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Numerical Simulations of How Economic Inequality Increases in Democratic Countries

Taiji HARASHIMA
Department of Economics
Kanazawa Seiryo University, Japan
harashim@seiryo-u.ac.jp

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Abstract:

It is not easy to perform a numerical simulation of the path to a steady state in dynamic economic growth models in which households behave by generating rational expectations. It is much easier, however, if households are assumed to behave according to a procedure based on the maximum degree of comfortability (MDC), where MDC indicates the state at which a household feels most comfortable with its combination of income and assets. In this paper, I simulate how economic inequality increases in democratic countries under the supposition that households behave according to the MDC-based procedure. The results indicate that high levels of economic inequality can be generated and even increase in a democracy. As causes, I postulate households' misunderstandings of the economic situation, a government against certain groups in the economy, or an upward trend in temporary rent incomes. I then present a criterion for establishing the socially acceptable level of economic inequality and point out a practical shortfall arising from the inability to distinguish temporary economic rents.

Keywords: democracy; economic rent; economic inequality; government transfer; heterogeneity; simulation.

JEL Classification: E17; E60; E63, E71; H2.

Introduction

It is not easy to perform a numerical simulation of the path to a steady state in dynamic economic growth models in which households behave by generating their own rational expectations. This is because there is no closed form solution in these models; however, Harashima (2022c) presented a completely different way to perform this simulation by using the concept of maximum degree of comfortability (MDC), where MDC indicates the state at which a household feels most comfortable with its combination of income and assets.

Usually, it is assumed that households behave by generating their own rational expectations to reach a steady state, but Harashima (2018a[1]) showed an alternative procedure for households to reach a steady state. In this procedure, households maintain their capital-wage ratio (CWR) at MDC, and their behaviour under the MDC-based procedure is equivalent to that of households who base their behaviour on rational expectations. That is, it is equivalent to the behaviour under a procedure based on the rate of time preference (RTP) (Harashima 2018a, 2021a, 2022a[2]). However, unlike the case of the RTP-based procedure, the path to a steady state can be easily simulated if households behave according to the MDC-based procedure because households are not required to do anything equivalent to computing a complex model.

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1 Harashima (2018a) is also available in Japanese as Harashima (2019).
2 Harashima (2022a) is also available in Japanese as Harashima (2022b).

Indeed, a numerical simulation of the path to a steady state under the MDC-based procedure shows that households can reach a steady state without generating any rational expectations (Harashima 2022c), which conforms with theoretical predictions (Harashima 2010[3], 2012[4], 2014) and that a government can achieve a steady state through appropriate intervention, although heterogeneous households cannot necessarily reach their intrinsic CWRs at MDC (a state known as "approximate sustainable heterogeneity (SH)"). Using the same method, Harashima (2023) simulated the effect of economic rents obtained heterogeneously among households. In these simulations, a household was assumed to increase or decrease its consumption according to simple formulas that are assumed to capture and represent a household's behavior under the MDC-based procedure.

The purpose of this paper is to examine how economic inequality increases in democratic countries using the simulation method employed in Harashima (2022c). Economic inequality has long been studied, and many empirical studies have shown that economic inequality has increased in industrialized democracies since the 1980s (Piketty 2003, 2013; Piketty and Saez 2003; Atkinson et al. 2011; Parker, 2014). Wealth inequality has also increased in those countries during the same period (Piketty 2013; Saez and Zucman 2016). Several explanations for the increase in economic inequality have been proposed (Katz and Murphy 1992; Autor et al. 1998, 2003; Card and DiNardo 2002; Leamer 1998; Goldberg and Pavcnik 2007; Helpman 2016; Piketty 2013), although no consensus has formed yet. However, simulations of the development of economic inequality have not been undertaken in the framework of dynamic economic growth models because it is difficult to simulate under the assumptions of the RTP-based procedure.

In theory, there are mechanisms that can make the level of economic inequality increase even in democratic countries (Harashima 2021d). For example, if households misunderstand the economic situation because of their inability to correctly discern the surrounding economic environment, they may not vote for measures to reduce economic inequality. In this paper, I simulate the development of economic inequality when such mechanisms take effect. I then discuss what level of economic inequality many people could agree is socially acceptable.

The results of simulations indicate that, as predicted theoretically in Harashima (2021d), high levels of economic inequality can be generated even in democratic countries and the levels can increase. The increases can result from households' misunderstandings about the economic situation, a government against certain groups in the economy, or an upward trend in temporary rent incomes. Conversely, these simulations indicate that even if a government, or equivalently a majority of voters, believe that an approximate SH has been well established even at a state with a high level of economic inequality, there is no guarantee that this state is socially optimal or acceptable. I discuss what kind of steady state a government should pursue to accomplish socially acceptable inequality, and present a possible criterion for a socially acceptable state. Nevertheless, because of the inability to distinguish temporary economic rents, this criterion may not necessarily be useful in practice.

1. Possible Causes of Increases in Economic Inequality

1.1. Heterogeneities that can cause economic inequality

Economic inequality must be generated by some kinds of heterogeneity among people because no difference in their states can emerge if people are identical. Furthermore, these heterogeneities will be exogenously given and cannot be easily removed because otherwise economic inequality would soon diminish.

Heterogeneous preferences

If RTP is heterogeneous among households, the most advantaged household (i.e., the one possessing the lowest RTP) eventually owns all the capital in an economy, and an extreme economic inequality can be generated (Becker 1980). Harashima (2010, 2012, 2014) showed that, in this case, if advantaged households voluntarily restrain themselves from accumulating capitals to the extent appropriate, a steady state can be reached in which all households achieve all of their optimality conditions (a state called "sustainable heterogeneity" (SH)), but SH is politically vulnerable and generally requires government interventions for its realization. Conversely, in cases where government intervention is not appropriate or feasible, the level of economic inequality can increase. Note that not only can heterogeneous RTPs lead to high economic inequality, but also, by a similar mechanism, to heterogeneous degrees of risk aversion among households (Harashima 2010, 2012, 2014).

[3] Harashima (2010) is also available in Japanese as Harashima (2017b).
[4] Harashima (2012) is also available in Japanese as Harashima (2020a).

Heterogeneous persistent economic rents

Harashima (2016[5], 2017a, 2018c[6], 2018d, 2020b, 2021a) showed that heterogeneity in obtaining economic rents also plays a similar role in the development of economic inequality. Harashima (2016, 2018c) also showed the existence of a type of economic rent that had not been discussed previously: monopoly profits (rents) derived from people's ranking preferences. These rents enable some individuals to be superstars in the world of sport, art, or music (Harashima 2016, 2018c) and enable some corporate executives to earn extremely high compensation (Harashima 2018d). Ranking preference is an important element in product differentiation that allows companies to accrue large amounts of monopoly rent (Harashima 2017a). As a result, product differentiation is one of the most important strategies a company uses to prosper (Porter 1980, 1985), and monopoly rents derived from product differentiation owing to ranking preference are highly likely to be found in an economy.

Furthermore, there is another important kind of economic rent that arises from heterogeneity: mistakes made in business (Harashima 2020b). Here, a "mistake" means, for example, that a household purchases a product at a price that is higher than the cost to produce it plus a normal margin, or that a worker accepts a wage that is lower than their marginal productivity would indicate is appropriate. Because people are certainly heterogeneous in their ability to make fewer mistakes in business dealings, economic rents derived from mistakes are likely to exist ubiquitously and, on an economy, -wide scale (Harashima 2020b).

Family lines consist of households that share similar traits because they have members descended from common ancestors. In addition, in accordance with local customs and for various other reasons, many people marry within the same or a similar group. It is therefore highly likely that abilities such as those related to obtaining economic rents are exogenously and unevenly distributed (Harashima 2020c, 2020d). This means that the average abilities of people in a given group (or family line) will remain different from those in other groups and that some groups (or family lines) will obtain persistent economic rents indefinitely (Harashima 2020c, 2020d). Similarly, there are groups (or family lines), whose economic resources are reduced persistently because of these persistent economic rents. As a result, many economic rents will be enjoyed persistently by only a small number of households and family lines; that is, the persistent economic rents are distributed very unevenly. Moreover, they will be widespread across an economy.

Heterogeneous productivities and success rates of investment

In the production function indicated by equation (1) in Section 2.1, total factor productivity consists not only of technology (A_t) but also of individual (labor) productivity (ω_i) (Harashima 2010, 2012, 2014). Technology can reflect scientific knowledge, for example, and is common to all households (laborers), but individual productivities are heterogeneous across households (laborers).

Harashima (2010, 2012, 2014) showed that, unlike the case of heterogeneous preference, a steady state is naturally achieved without government intervention in the case of heterogenous productivities, and extreme economic inequality does not result. This does not mean that no economic inequality is generated, however. Modest and stable economic inequality is generated so that the capital and consumption of households are heterogeneous and positively proportional to their productivities at steady state, but the level of inequality does not increase beyond this level.

In addition, Harashima (2021c) showed that, if the success rates of investment are heterogeneous across households, the same pattern of economic inequality occurs as in the case of heterogeneous preferences.

1.2. Temporary economic rents

An important element that can also generate high economic inequality is temporary economic rents. Here, "temporary" means that the sum of economic rents obtained by and extracted from an infinitely living household (or family line), i.e., net economic rents, is zero in the long-run for any household (family line), or equivalently the probability of obtaining economic rents is identical for all households, whereas "persistent" means that the sum of economic rents (i.e., net economic rents) is positive for some households (family lines) or the probabilities are heterogeneously distributed (2020e[7]). Economic rents of a temporary nature will certainly exist. Examples include rents from a lottery, gambling, stock speculation, and land or war profiteering. Unlike in the case of persistent economic rents, no household has any intrinsic advantage in obtaining temporary economic rents. Hence,

[5] Harashima (2016) is also available in Japanese as Harashima (2018b).

[6] Harashima (2018c) is also available in Japanese as Harashima (2021b).

[7] Harashima (2020e) is also available in Japanese as Harashima (2021f).

temporary economic rents have no persistent effect on economic inequality in the long run, at least not unless the amount of temporary economic rents is large.

On the other hand, temporary economic rents do have a large instantaneous effect on economic inequality. For example, a household that obtains a temporary economic rent through a stroke of luck can be rich, or a household can randomly become rich because of temporary economic rents. This means that the level of economic inequality superficially looks high. However, the richest set of households can change from period by period, and any given household has an equal chance of becoming the richest at some point in time. Therefore, there is no economic inequality among households in the case of temporary rents because no household is in a more advantageous position economically than any other household with regard to economic rents. To portray this point, I describe economic inequality from which the effect of temporary economic rents is removed as "real" economic inequality, and economic inequality from which it is not removed as "nominal" economic inequality. The estimated levels of economic inequality will be very different depending on whether temporary economic rents are excluded when this inequality is estimated.

In most studies, the level of economic inequality is estimated without excluding temporary economic rents, most likely because of the difficulty in distinguishing between temporary and persistent rent incomes and between rent incomes and non-rent incomes. Hence, the results of these studies may be misleading. Even if a researcher found nominal economic inequality to be high, real economic inequality could still be low. The possibility that the calculated high economic inequality is mostly caused by temporary economic rents (i.e., it is superficial) cannot be denied *a priori*. This element must therefore be treated with great care when examining economic inequality.

1.3. Government interventions and approximate sustainable heterogeneity

As shown in Section 1.1, SH can be achieved if a government intervenes appropriately towards this end, but the best a government can actually do is to achieve an approximate SH, whereby the number of votes cast in an election in response to increases in the level of economic inequality is roughly equal to those cast in response to decreases (Harashima 2018a).

1.4. Factors that can lead to greater economic inequality

Although many studies have concluded that economic inequality has increased recently (Harashima, 2021d), in a democratic country with free and fair elections, an approximate SH will be achieved and therefore economic inequality will stabilize. This means that the prediction that SH can be achieved seems to contradict the reality. However, because an approximate SH crucially depends on votes in elections, the level of economic inequality can increase even in a democratic country (Harashima 2021d). If people cannot correctly know the current economic situation, particularly concerning economic inequality, their votes may be biased or distorted. Even if economic inequality is actually increasing, a majority may misunderstand the situation and believe that the level of economic inequality has stabilized. I now briefly explain the factors that can make economic inequality increase in democratic countries, following Harashima (2021d).

Misunderstanding Type-1: Finite time horizon

Ordinary households will be very anxious about their lives if their incomes continue to decline. However, as long as their incomes continue to increase even a little, they may not mind the current economic situation and its increasing inequality. Furthermore, even if the growth rate of consumption by ordinary households is zero, these households may not complain about the increase in economic inequality between rich households and themselves as long as their growth rate is nonnegative. As a result, ordinary households may not change their voting behaviour.

Misunderstanding Type-2: Limited spatial horizon

Many ordinary households will be very concerned about the economic state of neighbouring households who have standards of living similar to their own. On the other hand, they are probably not interested in the lives of the rich households other than superficially. Hence, ordinary people may show little reaction even if the wealth of a rich household substantially increases. As a result, it may not be until the level of economic inequality between rich households and ordinary households becomes very high that the votes for strengthening measures to decrease inequality exceed the votes against it.

Misunderstanding Type-3: Uncertainty

In an uncertain situation, ordinary households may simply believe that the current situation must be the "normal" one and behave accordingly, even if in reality it deviates considerably from the normal (i.e., they exhibit a normalcy bias; Kahneman et al. 1982). Combined with intrinsic uncertainties, the uncertainty caused by an inability to perceive the surrounding economic situation can lead to misunderstandings by households. For example, even though government intervention is insufficient, it is possible that households will not perceive the increase in economic inequality very well owing to uncertainty, and therefore they may not change their voting behaviour.

Discriminatory government actions

A government may predominantly favour a particular part of the electorate and discriminate against the rest. Even under a democratically elected government, part of the electorate may be favoured for a long period of time. An example of where such a situation may occur is in countries severely divided by factors such as culture, language, religion, and race.

Other possible factors

The amount of temporary economic rents in a period may change over time and furthermore may have an upward trend. The ratio of temporary economic rents to GDP may also have an upward trend. In such a case, nominal economic inequality will increase although real economic inequality is not affected.

Intuitively, it seems likely that a majority (one formed from average citizens or those in the middle tier of society) will support a considerable increase in the tax rate on rich households. However, given the current disparity in wealth between rich and average households, average citizens do not appear to have acted in this manner. A possible explanation for the disparity is that average citizens may think that even if additional taxes are imposed on rich households, the additional revenues will not be distributed to the average household but only to poorer ones. Therefore, the average citizen may be indifferent to such a tax increase.

2. Simulation Method

Simulations in this paper are undertaken on the basis of the SH concepts presenting in Harashima (2010, 2012, 2014) and the MDC-based procedure developed in Harashima (2018a, 2021a, 2022a). These concepts are briefly summarized in Appendixes 1 and 2. The method of simulations is basically the same as that used in Harashima (2022c, 2023), which is briefly explained in Appendix 3,

2.1. Basic Simulation Assumptions

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical. The production function of Economy i ($1 \leq i \leq H$) is:

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \tag{1}$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of a household in Economy i in period t, respectively; ω_i is the productivity of a household in Economy i; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. In simulations, I set $\alpha = 0.65$, $A_t = 1$, and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

By equation (1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ .$$

The amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households although the amount of capital owned (not used) by each household can be heterogeneous. For any i,

$k_{i,t} = \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H}$, where $\breve{k}_{i,t}$ is the amount of capital a household in Economy i owns (not uses).

The capital income of a household in Economy i in period t ($x_{K,t}$) is calculated by

$x_{K,i,t} = r_t \breve{k}_{i,t}$, where r_t is the real interest rate in period t and $r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}}$.

The labour income of a household in Economy i in period t ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that: $x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H}$.

Household savings in Economy i in period t ($s_{i,t}$) are calculated by $s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t}$, *where* $c_{i,t}$ is the consumption of a household in Economy i in period t. In period $t + 1$, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\breve{k}_{i,t+1} = \breve{k}_{i,t} + s_{i,t} \ .$$

The following simple consumption formula is used.

Consumption formula 1**:** The consumption of a household in Economy *i* in period *t* is:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} \ ,$$

and equivalently:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\dfrac{1-\alpha}{\alpha}}\right)^{\gamma} \ ,$$

where: $\Gamma_{i,t}$ is the capital-wage ratio (CWR) of a household in Economy *i* in period *t*, $\Gamma(\tilde{s}_i)$ is $\Gamma_{i,t}$ of a household in Economy *i* in period *t* when the household is at its MDC, and γ is a parameter. In this paper, I set the value of γ to be 0.5. It is assumed that the intrinsic $\Gamma(\tilde{s}_i)$ (i.e., CWR at MDC) of a household is identical across households and economies, and I set this common $\Gamma(\tilde{s}_i)$ to be 0.04 × 0.65/(1 − 0.65) = 0.0743, which corresponds to an RTP of 0.04.

In a heterogeneous population, Consumption formula 1 should be modified to Consumption formula 2. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy *i* in period *t* in a heterogeneous population, and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate CWR).

Consumption formula 2**:** In a heterogeneous population, the consumption of a household in Economy *i* in period *t* is:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t\dfrac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma},$$

and equivalently,

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma}$$

Let κ_i be the $\breve{k}_{i,t}$ that a government aims for in order to induce a household in Economy *i* to own capital at a steady state (i.e., κ_i is the target value set by the government). Under these conditions, the bang-bang control (two-step control) rule of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy *i* to a household in Economy *i* + 1 in period *t* is T_{low} if $\breve{k}_{i,t}$ is lower than κ_i, and T_{high} if $\breve{k}_{i,t}$ is higher than κ_i, where T_{low} and T_{high} are constant amounts of capital predetermined by the government, and if *i* = *H*, *i* + 1 is replaced with 1. In the simulations, T_{low} is set to be −0.1 and T_{high} to be 0.5. The value of κ_i is varied in each simulation depending on what steady state the government aims to achieve.

2.2. Economic Rents

If a household in Economy *i* is assumed to obtain economic rents, these rents are set to be added to the capital it owns. Let $\rho_{i,t}$ be the number of economic rents a household in Economy *i* obtains in period *t*. Consequently, the capital that a household in the other $H-1$ economies own is set to decrease by $\frac{\rho_{i,t}}{H-1}$. The number of economic rents each household in Economy *i* obtains is identical, and the amount of capital decrease in each household in the other $H-1$ economies is also identical. Economic rents may be obtained either each period or intermittently, and they may be obtained either deterministically or stochastically.

Hence,

$$\breve{k}_{i,t} = \breve{k}_{i,t-1} + s_{i,t-1} + \rho_{i,t} \qquad \text{and for any} \qquad j(\neq i), \quad \breve{k}_{j,t} = \breve{k}_{j,t-1} + s_{j,t-1} - \frac{\rho_{i,t}}{H-1} \ .$$

3. Simulation Results

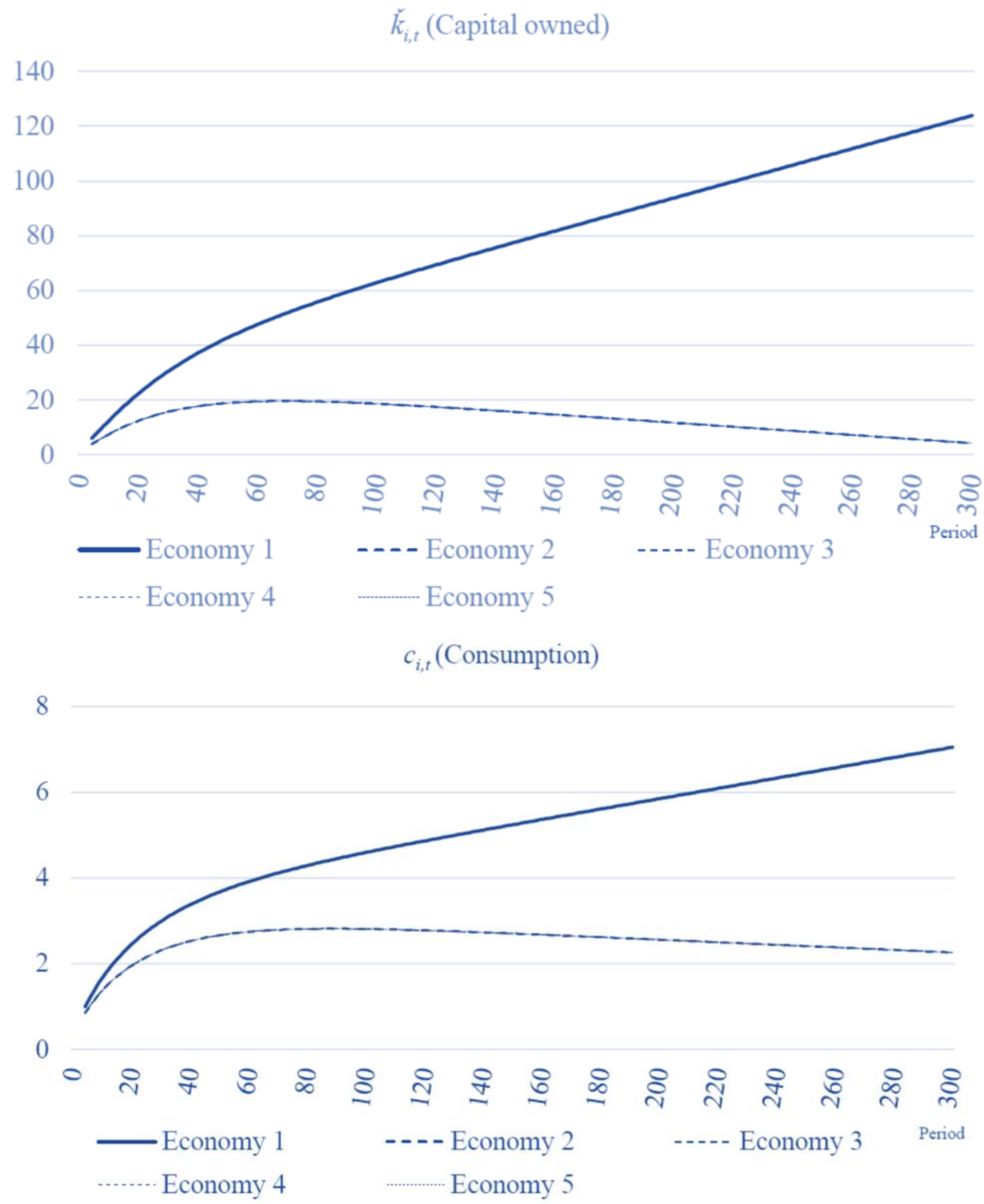
Suppose that there are five economies (Economy 1, 2, 3, 4, and 5) in a democratic country, that the number of households in each economy is identical, and that the households within each economy are identical. Only households in Economy 1 can obtain persistent economic rents, and the amount of capital owned by a household in each of the other four Economies (Economies 2, 3, 4, and 5) is reduced equally by one-fourth (the inverse of the number of economies in question) of the persistent economic rents that each household in Economy 1 obtains. For simplicity, each household in Economy 1 is assumed to obtain the same amount of persistent economic rents in every period (0.3 in this simulation).

3.1. Base Cases

No government intervention

Before examining the possibility that economic inequality may increase in a democratic country, I simulate the case in which a government takes no action about persistent economic rents. The simulated path over time clearly shows that a steady state is not achieved (Figure 1).

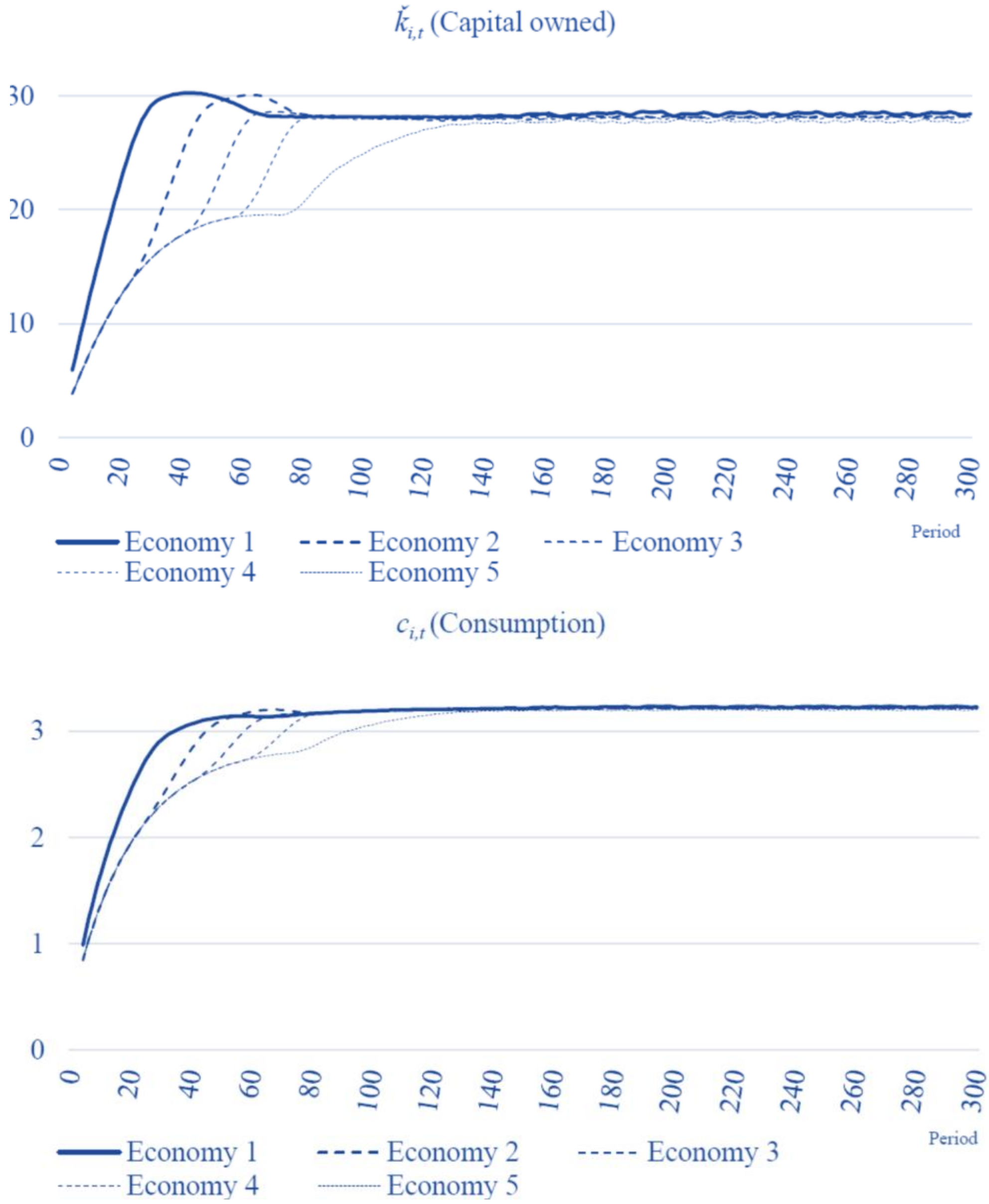
Figure 1. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) for heterogeneous households without government intervention



Sustainable heterogeneity

Conversely, I simulate the case in which a government makes an appropriate intervention and transfers the complete amount according to the transfer rule shown in Section 2.1. The simulated path clearly indicates a steady state, and therefore SH is eventually achieved (Figure 2).

Figure 2. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) for heterogeneous households with appropriate government interventions to achieve an SH

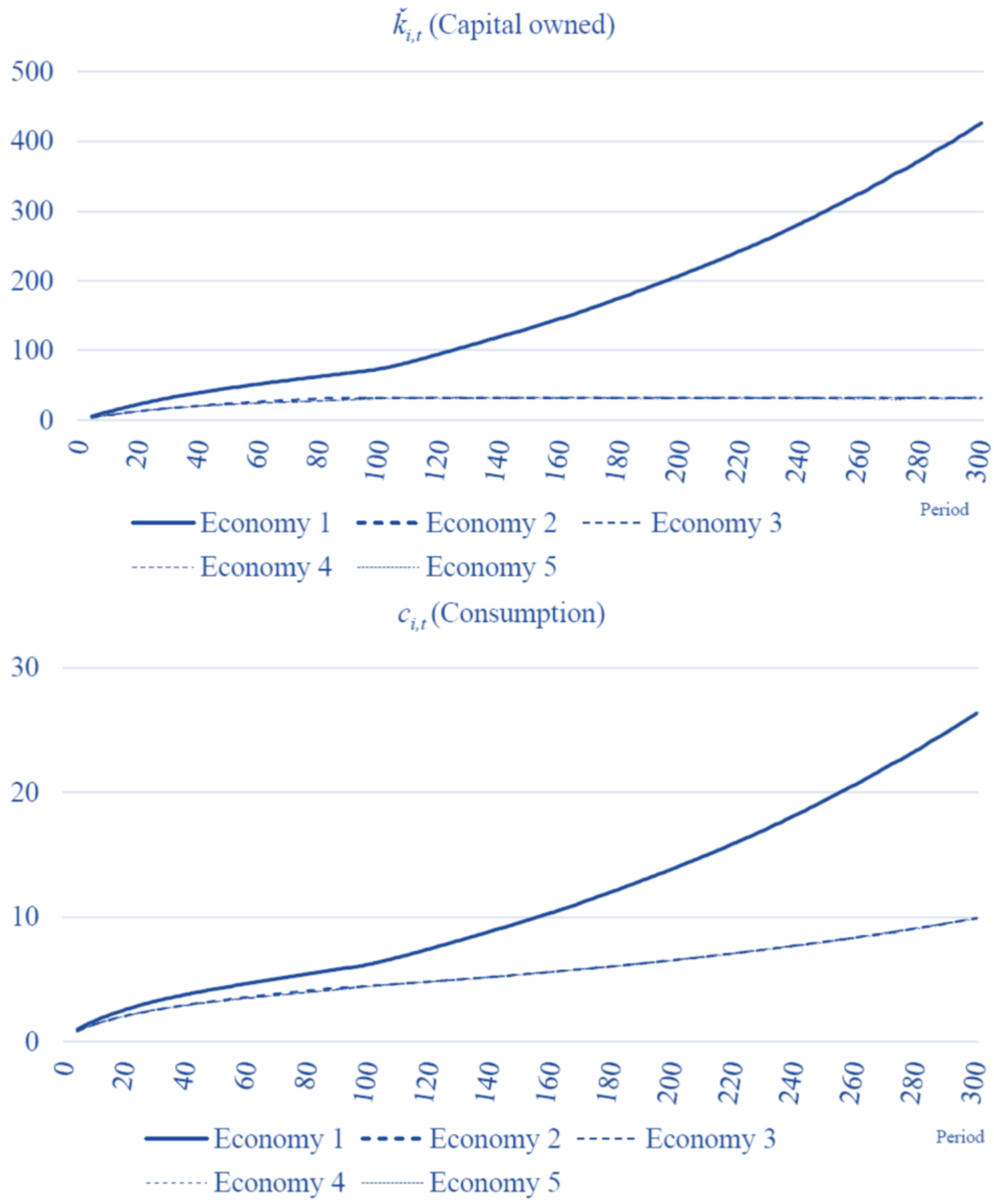


3.2. Misunderstanding Type-1

First, I simulate the case that households have Misunderstanding Type-1: finite time horizons. In this case, I assume that technology (A_t in equation (1)) grows at a constant rate of 0.005 each period. I also assume that government transfers are inappropriate, and as a result, only the capital owned by households in Economy 1 grows, and the amounts owned by households in the other four economies neither grows nor declines (i.e., their growth rates are zero). Despite this, the households in Economies 2, 3, 4, and 5 do not complain about this situation and do not vote against the current level of government transfers because of Misunderstanding Type-1, as long as the growth rates of Economies 2, 3, 4, and 5 are not negative.

The simulated path of this case is clearly not a stable one (Figure 3) in the sense that the growth rate of capital owned is not identical among households (without this misunderstanding, it would be the same for each period for all economies). Nevertheless, if Misunderstanding Type-1 takes effect, the voting behaviors of the majority will not change and the current level of government transfers will be maintained.

Figure 3. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) for heterogeneous households with inappropriate government interventions to achieve an SH because of Misunderstanding Type-1

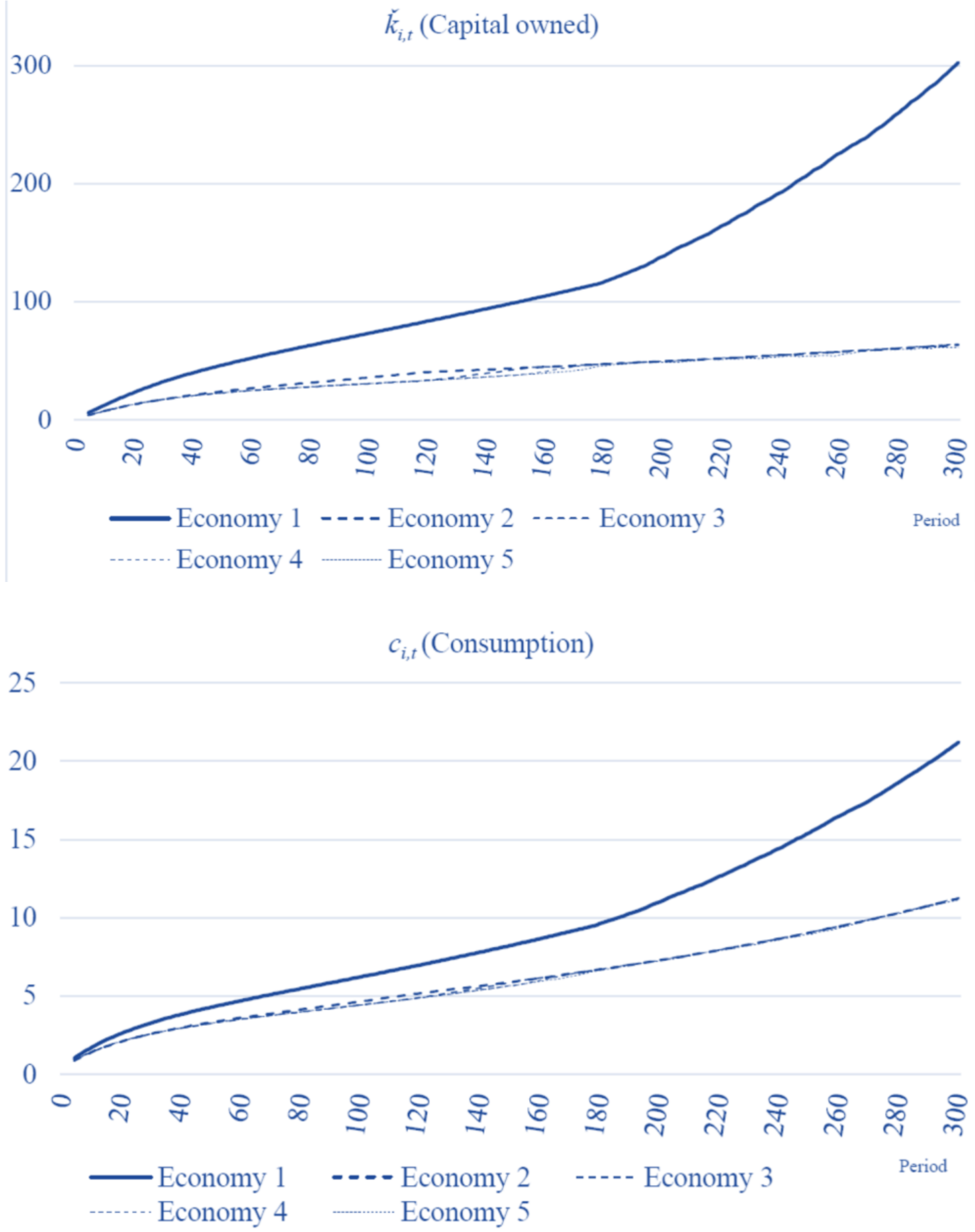


3.3. Misunderstanding Type-2

Next, I simulate the case in which households have Misunderstanding Type-2: limited spatial horizon. Here, I suppose again that technology (A_t) grows at a constant rate of 0.005 every period, but in this case, there is growth of the capital owned by households in all five economies. However, because government transfers would be deemed inappropriate under this misunderstanding, the capital owned by households grows at a higher rate in Economy 1 than in the other four economies, which have identical growth rates. Despite the disparity in growth rates, the households in Economies 2, 3, 4, and 5 do not complain about the situation and thus do not vote against the current level of government transfers because of Misunderstanding Type-2, given that the capital owned by households grows at an identical rate in Economies 2, 3, 4, and 5.

The simulated path in this situation is also clearly not stable (Figure 4). However, if Misunderstanding Type-2 takes effect, a voting majority and the government will maintain the current level of government transfers.

Figure 4. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) for heterogeneous households with inappropriate government interventions to achieve an SH because of Misunderstanding Type-2

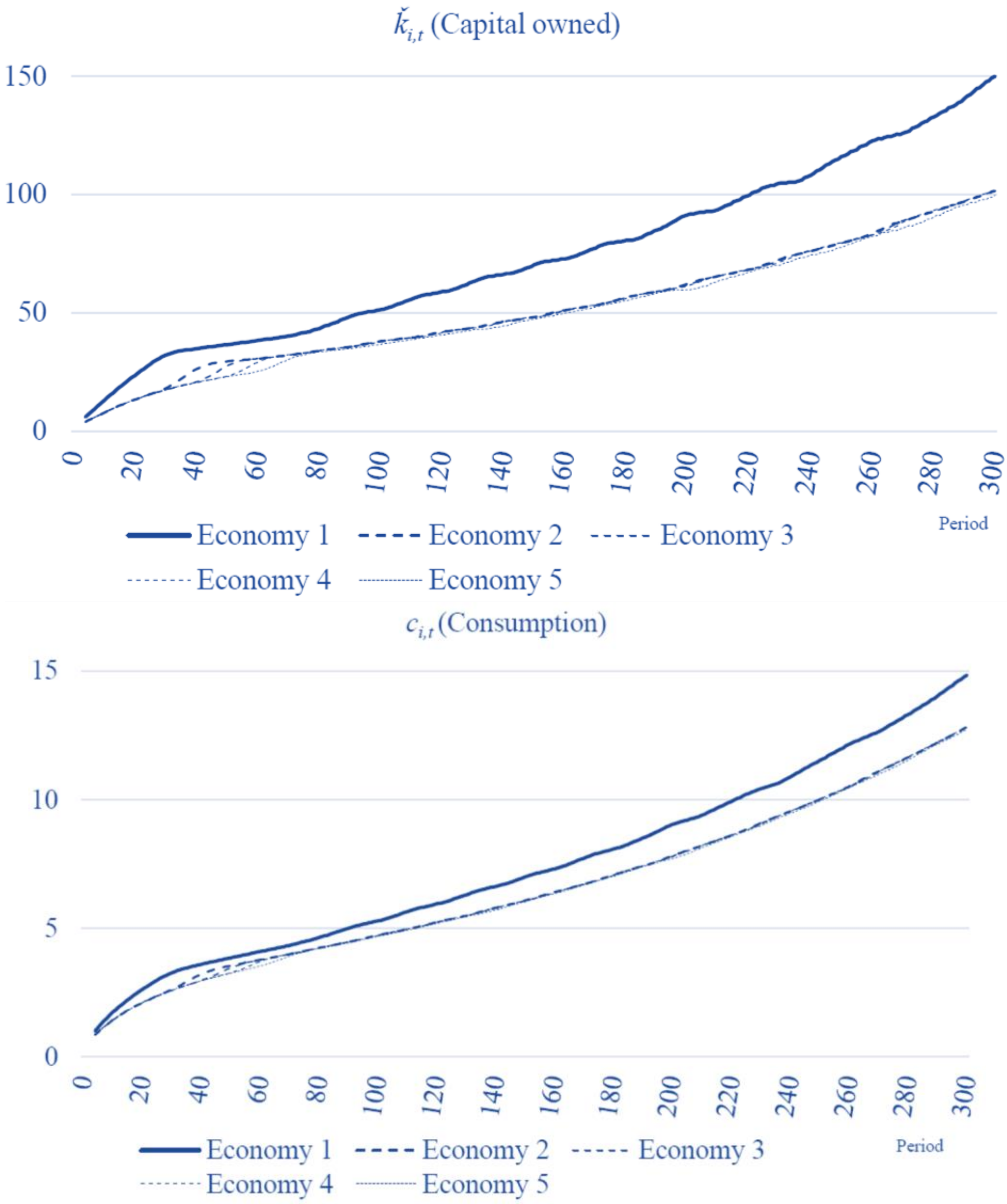


3.4. Misunderstanding Type-3

Thirdly, I simulate the case in which households have Misunderstanding Type-3. Technology (A_t) is assumed to grow constantly at the rate of 0.005 every period. The government transfers an inappropriate amount of income from households in Economy 1 to those of the other four economies such that κ_i for Economies 2, 3, 4, and 5 are equally 20% smaller than the necessary amount for SH. As a result, an economic inequality between Economy 1 and the other four economies is generated. However, the households in Economies 2, 3, 4, and 5 cannot perceive this economic inequality very well and therefore do not complain about the situation and do not vote against the current level of government transfers because of Misunderstanding Type-3, uncertainty about the economic situation.

The simulated path under this transfer policy is also one in which economic inequality is generated (Figure 5), but if Misunderstanding Type-3 takes effect, the current level of government transfers will nevertheless be maintained.

Figure 5. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) for heterogeneous households with inappropriate government interventions to achieve an SH because of Misunderstanding Type-3

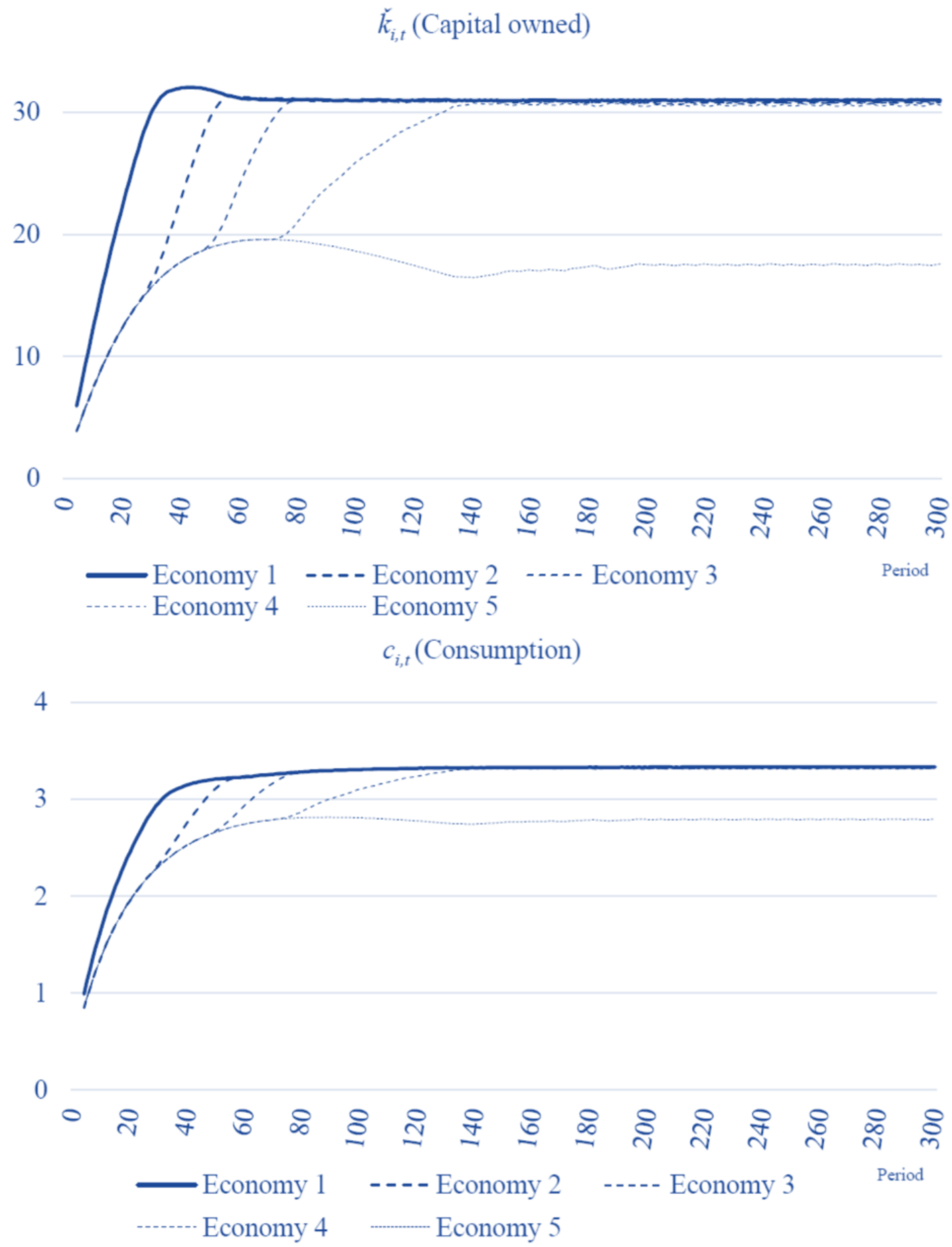


3.5. Discriminatory Government

In the fourth case, I simulate a government that is discriminatory and ignores the situation of one economy. Technology is assumed not to grow. I posit the case that the government intervenes to achieve an SH only among households in Economies 1, 2, 3, and 4, and it ignores Economy 5. In addition, the households in Economies 1, 2, 3, and 4 do not care about the situation of households in Economy 5.

The simulated path in this case clearly shows that economic inequality is generated (Figure 6). However, the discriminatory government (or equivalently, a majority of households) never objects to this inequality, and therefore the current level of government transfers is maintained.

Figure 6. Simulation of capital owned by each household ($\check{k}_{i,t}$) and consumption ($c_{i,t}$) for heterogeneous households with inappropriate government interventions to achieve an SH because the government is discriminatory



3.6. Upward Trend of Temporary Economic Rents

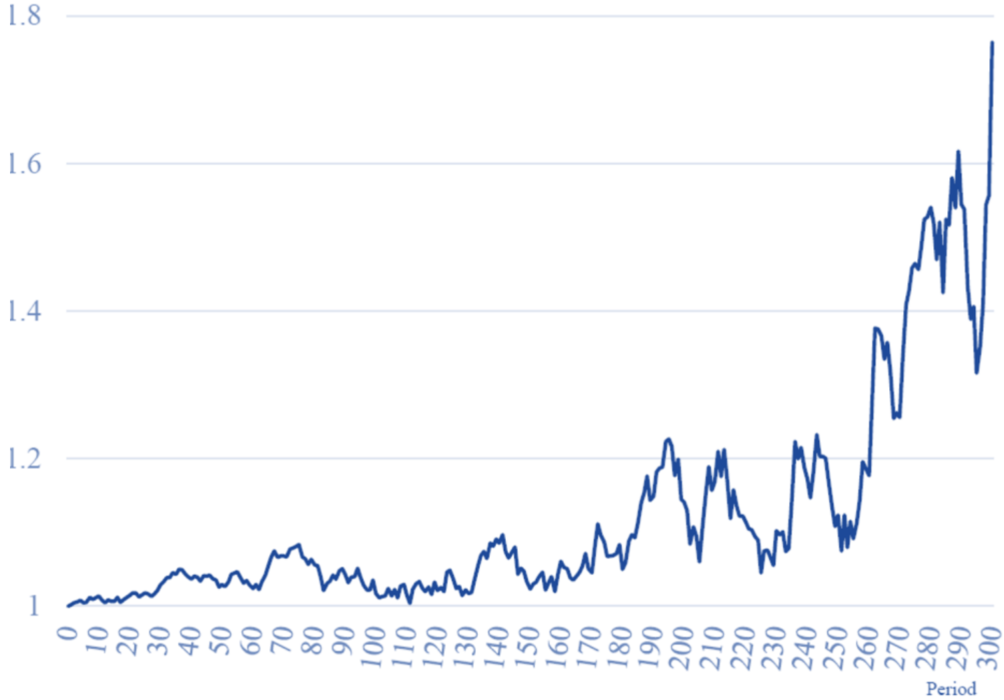
In the final case, I suppose that there is no persistent rent income in any of the five economies and that technology does not grow. However, households in any economy can obtain temporary rent incomes at the same probability. In addition, the amount of temporary rent income increases over time. Specifically, the amount of temporary rent income in period t ($R_{temp,\,t}$) is assumed to be:

$$R_{temp,t} = \lambda \frac{\sum_{i=1}^{5} \check{k}_{i,t}}{5\bar{k}} e^{\mu t}$$

where $\mu(>0)$ is a constant and represents the rate of increase in temporary rent income, $\bar{k}$ is the capital owned by a household at steady state, and $\lambda(>0)$ is a constant. Households do not have any of the specified types of misunderstanding, and the government is not discriminatory. The magnitude of nominal economic inequality is measured by the ratio of "the consumption of the household with the largest consumption among households" to "that of the household with the smallest consumption" in each period.

I set μ and λ to be 0.01 and 0.5, respectively. The simulated magnitudes of inequality indicate that nominal economic inequality increases because of the upward trend in temporary economic rents, whereas real economic inequality does not exist because no heterogeneity among households is assumed (Figure 7).

Figure 7. Simulation of nominal economic inequality when temporary rent incomes increase over time: ratio of "consumption of household with largest consumption" to "that of the household with the smallest consumption"



4. Discussion: Optimal Level of Economic inequality

4.1. State Most Accepted by Society

Problem of approximate sustainable heterogeneity

Whether SH is achieved or not may be a useful criterion for the "optimal" level of economic inequality. However, this criterion is problematic because a government can only achieve an approximate SH and to do so requires intervention. For an approximate SH, votes in elections are essential. If people cannot correctly recognize and utilize the information about the current situation of economic inequality, votes for and against measures on economic inequality can be biased or distorted and therefore government intervention can be also biased or distorted. Hence, even if a majority (or equivalently a government) believe that the economy is currently at an approximate SH, the level of economic inequality can be high and increasing, and furthermore, even if a majority may well know that some people are discriminated against, an approximate SH can be established, as shown in Section 3.

Multilateral state equivalent

Under both MDC- and RTP-based procedures, a multilateral state is uniquely determined (Harashima 2010, 2012, 2014, 2018a), where a multilateral state is an SH that is achieved not by government intervention but merely by households that behave intentionally and voluntarily so that all households can reach a steady state (i.e., they behave multilaterally). The unique determination of a multilateral state means that only one SH exists if a government does not intervene. On the other hand, there are states that are achieved by government intervention and that are equivalent to a multilateral state in that household consumption at such a state is identical to what it would be at a multilateral state (Harashima 2010, 2012, 2014, 2018a, 2022c, 2023). Therefore, if a state that is achieved by government intervention is equivalent to a multilateral state, this state may be labelled as optimal.

There is still one problem, however. Under the MDC-based procedure, even at a multilateral state, a household's CWR is not identical to its CWR at MDC, which is its target. This inability to meet targets means that a multilateral state under the MDC-based procedure may not always be "optimal" even though all optimality conditions of all heterogeneous households are satisfied at this state under the RTP-based procedure.

Socially acceptable state

The term "optimality" may give a strong impression that something is inviolable, but the term can be defined in various ways from different perspectives. For this reason, I depart from the point of view of optimality and now examine economic inequality more loosely as having a criterion that many people will likely agree upon.

A multilateral state means that advantaged households voluntarily restrain themselves from accumulating capital in order to make the entire economy sustainable. Conversely, to make the entire economy sustainable, advantaged households have to tolerate uncomfortable feelings up to the level suffered at a multilateral state. This also means that advantaged households need not tolerate these feelings beyond that level. In other words, the level of uncomfortable feelings at a multilateral state will indicate the maximum number of uncomfortable feelings advantaged households can be expected to tolerate socially. The additional uncomfortable feelings placed upon advantaged households to achieve a multilateral state can thus be seen as a kind of necessary and sufficient obligation, often in the form of taxes, imposed on advantaged households to make the entire economy sustainable.

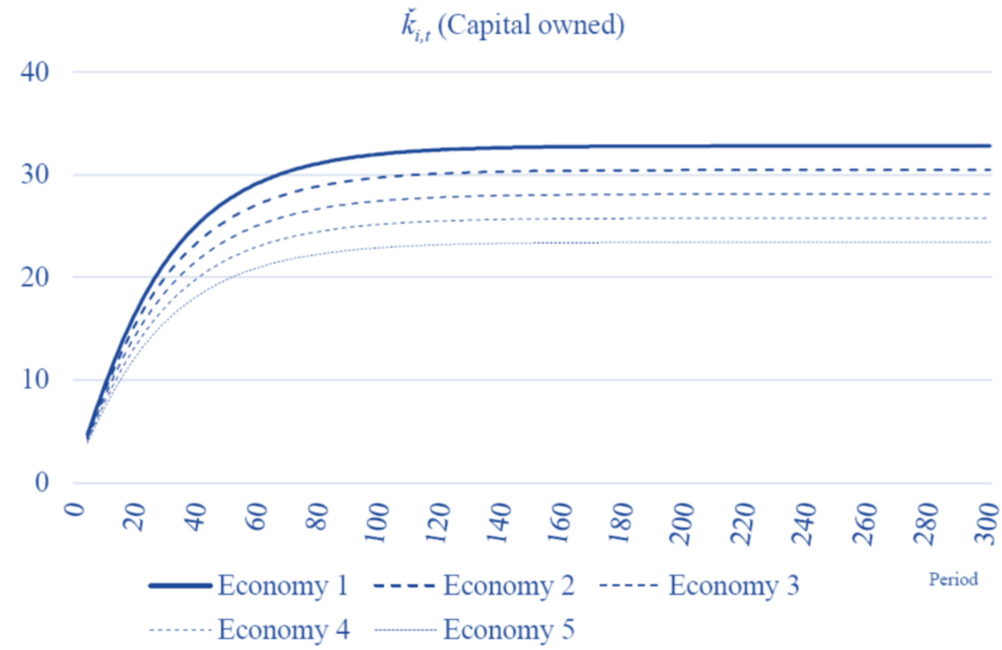
Therefore, a multilateral state likely indicates the steady state at which the combined uncomfortable feelings of all households that have to be tolerated to make the entire economy sustainable are minimized. (Note that this statement depends on how the number of uncomfortable feelings is defined and how the combined uncomfortable feelings are measured and summed.) Although a multilateral state is not perfect, it will be the best option. Based on this concept of a multilateral state, I submit the following criterion for the socially acceptable level of economic inequality: any household's consumption at a steady state is identical to that at a multilateral state.

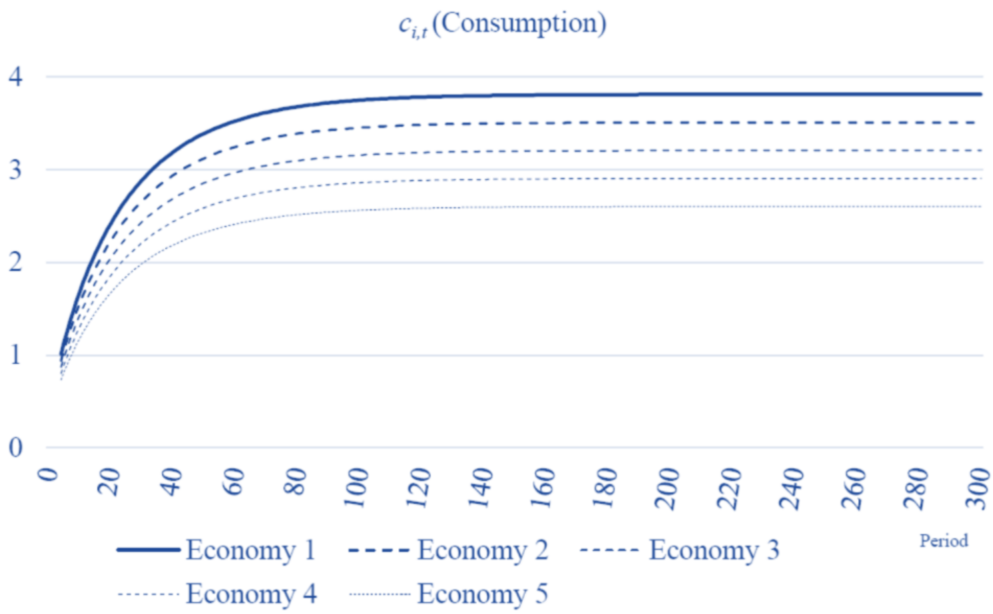
Under both the MDC- and RTP-based procedures, states that are achieved by governments and that are equivalent to a multilateral state, clearly meet this criterion. I would expect that many people could agree that the above criterion is socially acceptable. One upshot to note is that when such a state exists, any household's expected utility is identical to its expected utility at a multilateral state under the RTP-based procedure.

4.2. Heterogeneous Productivity

In the case of heterogeneous productivities, SH is naturally achieved without government intervention (Harashima 2010, 2012, 2014). Therefore, unlike the case of heterogeneous preferences and persistent economic rents, the criterion described in Section 4.1. is naturally and always satisfied.

Figure 8. Simulation of capital owned by each household ($\check{k}_{i,t}$) and household consumption ($c_{i,t}$) in the case of heterogeneous productivities





In the production function in equation (1), ω_i indicates the productivity other than A_t. Heterogeneous productivities means that the values of ω_i are heterogeneous across households (laborers). I simulate this case by supposing that the five economies are identical except for the value of ω_i, which I set to 1.2, 1.1, 1.0, 0.9, and 0.8 for ω_1, ω_2, ω_3, ω_4 and ω_5, respectively. There is no persistent or temporary rent income, technology does not change, all households behave unilaterally, and the government does not intervene. The simulated path (Figure 8) indicates that a steady state is naturally achieved without government intervention as predicted theoretically (Harashima 2010, 2012, 2014). Furthermore, because CWR at MDC of all households is identical (and also equal to the real interest rate) at this steady state, all households perceive that their CWR is equal to their CWR at MDC, i.e., they are satisfied at this steady state.

An important point is that although all households are satisfied, the levels of capital owned and consumption at the steady state differ among households and are almost in proportion to their values of productivity (ω_i), as also predicted theoretically (Harashima 2010, 2012, 2014). Therefore, economic inequality occurs naturally even though all households are satisfied. However, it is not only the case that all households are satisfied but also that the criterion in Section 4.1.3 is naturally satisfied, the economic inequality caused by heterogeneous productivity will be well accepted socially (i.e., it is likely that many people do not complain about this economic inequality).

4.3. A problem Remains: Temporary Rent Incomes

If the criterion presented in Section 4.1. is generally agreed upon as a socially acceptable state, the observed current economic inequality seems to be too high compared with what would be predicted by the criterion. This means that the current level of economic inequality should not be viewed as socially acceptable. If this is true, however, then why would a majority of voters not be in favour of measures to considerably reduce economic inequality? A possible reason for this incomprehensible voting behaviour is the existence of temporary rent incomes. As shown in Section 1.2, temporary economic rents are not related to real economic inequality, but they have considerable effects on nominal economic inequality. Even the household with the lowest productivity in Economy 5 can be super-rich nominally in certain periods thanks to temporary rent incomes.

An important point is that it is difficult to distinguish between temporary and persistent rent incomes and also between rent incomes and other incomes. Hence, even if nominal economic inequality is high, it is difficult to judge whether real economic inequality is also high (i.e., whether it is not at the socially acceptable level). If a majority think that the current high nominal economic inequality is largely due to temporary economic rents (i.e., real economic inequality is low), they may not vote for measures to considerably reduce economic inequality.

Furthermore, rent incomes are judged to be persistent if their amounts are positive on average in the long run. This means that, even though they are persistent, they need not necessarily be obtained constantly in every period. A huge amount of persistent rent income can be obtained in one period but not in other periods. Instead, they have part of their income extracted in conjunction with other households' persistent rent incomes. Like temporary rent incomes, a household's persistent rent incomes will also fluctuate largely. This characteristic makes it more difficult to distinguish between temporary and persistent rent incomes.

After all, even though some criterion for the socially most acceptable state may be presented conceptually like the criterion shown in Section 4.1, it is difficult to actually show whether the current level of economic inequality is socially acceptable. The only practical way to judge whether it is acceptable may still be to count votes in elections, although this presents many problems, including the three types of misunderstanding noted previously, a government against certain groups in the economy, or an upward trend of temporary rent incomes.

Concluding Remarks

Unlike the case of the RTP-based procedure, the path to a steady state will easily be simulated if we suppose that households behave under the MDC-based procedure. Harashima (2022c, 2023) numerically simulated the path to a steady state under the MDC-based procedure and showed that households can reach a steady state without generating any rational expectations—a result predicted theoretically (Harashima 2010, 2012, 2014)—and a government can achieve a steady state by appropriately intervening. In this paper, I examine how economic inequality increases in democratic countries by using the same simulation method.

Harashima (2021d) showed there are mechanisms that can make the level of economic inequality increase even in democratic countries. The results of simulations indicate that, as predicted theoretically (Harashima 2021d), a high economic inequality can be generated and its level can increase even in democratic countries because of households' misunderstandings about the economic situation, a government against certain groups in the economy, or an upward trend in temporary rent incomes. These simulations indicate that even if a government, or equivalently a voting majority, believe that an appropriate SH is well established, there is no guarantee that this state is socially optimal or most acceptable. I discuss what kind of steady state a government should pursue, and present a possible criterion. Nevertheless, because of the difficulty in distinguishing temporary economic rents, this criterion may not be practically useful.

Credit Authorship Contribution Statement

The contributions to this work are solely attributed to Taiji Harashima, who conceptualized the study, conducted the research, performed the experiments, analyzed the data, and composed the manuscript

Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest the work reported in the paper.

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APPENDIX 1

Sustainable heterogeneity

A1.1. Sustainable heterogeneity

Here, three heterogeneities—RTP, degree of risk aversion (DRA), and productivity are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labour is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for international transactions, but in this paper, this concept and the associated terminology are used even if each economy represents a group of identical households in a country.

The production function of Economy *i* (= 1, 2) is:

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \ ,$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of Economy *i* in period *t*, respectively; A_t is technology in period *t*; and α ($0 < \alpha < 1$) is a constant and indicates the labour share. All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance.

$\int_0^t \tau_s ds$, mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ is returns on investments, $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds$ and $\frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ represent income receipts or payments on the assets that an economy owns in the other economy. Hence, $\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ is the balance on goods and services of Economy 1, and $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$ is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that: $\tau_t = \kappa(k_{1,t}, k_{2,t})$.

This two-economy model can be easily extended to a multi-economy model. Suppose that a country consists of *H* economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2, … , Economy *H*). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy *i* in period *t*, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t} u_i''}{u_i'}$, ω_i, and u_i are the RTP, DRA, productivity, and utility function of a household in Economy *i*, respectively (i = 1, 2, …, *H*). The production function of Economy *i* is: $y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha}$.

In addition, $\tau_{i,j,t}$ is the current account balance of Economy *i* with Economy *j*, where *i*, *j* = 1, 2, … , *H* and $i \neq j$. Harashima (2010) showed that if, and only if,

$$\lim_{t \to \infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H} \omega_q}\right)^{-1} \left\{ \left[\frac{\varpi \alpha \sum_{q=1}^{H} \omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q} \right\} \qquad \text{(A1.1)}$$

for any *i* (= 1, 2, … , *H*), all the optimality conditions of all heterogeneous economies are satisfied, where *m*, *v*, and ϖ are positive constants. Furthermore, if, and only if, equation (A1.1) holds,

$$\lim_{t \to \infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \lim_{t \to \infty} \frac{\dot{k}_{i,t}}{k_{i,t}} = \lim_{t \to \infty} \frac{\dot{y}_{i,t}}{y_{i,t}} = \lim_{t \to \infty} \frac{\dot{A}_t}{A_t} = \lim_{t \to \infty} \frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}} = \lim_{t \to \infty} \frac{\frac{d \int_0^t \tau_{i,j,s} ds}{dt}}{\int_0^t \tau_{i,j,s} ds},$$

is satisfied for any *i* and *j* ($i \neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (A1.1) holds is SH by definition.

A1.2. Sustainable heterogeneity with government intervention

As shown above, SH is not necessarily naturally achieved, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ (H – 1) be the combined economy consisting of Economies 1, 2, …, and (H – 1). The population of Economy 1+2+… + (H – 1) is therefore (H – 1) times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+…+ (H – 1) in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ (H – 1) to households in Economy H, and $\bar{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is,

$$g_t = \bar{g}_t k_{1+2+\cdots,+(H-1),t} \ .$$

where: $\bar{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2010) showed that if:

$$\lim_{t\to\infty} \bar{g}_t = \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\omega_H}\right)^{-1} \left\{ \frac{\varepsilon_H \sum_{q=1}^{H} \omega_q - \sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H-1} \omega_q} \left[\frac{\varpi \alpha \sum_{q=1}^{H} \omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\varepsilon_H \sum_{q=1}^{H} \theta_q \omega_q - \theta_H \sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H-1} \omega_q} \right\},$$

is satisfied for any i (= 1, 2, …, H) in the case that Economy H is replaced with Economy i, then equation (A1.1) is satisfied (i.e., SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty} \bar{g}_t$ = constant.

Note that the amount of government transfers from households in Economy 1+2+ … + (H – 1) to a household in Economy H at SH is:

$$(H-1)g_t = (H-1)\, k_{1+2+\cdots+(H-1),t} \lim_{t\to\infty} \bar{g}_t \ .$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy 1+2+ · · · + (H – 1) and vice versa.

APPENDIX 2

The maximum degree of comfortability -based procedure

A2.1. "Comfortability" of capital-wage ratio

Let k_t and w_t be per capita capital and wage (labour income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ where $\tilde{k}_t$ and $\tilde{w}_t$ are household k_t and w_t, respectively. Let Γ be the subjective valuation of $\frac{\tilde{w}_t}{\tilde{k}_t}$ by a household and Γ_i be the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ of household i (i = 1, 2, 3, … , M). Each household assesses whether it feels comfortable with its current Γ (i.e., its combination of income and capital expressed by CWR). "Comfortable" in this context means "at ease," "not anxious," and other similar feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC). MDC therefore indicates the state at which the combination of revenues and assets is felt most comfortable. Let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household i's Γ_i when it is at $\tilde{s}_i$.

A2.2. Homogeneous population

I first examine the behavior of households in a homogeneous population (i.e., all households are assumed to be identical).

A2.2.1. Rules

Household i should act according to the following rules:

Rule 1-1: If household *i* feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any *i*.

Rule 1-2: If household *i* feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any *i*.

A2.2.2. Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period *t* and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state under the RTP-based procedure; that is, it is the steady state in a Ramsey-type growth model in which households behave based on rational expectations generated by discounting utilities by θ, where θ (> 0) is the RTP of a household. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: See Harashima (2018a).

Proposition 1 indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$. This means that both the MDC-based and RTP-based procedures can function equivalently and that CWR at MDC can be substituted for RTP as a guide for household behavior.

A2.3. Heterogeneous population

In actuality, however, households are not identical—they are heterogeneous—and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker 1980; Harashima 2010, 2012). However, Harashima (2010, 2012) has shown that SH exists under the RTP-based procedure. In addition, Harashima (2018a) has shown that SH also exists under the MDC-based procedure, although Rules 1-1 and 1-2 have to be revised, and a rule for the government should be added in a heterogeneous population.

Suppose that households are identical except for their MDCs (i.e., their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ and several other related values. Specifically, let $\Gamma_{R,i}$ be Γ_R of household *i*, *T* be the net transfer that a household receives from the government with regard to SH, and T_i be the net transfer that household *i* receives (*i* = 1,2,3, … , *M*).

A2.3.1. Revised and additional rules

Household *i* should act according to the following rules in a heterogeneous population:

Rule 2-1: If household *i* feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any *i*.

Rule 2-2: If household *i* feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any *i*.

At the same time, the government should act according to the following rule:

Rule 3: The government adjusts T_i for some *i* if necessary, so as to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases.

A2.3.2. Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (an approximate SH), and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure, that is, in a Ramsey-type growth model in which households that are identical except for their θs behave generating rational expectations by discounting utilities by their θs. Furthermore, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in an economy where households are identical except for their θs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.

Proof: See Harashima (2018a).

Proposition 2 indicates that we can interpret $\tilde{S}_{MDC,SH,ap}$ as being equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

APPENDIX 3: Simulation method

A3.1. Simulation assumptions

A3.1.1. Environment

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical.

A3.1.2. Production

The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \qquad \text{(A3.1)}$$

where ω_i is the productivity of a household in Economy i. Because α indicates the labour share, I set $\alpha = 0.65$. In addition, I set $A_t = 1$ and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

With $A_t = 1$ and $\omega_i = 1$, by equation (A3.1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ . \qquad \text{(A3.2)}$$

A3.1.3. Capitals

Because the marginal productivity is kept equal across economies within the country through arbitrage in markets, the amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households in all economies in any period; that is, $k_{i,t}$ is identical for any i although the amount of capital each household owns (not uses) can be heterogeneous. Hence, by equation (A3.2), the amount of production ($y_{i,t}$) is always identical across households and economies regardless of how much capital a household in Economy i owns, when $\omega_i = 1$. In addition, for any i,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H} \ ,$$

where $\check{k}_{i,t}$ is the amount of capital a household in Economy i owns (not uses). As shown above, I set the initial capital of a household owns to be 1 (i.e., $\check{k}_{i,0} = 1$ for any i) throughout simulations in this paper.

A3.1.4. Incomes

The capital income of a household in Economy *i* in period *t* ($x_{K,t}$) is calculated by:

$$x_{K,i,t} = r_t \check{k}_{i,t} \ ,$$

where r_t is the real interest rate in period *t* and,

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ . \qquad \text{(A3.3)}$$

Hence, by equations (A3.1) and (A3.3), the real interest rate r_t is calculated by:

$$r_t = (1-\alpha) k_{i,t}^{-\alpha} = (1-\alpha)\left(\frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H}\right)^{-\alpha} \ .$$

The labour income of a household in Economy *i* in period *t* ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that:

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H} \ .$$

Because the amount of capital used and the amount of labour inputted by a household is identical for any household in any economy when $\omega_i = 1$, household labour income is identical across economies. Note that if productivity ($\omega_{i.t}$) is heterogeneous among economies, production and labour income differ in proportion to their productivities. Note also that in a homogeneous population, the labour income becomes equal to $\alpha y_{i,t}$ for any household.

A3.1.5. Savings

Household savings in Economy *i* in period *t* ($s_{i,t}$) are calculated by:

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} \ .$$

In period *t* + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\check{k}_{i,t+1} = \check{k}_{i,t} + s_{i,t} \ .$$

A3.2. Cconsumption formula

A3.2.1. Consumption formula in a homogeneous population

For a simulation to be implemented, the consumption formula that describes how a household adjusts its consumptions needs to be set beforehand. However, under the MDC-based procedure, there is no strict consumption formula for households. A household just has to behave roughly feeling and guessing (i.e., not exactly calculating) its CWR and CWR at MDC in each period. It increases its consumption somewhat if it feels that $\Gamma(\tilde{s}_i)$ is larger than $\Gamma_{i,t}$ and decreases its consumption somewhat if it feels the opposite way. The amount of the increase/decrease will differ by period. In this sense, the actual formula of consumption under the MDC-based procedure is lax and vague; therefore, it is difficult to set a strict consumption formula with a mathematical functional form.

Nevertheless, if we consider the average consumption over some periods (i.e., moving averages), it will be possible to describe a mathematical form of the consumption formula because households will behave in a similar manner on average. Considering this nature, I introduce the following simple consumption formula because it seems to simply but correctly capture the behaviour of households under the MDC-based procedure on average. Please note that that this consumption formula is not the only possible choice. Other, possibly more complex and subtle, functional forms could be chosen.

Consumption formula 1**:** The consumption of a household in Economy *i* in period *t* is:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} \ , \qquad \text{(A3.4)}$$

where $\Gamma_{i,t}$ is the CWR of household in Economy *i* in period *t* and γ is a parameter.

Because,

$$\theta_i = \left(\frac{1-\alpha}{\alpha}\right) \Gamma(\tilde{s}_i) \ , \tag{A3.5}$$

as shown in Harashima (2018a, 2021a, 2022a), by equation (A3.5), equation (A3.4) is equal to

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right) \left(\frac{\theta_i}{\Gamma_{i,t} \frac{1-\alpha}{\alpha}}\right)^{\gamma} .$$

Athough a household is set to precisely follow equation (A3.4) in the simulations, in reality, they do not behave by calculating equation (A3.4). Furthermore, they are not even aware of Consumption formula 1 itself and cannot know the exact numerical value of each $\Gamma(\tilde{s}_i) = \theta_i \alpha/(1-\alpha)$. Instead, households feel and guess whether they should increase or decrease consumption considering their income and wealth.

That is, Consumption formula 1 is set only for the convenience of calculation in the simulation. It seems to well capture the essence of household behavior in that it increases or decreases consumption depending on a household's feelings with regard to $\Gamma_{i,t}$ and $\Gamma(\tilde{s}_i)$. In this context, the value of parameter γ represents the average adjustment velocity of increase or decrease in consumption.

Consumption formula 1 means that a household's consumption is roughly equal to the sum of its incomes $(x_{L,i,t} + x_{K,i,t})$. The reason for this equality is that there is no technological progress and capital depreciation, so savings stay around zero at the stabilized (steady) state. As mentioned above, the adjustment velocity of consumption in each period is determined by the value of γ in equation (A3.4). As the value of γ is larger, a stabilized (steady) state can be achieved more quickly (if it can be achieved). In this paper, I set the value of γ to be 0.5.

A3.2.2. Consumption formula in a heterogeneous population

As shown in Harashima (2018a, 2021a, 2022a), in a heterogeneous population, a household behaving under the MDC-based procedure does not use its CWR ($\Gamma_{i,t}$) to make decisions about its consumption. Instead, it uses an adjusted value of CWR considering the behaviors of other heterogeneous households and the government because the entire economic state of the country depends on these heterogeneous behaviors in a heterogeneous population. Accordingly, in a heterogeneous population, Consumption formula 1 has to be modified to accommodate the adjusted CWR. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy *i* in period *t* and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate capital-wage ratio).

A3.2.2.1. Consumption formula 2

Unilateral behavior implies that a household behaves supposing that other households must behave in the same manner as it does. In other words, it assumes that other households' preferences are almost identical to its preferences, or at least, its preferences are not exceptional but roughly the same as the preferences of the average household (Harashima, 2018a). If all households behaved in the same manner as a household in Economy *i* did, the real interest rate (r_t) would be equal to the household's $\Gamma_{R,i,t}(1-\alpha)/\alpha$ and eventually converge at its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. Hence, if a household in Economy *i* behaves unilaterally in a heterogeneous population, it feels and guesses that its $\Gamma_{R,i,t}$ $(1-\alpha)/\alpha$ is roughly identical to the real interest rate (r_t). That is, the real interest rate will be used as $\Gamma_{R,i,t}(1-\alpha)/\alpha$, and $r_t\alpha/(1-\alpha)$ will be used as its adjusted CWR ($\Gamma_{R,i,t}$).

Therefore, even if a unilaterally behaving household's raw (unadjusted) CWR is accidentally equal to its CWR at MDC, the household does not feel that it is at its MDC unless at the same time r_t is accidentally equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household will instead feel that the value of r_t will soon change, and accordingly, its raw (unadjusted) CWR will also change soon. That is, it feels and guesses that the entire economic state of the country is not yet stabilized because r_t is not equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. As a result, the household will still continue to change its consumption to accumulate or diminish capital (see Lemma 2 in Harashima, 2018a).

Considering the above-shown nature of the adjusted CWR, Consumption formula 1 can be modified to Consumption formula 2 to use in simulations with a heterogeneous population.

Consumption formula 2: In a heterogeneous population, consumption of a household in Economy *i* in period *t* is:

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t\frac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\frac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma} \tag{A3.6}$$

and equivalently, by equations (A3.5) and (A3.6),

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma}$$

As with $\Gamma_{i,t}$ in Consumption formula 1, the use of r_t in equation (A3.6) does not mean that households always actually behave by paying attention to r_t. What Consumption formula 2 means is that, on average, unilaterally behaving households will feel and guess that r_t represents their adjusted CWRs.

Under the RTP-based procedure, a household changes its consumption according to:

$$\frac{\dot{c}_{i,t}}{c_{i,t}}=\varepsilon^{-1}(r_t-\theta_i),$$

where ε is the degree of relative risk aversion. That is, a household changes its consumption by comparing r_t and its $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household changes consumption as r_t increasingly differs from $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. This household's behaviour under the RTP-based procedure is very similar to that according to Consumption formula 2, which means that the formula is basically consistent with a household's behaviour under the RTP-based procedure.

In addition, in a homogeneous population, r_t is always equal to a homogenous household's $\Gamma_{i,t}(1-\alpha)/\alpha$ because all households behave in the same manner. Hence, equation (A3.4) is practically identical to equation (A3.6) (i.e., Consumption formula 1 is practically identical to Consumption formula 2) because $\Gamma_{i,t}$ in equation (A3.4) can be replaced with $r_t\frac{\alpha}{1-\alpha}$.

A3.2.2.2. Consumption formula 2-a

In Consumption formula 2, a household is supposed to feel that its preferences are not exceptional and almost the same as the preferences of the average household, but it may not actually feel that way. It may instead feel that its preferences are different from those of the average household. In this case, the household will not only feel its preferences are different, but it will also have to guess how far its preferences are from the average (i.e., by how much its adjusted CWR is different from the real interest rate).

For example, a household in Economy *i* may feel and guess that its adjusted CWR is:

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}(r_t+\chi_i) \tag{A3.7}$$

instead of $\Gamma_{R,i,t}=r_t\frac{\alpha}{1-\alpha}$ in Consumption formula 2, where χ_i is a constant and $\chi_i\neq\chi_j$ for any *i* and *j*. χ_i represents the magnitude of how much a household in Economy *i* feels it is different from the average household. I refer to a modified version of Consumption formula 2 in which $r_t\frac{\alpha}{1-\alpha}$ is replaced with $\frac{\alpha}{1-\alpha}(r_t+\chi_i)$ shown in equation (A3.7) as Consumption formula 2-a. In this case, a household in Economy *i* behaves feeling that:

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}(r_t+\chi_i)=\Gamma_{i,t} \tag{A3.8}$$

holds at a stabilized (steady) state that will be realized at some point in the future.

A3.2.2.3. Consumption formula 2-b

In both Consumption formulae 2 and 2-a, the raw (unadjusted) CWR is not included and therefore plays no role. Nevertheless, a household may utilize a piece of information derived from its raw (unadjusted) CWR because past behaviour may contain some useful information for guiding future behaviour. As indicated in Section A3.2.2.2, χ_i is a parameter that indicates how far a household is from the average household. In general, the value of the parameter should be adjusted if households obtain any new and additional pieces of information. This implies that a piece of information derived from the raw (unadjusted) CWR may be used to adjust the value of parameter χ_i.

For example, a household in Economy *i* may use its raw (unadjusted) CWR ($\Gamma_{i,t}$) to adjust the value of χ_i such that:

$$\chi_{i,t}=\chi_{i,t-1}+\zeta_i\left(\Gamma_{i,t}\frac{1-\alpha}{\alpha}-r_{t-1}-\chi_{i,t-1}\right), \tag{A3.9}$$

where $\chi_{i,t}$ is χ_i in period *t*, and ζ_i is a positive constant and its value is close to zero. Equation (A3.9) means that a household in Economy *i* increases the value of $\chi_{i,t}$ a little if its raw (unadjusted) CWR is higher than its adjusted CWR ($r_{t-1}+\chi_{i,t-1}$) in the previous period and vice versa. It fine-tunes $\chi_{i,t}$ in this manner because it feels that equation (A3.8) will eventually hold at some point in the future, as shown in Section A3.2.2.2. The value of ζ_i is close to zero because $\Gamma_{i,t}$ is highly likely to be almost equal to $\Gamma_{i,t-1}$, and therefore, the guess of $\chi_{i,t}$ in period *t* will not change largely from that of $\chi_{i,t-1}$ in period $t-1$. I refer to the modified version of Consumption formula 2-a in which χ_i is replaced with $\chi_{i,t}$ shown in equation (A3.9) as Consumption formula 2-b.

A3.3. Rule of government transfer

Although governments implement transfers among households in complex and subtle manners, a simple bang-bang control is adopted in simulations in this paper as the rule of government transfer for simplicity. In addition, government transfers in each period are assumed to be added to or extracted from the capital of each relevant household in the next period.

In simulations with government transfers, it is assumed for simplicity that there are two economies (Economies 1 and 2) in a country, the economies are identical except for each $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$, and all households in each economy are identical. Let κ be the $\check{k}_{1,t}$ that a government aims for to force a household in Economy 1 to own capital at a stabilized (steady) state (i.e., κ is the target value set by the government). Under these conditions, the bang-bang control of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy 1 to a household in Economy 2 in period t is T_{low} if $\check{k}_{1,t}$ is lower than κ and T_{high} if $\check{k}_{1,t}$ is higher than κ, where T_{low} and T_{high} are constant amounts of capital predetermined by the government.

In the simulations, I set T_{low} to be -0.1 and T_{high} to be 0.5. The value of κ is varied in each simulation depending on what stabilized (steady) state the government is aiming to achieve. Note that because of the discontinuous control signal in bang-bang control, flow variables may show discontinuous zigzag paths but stock variables can move relatively smoothly. These zigzag paths may look unnatural, but they are generated only because of the bang-bang control method that is adopted for simplicity.

Even if a household knows about the existence of government transfers, it still behaves based on Consumption formula 2 (or 2-a and 2-b) with no government transfer. That is, a household uses $x_{L,i,t} + x_{K,i,t}$, not $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}), as the "base" consumption in determining whether it should increase or decrease its consumption. This behaviour superficially may mean that a household does not consider government transfers in the process of adjusting its CWR. However, it is implicitly assumed that a household knows that government transfers exist and that they are an exogenous factor. Therefore, the household feels that the transfers should be removed from the elements that it can change or control freely. Furthermore, it is implicitly assumed that a household correctly knows the exact amount of government transfers.

However, these assumptions may be oversimplifications, and they can be relaxed to allow for incorrect guesses on the amount of government transfers. This relaxation enables a household to use $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}) instead of $x_{L,i,t} + x_{K,i,t}$ in determining its consumption.