considerations (Ben-Tal and Nemirovsky 1999, Belton and Stewart 2002, Chen, Kilgour and Hipel 2002)

Multiple criteria decision analysis has been studied for helping decision makers to make their final decisions in MCDM (Multiple Criteria Decision Making) problems. One of the main tasks in this research is how to incorporate value judgments of decision makers in decision support systems. If decision makers can make their decisions by seeing efficiencies (or inefficiencies) of alternatives, the idea of DEA (Data Envelopment Analysis) can be applied to MCDM problems. Data envelopment analysis (DEA) is an increasingly popular managerial decision tool that was initially proposed by Charnes, Cooper and Rhodes (1978). As a nonparametric method for estimating production frontiers, DEA measures relative performance of a set of producers or decision making units where the presence of multiple inputs and outputs makes comparisons difficult. A comprehensive survey of DEA research covering its 30 years of history (1978-2008) is presented in Emrouznejad, Parker and Tavares (2007).

Data Envelopment Analysis (DEA) uses the best favorable weight set for the inputs and outputs of each decision-making unit (DMU) to obtain its best possible score. However, current DEA models are difficult to discriminate decision-making units through articulating the decision makers' preferences Banker 1980).

Fuzzy set theory might provide the flexibility needed to represent the uncertainty resulting from the lack of knowledge. There exist numerous opportunities to apply fuzzy sets theory in decision making. Numerous authors have significantly contributed with their works to a better understanding of group decision making or social choice theory and fuzzy multiple criteria decision making theory (Xu 2003)

This paper puts forward a fuzzy Goal Programming approach which is both practical and intellectually compatible with the DEA philosophy. We develop a procedure for incorporating fuzzy incomplete preference information in a novel way in the efficiency analysis of DMU by aiding the Decision Makers in using fuzzy incomplete terms instead of numerical values in order to express their fuzzy incomplete preferences.

The remaining part of this paper is planned as follows: the next section highlights motivations of incorporating value judgments in DEA models, and reviews some existing approaches; we further discuss the approaches related to weights restrictions. In section 2, we propose a new fuzzy DEA/Goal Programming method which allows group members to express their fuzzy incomplete preferences. In section 3 we will illustrate our formulation through a numerical example. Some concluding remarks will be formulated within last section.

1. Incorporation of Value Judgments in Data Envelopment Analysis Models

Charnes *et al.* (1978) present a description of a mathematical programming formulation (CCR model) for the empirical evaluation of relative efficiency of a DMU. This presentation rests mainly on the observed quantities of inputs and outputs for a group of similar referent DMU. Banker *et al.* (1984) provided a formal link between DEA and estimation of efficient production frontiers via constructs used in production economics.

A major feature of the ratio formulation of CCR model is the reduction of multiple outputs as well as multiple inputs for each DMU into a single 'virtual' output and single 'virtual' input. This ratio allows for an efficiency measurement of each DMU. Indeed, this ratio is maximized by forming an objective function for a particular DMU which we are going to refer to as DMU_0.

Provided that input savings lend themselves to estimation in practical applications, DMU managers have to deal with some inputs such as the level of advertising, median income in service area, number of competitors that are out of control. In such cases, data about the extent to which an exogenously fixed input variable may be reduced holds little interest for the DMU manager.

It is imperative that we further extend the DEA models in order to estimate the extent to which the controllable or discretionary inputs can be reduced by the DMU manager while keeping the exogenously fixed inputs at their current level. Banker (1980) specified explicit postulates, such as convexity and monotonicity, for the underlying production possibility set employed for estimating the relative efficiency of individual DMUs. This characterization of the production possibility set will be adopted in order to estimate the relative efficiencies when some of the inputs elude control. By the way, this is one of the DEA model restrictions which are regarding the weights fixation of the inputs and outputs.

Thereby, Pedraja-Chaparro *et al.* (1997) revealed significant variations in the coefficients from year to year even when the basic input-output data are supplied in constant values. It appears that only a minor fraction of the variations may be attributable to technological change.

Sinuany-Stern and Friedman (1998) developed a new method which provides for given inputs and outputs the best common weights for all the units that discriminate optimally between the efficient and inefficient units as given by the DEA. Their approach allows ranking all the units on the same scale. This new method, Discriminant Data Envelopment Analysis of Ratios (DR/DEA), presents a further post-optimality analysis of DEA for organizational units when their multiple inputs and outputs are given. They construct the ratio between the composite output and the composite input, where their common weights are computed by a new non-linear optimization of goodness of separation between the two groups.

Roll *et al.* (1991) argued that DEA is a mathematical programming approach to assessing relative efficiencies within a group of Decision Making Units (DMUs). An important outcome of such an analysis is a set of virtual multipliers or weights accorded to each (input or output) factor considered. These sets of weights are, typically, different for each of the participating DMUs. A constrained version of the DEA model is offered where bounds are imposed on weights, reducing, thus, the variation of the importance accorded to the same factor by the various DMUs.

Pedraja-Chaparro, Salinas-Jimenez and Smith (1998) study the role that weight restrictions play in DEA. Arguably, the decision to include a factor (input or output) in a DEA model represents an implicit judgment that the factor has a non-trivial weight. For that reason, it seems perverse to allow DEA to assign a trivial weight to that factor in assessing the efficiency of a unit. As a result, there is a strong reason for imposing restrictions on factor weights. Conversely, many existing methods of weight restriction are in practice unwieldy. Due to the complexity and uncertainty involved in real world decision problems, the determination and interpretation of the weights given to the output remain a challenging task for the decision makers.

2. Determination of the Fuzzy Weight Given to the Output

Refers to Charnes *et al.* (1978), the CCR formulation developed to evaluate the technical (output) efficiency measure for a DMU target is given by the following linear programming sets:

$$\text{Max } Z_0 = \phi$$

$$\text{Subject to: } \phi.\, Y_0 - Y.W + s^+ = 0 \qquad (1)$$

$$X\, W + s^- = X_0 \text{ and } W, s^+, s^- \geq 0.$$

where: ϕ represents the efficiency measure in the output-oriented CCR model. It is calculated by running the above linear programming algorithm once for each firm in the sample. If $\phi = 1$, firms are considered efficient, while if $\phi < 1$ firms are considered inefficient and ϕ measures how much each output should be expanded for every firm to be considered technically efficient; X is the input matrix used by all firms in the sample; Y is the output matrix produced by all the firms in the sample; X_0 is the input vector consumed by the DMU_0 to produce Y_0; $\mathbf{s}^+$ and $\mathbf{s}^-$ denote the output and input slacks vectors respectively. These slacks allow a handling reduction and an increase in inputs or outputs to reach the boundary of a production frontier; *W* is the weighting vectors of an evaluated DMU_0.

These weights (W vector) reflect the DM's preferences. As it is known, decision making is a basic human activity, in which most decision processes are based on preference relations. Up to now, some studies dealt with the DM's preference where the parameters are usually subjectively fixed and considered as crisp values (Foroughi and Aouni 2012, Pedraja-Chaparro, Salinas-Jimenez and Smith 1997)

In some practical situations, due to either the vague nature of human judgment, high order of the preference relation presented by multiple decision makers, the DMs may obtain some preference relations with entries being fuzzy. Also, and sometimes, because of time pressure, lack of knowledge, and the DM's limited expertise related with the decision making problem, the DMs may develop an incomplete fuzzy preference relation in which some of the elements cannot be provided. In this paper, we use the goal programming method to reflect the decision making preferences in the process of assessing efficiency, such that several incomplete fuzzy preference relations of the decision maker are considered. Therefore, in order to elucidate these preferences, we put forward the following definitions.

Definition 1:

Let $L_d = (l_{jkd})_{m\times m}$ be a preference relation, and then *L* is called an incomplete fuzzy preference relation, if some of its elements cannot be given by the decision maker d, which we denote by the unknown number (π), and the others can be provided by the DM, which satisfy (Chen, Kilgour and Hipel 2011)

$$l_{jkd} \in [0,1] \tag{2}$$

$$l_{jkd} + l_{kjd} = 1\ \forall j,k \in M;\ \forall d \in D; \tag{3}$$

$$l_{jjd} = 0.5 \tag{4}$$

where l_{jkd} Represents the preference degree provided by the decision maker *d*.

Definition 2:

Let $L_d = (l_{jkd})_{m\times m}$ be an incomplete fuzzy preference relation, then *L* is called an additive consistent incomplete fuzzy preference relation, if all the known elements of *L* satisfy the additive transitivity:

$$l_{jkd} = l_{jzd} - l_{kzd} + 0.5\ \forall j,k,z \in M;\ \forall d \in D; \tag{5}$$

For the convenience of computation, we construct an indication matrix $\Delta = (\varsigma_{jkd})_{m\times m}$ of the incomplete fuzzy preference relation $L_d = (l_{jkd})_{m\times m}$, *where*:

$$\varsigma_{jkd} = \begin{cases} 0 & Si \quad l_{jkd} = \pi \\ 1 & Si \quad l_{jkd} \neq \pi \end{cases} \quad \forall j,k,\in M;\ \forall d \in D; \tag{6}$$

Let $W = (w_1, w_2, \ldots w_m)^T$ be the weights vector of the incomplete fuzzy preference relation $L_d = (l_{jkd})_{m\times m}$, *where*:

$$\sum_{j=1}^{m} w_j = 1 \quad \forall j = 1,2,...,m; \qquad w_j \geq 0 \tag{7}$$

If $L_d = \left(l_{jkd}\right)_{m\times m}$ is an additive consistent incomplete fuzzy preference relation, then such a preference relation must satisfy (Xu, Da and Liu 2009):

$$\varsigma_{jkd}(l_{jkd}) = \varsigma_{jkd}\left[\frac{m}{2}\left(w_j - w_k\right) + 0.5\right] \forall j,k,\in M; \forall d \in D; \tag{8}$$

However, in the general case, eq. (8) does not hold. Refers to (Xu, Da and Liu 2009), we shall relax eq. (8) by looking for the weights vector of the incomplete fuzzy preference relation $L_d = \left(l_{jkd}\right)_{m\times m}$ that approximates eq. (8) by minimizing the error $\varepsilon_{\mathrm{jkd}}$, *where*:

$$\varepsilon_{jkd} = \left|\ \varsigma_{jkd} l_{jkd} - \varsigma_{jkd}\left[\frac{m}{2}\left(w_j - w_k\right) + 0.5\right]\ \right| = \varsigma_{jkd}\left|l_{jkd} - \frac{m}{2}\left(w_j - w_k\right) - 0.5\right| \quad \forall j,k,\in M;; \forall d \in D \tag{9}$$

Thus, we can construct the following multi-objective programming model:

$$\min \varepsilon_{jkd} = \varsigma_{jkd}\left|l_{jkd} - \frac{m}{2}\left(w_j - w_k\right) - 0.5\right| \qquad \forall j,k,\in M; \forall d \in D;$$

S/C: (10)

$$\sum_{j=1}^{m} w_j = 1 \quad \forall j = 1,2,...,m; \ w_j \geq 0$$

The problem of finding a weights vector can also be formulated as the following programming model:

$$\min \sum_{d=1}^{D} \sum_{j=1}^{m} \sum_{k=1,k\neq j}^{m} = \left|\varsigma_{jkd}\left|l_{jkd} - \frac{m}{2}\left(w_j - w_k\right) - 0.5\right|\right| \forall j,k,\in M; \forall d \in D;$$

S/C: (11)

$$\sum_{j=1}^{m} w_j = 1 \quad \forall j = 1,2,...,m; \ w_j \geq 0.$$

Solution to the above minimization problem is found by solving the following goal programming model:

$$Minimiser\left(\delta^{+}_{jkd} + \delta^{-}_{jkd}\right)$$

S/C (12)

$$\varsigma_{jkd}\left[l_{jkd} - \frac{m}{2}\left(w_j - w_k\right) - 0.5\right] = \delta^{-}_{jkd} - \delta^{+}_{jkd} \qquad \forall j,k,\in M; \ \ \forall j \neq \mathrm{k}; \ \forall d \in D;$$

$$\sum_{j=1}^{m} w_j = 1 \quad \forall j = 1,2,...,m;$$

$$\sum_{j=1}^{m} w_j = 1$$

$$\delta^{+}_{jkd}, \delta^{-}_{jkd} \geq 0 \ \forall j,k,\in M;$$

The model (eq. 12) can be approached with the following fuzzy multiple objective programming model:

$$Max\lambda$$

S/C: (13)

$$1-(\delta_{jkd}^{-}-\delta_{jkd}^{+})\geq\lambda$$

$$\varsigma_{jkd}\left[l_{jkd}-\frac{m}{2}\left(w_j-w_k\right)-0.5\right]=\delta_{jkd}^{-}-\delta_{jkd}^{+}\quad\forall j,k,\in M;\quad\forall j\neq k;\ \forall d\in D;$$

$$\sum_{j=1}^{m}w_j=1\quad\forall j=1,2,\ldots,m;\ w_j\geq 0$$

$$\delta_{jkd}^{+},\delta_{jkd}^{-}\geq 0\ \forall j,k,\in M;\ \forall d\in D;$$

The weight vector $W=\left(w_1,w_2,\ldots w_m\right)^T$ of the incomplete fuzzy preference relation $L_d=\left(l_{jkd}\right)_{m\times m}$ can be obtained by solving the mathematical model (eq. 13).

Each decision maker *d* who is a member of the group decision making provides fuzzy incomplete preferences instead of precise preferences for a pairwise comparison matrix l_d (for example in the case that we consider 3 DMUs):

$$l_d=\begin{bmatrix}0.5 & l_{12d} & l_{13}\\ l_{21d} & 0.5 & l_{23d}\\ l_{31} & 0.l_{32d} & 0.5\end{bmatrix}\qquad(14)$$

where: l_{12d} represents the preference degree of DMU1 to DMU2, the degree is provided by the decision maker *d*.

Combining models (eq. 1) and (eq. 13) will result to a fuzzy DEA GP model that integrates explicitly the fuzzy preference of the FDM, as follows:

$$Max[\phi+\lambda]$$

S/C: (15)

$\phi.\,Y_0 - Y.W + s^{+} = 0$

$X\,W + s^{-} = X_0 \qquad s^{+},\ s^{-}\geq 0$

$$1-(\delta_{jkd}^{-}-\delta_{jkd}^{+})\geq\lambda$$

$$\varsigma_{jkd}\left[l_{jkd}-\frac{m}{2}\left(w_j-w_k\right)-0.5\right]=\delta_{jkd}^{-}-\delta_{jkd}^{+}\quad\forall j,k,\in M;\quad\forall j\neq k;\ \forall d\in D;$$

$$\sum_{j=1}^{m}w_j=1\quad\forall j=1,2,\ldots,m;$$

$$w_j\geq 0$$

$$\delta_{jkd}^{+},\delta_{jkd}^{-}\geq 0\ \forall j,k,\in M;\ \forall d\in D;$$

The software LINGO package can be used to solve program (15).

3. Numerical Example

In this section, a numerical example will be utilized to illustrate the application of the developed model. Suppose that we consider the case of three decision maker who have given their preferences on the three DMUs in three different formats, *i.e.:*

$$l_1 = \begin{bmatrix} 0.5 & 0.6 & \pi \\ 0.4 & 0.5 & 0.3 \\ 1-\pi & 0.7 & 0.5 \end{bmatrix} \qquad l_2 = \begin{bmatrix} 0.5 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.4 \\ 0.6 & 0.6 & 0.5 \end{bmatrix} \qquad l_3 = \begin{bmatrix} 0.5 & \pi & 0.3 \\ 1-\pi & 0.5 & 0.5 \\ 0.7 & 0.5 & 0.5 \end{bmatrix}$$

Concluding Remarks

In this paper, the procedure for incorporating fuzzy preference information is proposed for evaluating the efficiency analysis of Decision Making Units. The obtained efficiencies are fuzzy numbers to reflect the inherent fuzziness in evaluation problems by aiding the Decision Makers in using incomplete fuzzy preferences relations instead of numerical values in order to express their fuzzy preferences.

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