The Complex Political Game of Government Formation: A Nash Non-Cooperative Game Perspective

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Abstract

We use the Nash Equilibrium to solve the complex Lebanese political game of forming a government. We solve for a probable and logical outcome given each player's priority issues relative to each other player's priority issues. The results indicate that there is a global best response equilibrium between Hizballah/Amal ("H/A"), Other National Parties ("ONP"), and France.

The equilibrium requires a cooperative approach between H/A and ONP to find a solution that satisfies both, particularly in respect of control over the Ministry of Finance ("MOF"), which represents an important executive position in the country's domestic political system. For example, H/A (given its relatively higher utility for this variable) maintains nominal control over MOF while ONP shares in some manner in the nomination. This would ensure stability of the political regime, which could then facilitate at least some economic reforms. Under this scenario, France is the biggest winner in respect of its regional interests as the success or failure of its initiative in Lebanon may have significant consequences on its credibility in the East Mediterranean region. However, this equilibrium is sensitive to national and regional variables.

Further analysis of the statistics indicates that France, in this game, is not the primary player, as the USA has the capability to sway the game in its favor. The results further indicate a clear conflict between the regional interests of France and the USA. The USA's payoff function was not clear related to the other players and their preferential interests. This may be due to the USA's main interests residing in other national and regional considerations not considered in this game.

Keywords: Nash equilibrium; best response function; Lebanese complex political game; international relations; conflicts; F-score; Z-score.

JEL Classification: C5; C57; C7; C72.

Introduction

The evolution of the French Initiative to form a government in Lebanon may be explained using the basic game theory of conflict assuming four players: Hizballah/Amal ("H/A"), Other National Parties (all major parties, include Patriarch - "ONP"), France, and the USA (Schelling 1981).

The Lebanese domestic political landscape is not, as in most countries, an organized democratic system within which political disputes are resolved. Domestic Lebanon politics may, therefore, be better understood using the tools available for analyzing anarchic systems (Roughgarden 2005), which govern relations between independent states (or between street gangs). It is a system without a "higher authority" to enforce agreements and comes with its own rules of the game. Consequently, there are no "rules" that can be enforced by the "higher authority", and thus the game becomes one of threats of violence or offers of incentives by the different players. In this environment, the credibility of the threats and incentives made are paramount.

Under the French Initiative, H/A were being asked to cede power (as represented by ceding control over the Ministry of Finance "MOF"). It is unclear why they would agree to such a concession. Power does not give itself up voluntarily in an anarchic system. It does so when it is threatened with force or offered appropriate incentives. We understood that the French threat was sanctions. For threats to be effective, they must be credible. Developing the credibility

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of threats is an entire field of study in international relations. It is not enough to simply state behind closed doors that sanctions are "on the table", particularly sanctions which the French do not, in the first place, control – the USA controls its own sanctions without any coordination with the French (and these sanctions are the more painful and plausible ones).

One may question the credibility of the French sanctions threat. The USA undermined that "stick" when it imposed its own sanctions against former ministers (associated with H/A) soon after the start of the French Initiative[3], followed by Secretary of State Pompeo's statement[4]. Moreover, there has been no discussion about EU-specific sanctions and, in any case, the EU has vocally opposed the US's demand to impose snapback sanctions against Iran for violating the JCPOA, suggesting less appetite than the USA for imposing unilateral sanctions.

For a threat to be credible, the threatening party has to be committed in a convincing way that it will execute the threat, but also able to not execute it if the opposing party complies with the demand of the threatening party. France lost on both ends. The USA undermined the French Initiative by demonstrating that France does not control the important sanction stick/carrot.

The French threatening sanctions behind closed doors, Macron's angry reaction towards the journalist who reported it, and the EU's opposition to JCPOA snapback sanctions suggest that France is not committed to executing the threat and has, in any case, left itself a way out of having to carry it out.

Threats are also only effective if the adversary is rational. At times, irrationality is the rational position for a party. "Madmen, like small children, can often not be controlled by threats." The more sophisticated player often plays at a disadvantage because it cannot be reckless. "Give me $10 or I'll cut myself" is a threat only the madman can make. France has an interest in stability in Lebanon (and therefore a disincentive to impose sanctions) and the relatively less sophisticated H/A side has the capability (and incentive) to act the "madman".

Finally, chaos in Lebanon is not in France or the EU's interest (*e.g.,* concern over a potential flow of refugees to Europe, anti-terrorism concerns). Will, then, France seeks to impose crippling sanctions against Lebanese politicians, thereby risking such instability? What does France get out of it? France left itself considerable room to avoid carrying out the threat, rendering it ineffective.

Sanctions would only be effective if the cost on the party being threatened is greater than the cost of complying with the demands. H/A have publicly and loudly committed to the need for them to control MOF as a fundamental part of the power sharing structure. Such public pronouncements are done on purpose in order for the party to binds itself into a position from which it cannot afford to deviate without losing face, thereby making its position "binding" or final and non-negotiable. In an anarchic system, losing face is a substantial cost. "Losing face" is a show of weakness (conceding on a point labeled as fundamental) that reduces the player's public support and suggests to adversaries that further concessions are likely in the future. The conceding party's "firm" positions are no longer firm/credible if it concedes (without rationalization) on a fundamental point.

The dispute over MOF (which, to be clear, represents a battle between the Lebanese political factions over their relative share of power in the country's complex power-sharing system) became even more challenging to resolve after the two domestic players (H/A on one side and the ONP backed by France on the other) have further dug in their heels and locked in their positions in public pronouncements. Each side has firmly committed to its position using all of its symbols of national/religious prestige (*e.g.,* Maronite Patriarch + ONP's leadership vs. Grand Shiite Mufti Qabalan + H/A leadership). Both sides have made it an existential issue to them (Sandholm 2010).

This means, in effect, that neither side is able to back down anymore without a costly loss of face or "national prestige", which would have long-term consequences on its ability to bargain in the future by showing its "firm" positions aren't so firm after all. This is the fundamental problem with and benefit of a player taking a position and "burning the bridge behind it." The player forces the opposing side to "take it or leave it" by rendering itself unable to concede. But if the other player has done the same, then the result is stalemate or escalation of threats, and possibly conflict. Given that Biden currently has an approximately 82% chance of winning the USA elections (as of 6th of October, 2020), and H/A betting on beneficial geopolitical changes in the region if Biden wins (US-Iran deal, loosening of the maximum pressure campaign), this suggests that one side clearly has a greater incentive for stalemate and/or escalation.

Conversely, the French "incentive" was international aid to avoid an economic meltdown. But is this also credible? How much aid? Which aid? Will the USA support or obstruct such aid? Do certain Lebanese sides even need or want this aid at the cost of ceding power domestically and all of the costs that entails for them? In any case, the economic meltdown or possibility of aid may not be sufficient to cause one side or the other to concede on a

[3] https://home.treasury.gov/policy-issues/financial-sanctions/recent-actions/20200908
[4] https://lb.usembassy.gov/designation-companies-and-official/

point labeled as “fundamental” because the idea that "the other side can concede instead of us" is what we're seeing happen at the moment. Why should we expect this to change?

Finally, the failure of the French Initiative is costly for French credibility, not only for its ability to influence events in Lebanon, but elsewhere in the Eastern Mediterranean region (especially if it doesn't follow through on its threats). Countries like Turkey will be observing whether France is willing to carry out its threats. The implications go further than just French credibility in Lebanon. It could affect the country’s ability to do statecraft in the region.

Thus, it would seem that the French have the bigger incentive to compromise and concede on the issue of the MOF (and to pressure the ONP to concede). France put much of its credibility at stake. H/A can afford strategic patience and call out France's bluff. H/A have already committed to their public not to budge. And the French threats and incentives appear insufficient to change their position for reasons discussed above. What further threats or incentives may France offer to reach an agreement and ensure the success of its initiative?

Whichever side concedes will need a way to "rationalize" such concession to save face and avoid losing credibility. Which side will "win"? Or there will be further stalemate. Finally, the loss of face of the side that concedes could lead to it taking steps to avoid such a scenario re-occurring. Does each "party" then have an incentive (if it can) to increase its power and improve its future bargaining position when a deal is eventually struck? Given these circumstances, it appears that every party in Lebanon is incentivized to build up as much power on the ground to improve its bargaining power in any future deal. What is power on the ground in an anarchic system? It is military power. The implications of the incentive structure that is being created in Lebanon are troubling (Sandholm 2010).

Parties will also continue to be incentivized into locking themselves into very firm positions on important political issues (burning the bridge behind them) (Alos-Ferrer and Netzer 2010, Monderer and Shapley 1996). By increasing their military power and locking in their positions, the players improve their bargaining power in any future deal, but also increase the risk of armed conflict. We have laid out the political landscape above. We will now attempt to find a Nash Equilibrium solution in respect of the government formation initiative, which is ongoing.

1. Methodology

The complex Lebanese political game in respect of forming a government. To solve the complex Lebanese political game, we use the Nash Equilibrium to predict a probable and logical outcome given each player’s priority issues relative to each other player’s priority issues (Alos-Ferrer and Netzer 2010, Rosenthal 1973).

1.1. Hypothesis

- We consider a finite game $G \overset{\text{def}}{=} (i, \mathrm{p}, u)$ consisting of:
  - a finite set of players $i = \{1, \dots, n\}; n = 4$;
  - a finite set of pure strategy $\mathrm{p}_i$for each payer i; the set of profiles of the game is $p \overset{\text{def}}{=} \prod_i p_i$;
  - the payers’ payoff functions $u_i : p \to \mathbb{R}$ that players seek to maximize;
- We define the uniqueness Best Response $BR_i(p) \overset{\text{def}}{=} \left\{ \underset{\alpha \in p_i}{\operatorname{argmax}}\, u_i(\alpha; p_{-i}) \right\}$. A NE is a fixed point of the correspondence i.e., a profile p^*such that $p_i^* \in BR_i(p^*)$ for each player i;
- A Nash Equilibrium wherein no player is incentivized to deviate unilaterally, given the strategies played by other players (Monderer, Shapley 1996);
- The main property of initial time of games where players do not interact with one another is $BR_i(\alpha) = BR_1\big(BR_2(\alpha)\big) = BR_2\big(BR_1(\alpha)\big)$. In other words, letting players play simultaneously or one after the other leads to the same state (Rosenthal 1973).
- At time T_0, all parameter of strategies of all players are strictly independent and identically distributed variables (“IID”) (i.e., players have independent objectives).

Definition: Nash Equilibrium of payoff function in finite game; $p^* = (p_1^*, \dots, p_n^*)(p_i^* \in P_i, i = 1 \dots n)$; Which p_i^* is the profile of player i strategy while other players continue to play the profile p_{-i}^* : p^* is the Nash Equilibrium; p_i^* is Nash Equilibrium in mixt strategies if: $u_i(p_i^*, p_{-i}^*) > u_i(p_i, p_{-i}^*), \forall p_i \in P_i, \forall\, i = 1 \dots n$

Definition: Function of best answer (reaction function as Nash Equilibrium) in finite game; Assuming that the structure of the game is a payoff function, we determine which strategies of player i correspond to the greatest satisfaction for such player facing any profile s_{-i} (Coucheny *et al.* 2014, Monderer & Shapley 1996, Rosenthal 1973). This strategy corresponds to the best situation that player $i$could obtain in the face of p_{-i}.

Definition: With a game of i players, the best solution for player i, $R_i(p_{-i})$, associated with each combination of strategies of other players p_{-i}; the strategy of player 'i' which maximize its gains: $u_i(R_i(p_{-i}), p_{-i}) \geq u_i(p_i, p_{-i}), \forall p_i \in P_i, p_{-i} \in P_{-i}$; Therefore, s^* is a Nash satisfaction equilibrium, $s_i^* = RB_i(s_{-i}^*), \forall i = 1 \ldots n$.

Proof: $RB_i(s_{-i}^*)$ maximize $u_i(s_i, s_{-i}^*)$ for player $i \rightarrow$ no player has an interest to deviate unilaterally from its strategy s_i^*. The Nash Equilibrium is the interaction between the best response functions of all the players (Rosenthal 1973).

1.2. Best Response Game Protocol

- There are four players: H/A, ONP, France, and the USA;
- Each player possesses '*n*' own strategies which initially are IID;
- Parameters represent the "objectives" of each player (see Table 1 below: Θ, μ, π, φ): Θ: Control over MOF; μ: Regime stability; π: Regional interests; φ: Economic reforms;
- *t*, *q*, *f*, *a* are weighing parameters (*i.e.,* frequencies of strategies of each player or the relative priority each player assigns to each parameter). Weighing parameters are initially weighted randomly at 0.25. Nonlinear optimization method for mixed variables is used to weight the strategy of each player.

Sanctions (Λ) is a conditional dummy variable used to constraint the choice of each player (no sanctions: 0; sanctions: 1).

Table 1. Classification Based on Player's Utility Function (by order of utility of each)

H/A	ONP	France	USA
MOF (Θ)	MOF (Θ)	REGIM (μ)	Sanctions (Λ)
REGIM (μ)	REGIM (μ)	Regional (π)	Regional (π)
Regional (π)	Sanctions (Λ)	Eco Reform (φ)	REGIM (μ)
Sanctions (Λ)	Eco Reform (φ)	MOF (Θ)	Eco Reform (φ)
Eco Reform (φ)	Regional (π)	Sanctions (Λ)	MOF (Θ)

- There are four players (i=4): H/A, ONP, France, and the USA.
- Mixed Strategies: $p_{AH}(t, 1-t)$, $p_{ONP}(q, 1-q)$, $p_{France}(f, 1-f)$, and $p_{USA}(a, 1-a)$
- Which 't' is the frequency choice of strategy one by player H/A; 'q' is the frequency choice of strategy one by player ONP; 'f' is the frequency choice of strategy one by player France; 'a' is the frequency choice of strategy one by player USA; with $t, q, f, a \in [0, 1]$.
- (Θ, μ, π, φ) are parameters which constitutes the objectives of player 'i' (strategy). These parameters are classified based on the utility of each player (Table 1). All parameters are indexed to the frequency choice of each player's strategy. Λ is a conditional dummy variable used to constraint the choice of each player (no sanctions: 0; sanctions: 1).

Thus, (Θ, μ, π, φ; Λ) represent the final profile/objectivesp_i of player 'i', depending on sanctions variable Λ. To simplify, first we replace the profile's parameters (Θ, μ, π, φ; Λ) by parameter β indexed to the frequency choice of each player 'i'. To estimate the best response function of player 'i', we proceed as follow:

(a) We set the initial condition randomly for utility parameters (Θ, μ, π, φ; Λ) at 0.25 for each parameter and '0' for the sanction's parameter, meaning that the initial function of all players at T_0 is $(0.25, 0.25, 0.25, 0.25; 0)$, respectively;

(b) Then, we start to estimate the utility function of player 'i', with mixed strategy (O, N) at frequencies (t, q, f, a) by activating parameters in respect of the preferential order presented in Table 1;

(c) Once all the parameters of the strategy are estimated, an optimal mixed technique is used to optimize the best weighting of the parameters and then to quantify the frequented strategies;

(d) To compute the best answer of player 'i', we used the F-score measurement;

(e) Finally, to predict the optimal outcome, we use an analytical schema based on the estimated Z-scores of strategies relative to $u_i(R_i(p_{-i}), p_{-i}) \geq u_i(p_i, p_{-i}), \forall p_i \in P_i, p_{-i} \in P_{-i}$. * is a Nash satisfaction equilibrium; $s_i^* = R_i(s_{-i}^*), \forall i = 1 \ldots n$.

Table 2. Mixed strategies of players

		H/A		ONP		France		USA	
		O t	N 1-t	O Q	N 1-q	O f	N 1-f	O a	N 1-a
H/A	O t	1	1	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)
	N 1-t	1	1	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)
ONP	O q	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	1	1	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)
	N 1-q	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	1	1	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)
France	O f	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	1	1	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)
	N 1-f	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	1	1	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)
USA	O a	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	1	1
	N 1-a	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	(Θ, μ, π, φ ; Λ)	1	1

Table 3. Weighting parameters[5]

		H/A		ONP		France		USA	
		O t	N 1-t	O q	N 1-q	O f	N 1-f	O a	N 1-a
H/A	O t	1	1	(0.3, 0.32, 0.19, 0.17 ; 1)	(0.45, 0.18, 0.22, 0.15 ; 0)	(0.27, 0.30, 0.22, 0.21 ; 0)	(0.35, 0.35, 0.20, 0.1, 0)	(0.36, 0.35, 0.2, 0.09 : 0)	(0.39, 0.18, 0.19, 0.22 ; 1)
	N 1-t	1	1	(0.21, 0.31, 0.29, 0.19 : 1)	(0.6, 0.36, 0.02, 0.02 ; 1)	(0.3, 0.3, 0.3, 0.1 ; 1)	(0.03, 0.4, 0.04, 0.53 ; 1)	(0.4, 0.31, 0.28, 0.01 ; 1)	(0.56, 0.38, 0.04, 0.02 ; 1)
ONP	O q	(0.34, 0.22, 0.23, 0.19 ; 0)	(0.29, 0.37, 0.12, 0.22 ; 0)	1	1	(0.33, 0.28, 0.25, 0.14 ; 0)	(0.21, 0.29, 0.35, 0.15 ; 1)	(0.34, 0.3, 0.2, 0.16 ; 0)	(0.53, 0.15, 0.18, 0.1 ; 1)
	N 1-q	(0.29, 0.28, 0.22, 0.21 ; 0)	(0.37, 0.19, 0.15, 0.29 ; 0)	1	1	(0.25, 0.44, 0.3, 0.01 ; 0)	(0.29, 0.31, 0.18, 0.22 ; 0)	(0.21, 0.25, 0.14, 0.4 ; 0)	(0.63, 0.14, 0.12, 0.11 ; 1)
France	O f	(0.36, 0.3, 0.1, 0.24 ; 0)	(0.63, 0.25, 0.11, 0.01; 0)	(0.25, 0.28, 0.31, 0.16 ; 0)	(0.29, 0.15, 0.17, 0.39 ; 0)	1	1	(0.48, 0.22, 0.25, 0.05 ; 0)	(0.55, 0.2, 0.15, 0.15 ; 1)
	N 1-f	(0.29, 0.25, 0.2, 0.24 ; 0)	(0.34, 0, 45, 0.2, 0.01 ; 1)	(0.36, 0.34, 0.27, 0.03 ; 0)	(0.25, 0.22, 0.22, 0.31 ; 1)	1	1	(0.25, 0.35, 0.22, 0.18)	(0.2, 0.39, 0.25, 0.16 ; 0)
USA	O a	(0.3, 0.3, 0.2, 0.2 ; 0)	(0.3, 0.3, 0.15, 0.25 ; 0)	(0.33, 0.24, 0.26, 0.07 ; 0)	(0.42, 0.25, 0.32, 0.03 ; 0)	(0.29, 0.35, 0.2, 0.16 ; 0)	(0.15, 0.33, 0.23, 0.29 ; 0)	1	1
	N 1-a	(0.35, 0.26, 0.22, 0.17 ; 1)	(0.65, 0.11, 0.12, 0.13 ; 1)	(0.29, 0.22, 0.33, 0.16 ; 0)	(0.2, 0.33, 0.32, 0.15 ; 1)	(0.4, 0.21, 0.3, 0.09 ; 0)	(0.63, 0.14, 0.12, 0.11 ; 1)	1	1

[5] The figures represent weighing parameters for the player in the respective row based on the response strategy of the player in each column. This is how all of the tables in this paper should be read: the figures in the table are in respect of the player in each row.

Table 4a. F-Score measurement to attribute a score for player strategies*

4a. Player H/A		H/A	
		O t	N 1-t
ONP	O q	(0.46, 0.39)	(0.31, 0.22)
	N 1-q	(0.16, 0.1)	(0.1, 0.07)
France	O f	(0.62, 0.55)	(0.29, 0.14)
	N 1-f	(0.17 0.16)	(0.11, 0.38)
USA	O a	(0.27, 0.2)	(0.33, 0.16)
	N 1-a	(0.45, 0.13)	(0.29, 0.15)

Note: *based on weighted parameters in Table 3

For player H/A, the expected best response in a mixed strategy are:

- Strategy O: $q * 0.39 + (1 - q) * 0.1 + f * 0.55 + (1 - f) * 0.16 + a * 0.22 + (1 - a) * 0.09$;
- Strategy N: $q * 0.22 + (1 - q) * 0.07 + f * 0.14 + (1 - f) * 0.38 + a * 0.09 + (1 - a) * 0.09$.

The expected utility of H/A in the face of ONP:

- Face the mixed strategy of ONP $(\mathrm{p_{ONP}})$; $\mathrm{U_{AH}(p_{AH}, p_{-ONP}) = E_{U_{AH}} = q * 0.39 + (1 - q) * 0.1 \geq q * 0.22 + (1 - q) * 0.07 \Rightarrow q \geq 0.214}$;
- If H/A plays strategy O, its expected of gains in the face of ONP should be: $\mathrm{E_{U_{AH,O}} \geq 0.34}$;
- If H/A play strategy N, its expected of gains in the face of ONP should be: $\mathrm{E_{U_{AH,N}} \geq 0.46}$.

The expected utility of H/A in the face of France:

- Face the mixed strategy of France (p_{France}); $U_{AH}(p_{AH}, p_{-Fr}) = E_{AH} = f * 0.55 + (1 - f) * 0.16 \geq f * 0.14 + (1 - f) * 0.38 \Rightarrow f \geq 0.33$;
- For H/Ato play strategy O, its expected gains face of France should be: $E_{U_{Fr,O}} \geq 0.41$;
- For H/Ato play strategy N, its expected gains face of France should be: $E_{U_{AH,N}} \geq 0.53$.

The expected utility of H/A in the face of the USA:

- Face the mixed strategy of USA (p_{USA}); $U_{AH}(p_{AH}, p_{-USA}) = E_{U_{AH}} = a * 0.2 + (1 - a) * 0.13 \geq a * 0.16 + (1 - a) * 0.15 \Rightarrow a \geq 0.33$;
- For H/Ato play strategy O, its expected gains in the face of USA should be: $E_{U_{AH,O}} \geq 1.8$;
- For H/Ato play strategy N, its expected gains in the face of USA should be: $E_{U_{AH,N}} \geq 150$.

Table 4b. Player France

4b. Player France		France	
		O f	N 1 – f
H/A	O t	(0.55, 0.62)	(0.14, 0.29)
	N 1-t	(0.16, 0.17)	(0.38, 0.11)
ONP	O q	(0.37, 0.39)	(0.55, 0.63)
	N 1-q	(0.09, 0.16)	(0.13, 0,22)
USA	O a	(0.43, 0.24)	(0.25, 0.22)
	N 1-a	(0.06, 0.19)	(0.15, 0.39)

For player France, the expectation of the best response in a mixed strategy are:

- Strategy O: $t * 0.62 + (1 - t) * 0.17 + q * 0.33 + (1 - q) * 0.21 + a * 0.24 + (1 - a) * 0.19$;
- Strategy N: $t * 0.29 + (1 - t) * 0.11 + q * 0.14 + (1 - q) * 0.28 + a * 0.22 + (1 - a) * 0.39$.

The expected payoff of France in the face of H/A:

- Face the mixed strategy of H/A $(p_{\frac{H}{A}})$; $U_{FR}\left(p_{FR}, p_{\_\frac{H}{A}}\right) = E_{Fr} = t * 0.62 + (1-t) * 0.17 \geq t * 0.29 + (1-t) * 0.11 \Rightarrow t \geq 0.22$;
- For France to play strategy O, its expected gain in the face of H/A should be: $E_{U_{Fr,O}} \geq 0.37$;
- For France to play strategy N, its expected gain in the face of H/A should be: $E_{U_{F,N}} \geq 0.61$.

The expected payoff of France in the face of ONP:

- Face the mixed strategy of ONP (p_{ONP}) ; $U_{FR}(p_{Fr}, p_{-ONP}) = E_{U_{Fr}} = q * 0.39 + (1-q) * 0.16 \leq f * 0.63 + (1-f) * 0.22 \Rightarrow q \geq 0.33$;
- For France to play strategy O, its expected gain in the face of ONP should be: $E_{U_{Fr,O}} \geq 0.51$;
- For France to play strategy N, its expected gain in the face of ONP should be: $E_{U_{Fr,N}} \geq 0.53$.

The expected payoff of France in the face of the USA:

- Face the mixed strategy of USA (p_{USA}) ; $U_{FR}(p_{Fr}, p_{-USA}) = E_{Fr} = a * 0.24 + (1-a) * 0.19 \geq a * 0.22 + (1-a) * 0.39 \Rightarrow a \geq 0.83$;
- For France to play strategy O, its expected gain in the face of USA should be: $E_{U_{Fr,O}} \geq 3.8$;
- For France to play strategy N, its expected gain in the face of USA should be: $E_{U_{Fr,N}} \geq 2.29$.

Table 4c. Player ONP

4c. Player ONP		ONP	
		O Q	N 1-q
H/A	O t	(0.39, 0.46)	(0.22, 0.31)
	N 1-t	(0.1, 0.13)	(0.07, 0.1)
France	O f	(0.39, 0.37)	(0.63, 0.55)
	N 1-f	(0.16, 0.09)	(0,22, 0.13)
USA	O a	(0.42, 0.015)	(0.57, 0.38)
	N 1-a	(0.39, 0.05)	(0.25, 0.1)

For player ONP, the expectation of the best response in a mixed strategy are:

- Strategy O: $t * 0.46 + (1-t) * 0.13 + f * 0.37 + (1-f) * 0.09 + a * 0.15 + (1-a) * 0.05$;
- Strategy N: $t * 0.31 + (1-t) * 0.1 + f * 0.55 + (1-f) * 0.13 + a * 0.38 + (1-a) * 0.1$.

The expected payoff of ONP in the face of H/A:

- Face the mixed strategy of H/A (p_{AH}) ; $U_{ONP}(p_{ONP}, p_{-AH}) = E_{ONP} = t * 0.46 + (1-t) * 0.13 \geq t * 0.31 + (1-t) * 0.10 \Rightarrow t \geq 0.25$;
- For ONP to play strategy O, its expected gain in the face of H/A should be: $E_{U_{ONP,O}} \geq 0.39$;
- For ONP to play strategy N, its expected gain in the face of H/A should be: $E_{U_{ONP,N}} \geq 0.66$.

The expected payoff of ONP in the face of France:

- Face the mixed strategy of France (p_{Fr}) ; $U_{ONP}(p_{ONP}, p_{-Fr}) = E_{ONP} = f * 0.37 + (1-f) * 0.09 \geq f * 0.55 + (1-f) * 0.13 \Rightarrow f \geq 0.28$;
- For ONP to play strategy O,its expected gain in the face of France should be: $E_{U_{ONP,O}} \geq 0.32$;
- For ONP to play strategy N, its expected gain in the face of France should be: $E_{U_{ONP,N}} \geq 0.31$.

The expected payoff of ONP in the face of USA

- Face the mixed strategy of USA (p_{USA}) ; $U_{ONP}(p_{ONP}, p_{-USA}) = E_{ONP} = a * 0.15 + (1-a) * 0.05 \geq a * 0.38 + (1-a) * 0.10 \Rightarrow a \geq 0.28$;
- For ONP to play strategy O, its expected gain in the face of USA should be: $E_{U_{ONP,O}} \geq 0.5$;
- For ONP to play strategy N, its expected gain in the face of USA should be: $E_{U_{ONP,N}} \geq 0.35$.

Table 5. Mixed strategies which respect the Nash equilibrium

		H/A		ONP		France		USA	
		O t	N 1-t	O Q	N 1-q	O f	N 1-f	O A	N 1-a
H/A	O t	1	1	0.39	0.22	0.55	0.14	0.2	0.16
	N 1-t	1	1	0.1	0.07	0.16	0.38	0.13	0.15
ONP	O q	0.46	0.31	1	1	0.37	0.55	0.015	0.38
	N 1-q	0.16	0.1	1	1	0.09	0.13	0.05	0.1
France	O f	0.62	0.29	0.39	0.63	1	1	0.24	0.22
	N 1-f	0.17	0.11	0.16	0.22	1	1	0.19	0.39
USA	O a	0.27	0.33	0.42	0.57	0.43	0.25	1	1
	N 1-a	0.45	0.29	0.39	0.25	0.06	0.15	1	1

Note: Yellow Marks are coefficients accepted at 5% of significance level

Table 6. Mixed strategies which respect the NE and the best response of individual strategies of players

		H/A		ONP		France		USA	
		O T	N 1-t	O Q	N 1-q	O f	N 1-f	O A	N 1-a
H/A	O t	1	1	0.39	0.22	0.55	0.14	0.2	0.16
	N 1-t	1	1	0.1	0.07	0.16	0.38	0.13	0.15
ONP	O q	0.46	0.31	1	1	0.37	0.55	0.015	0.38
	N 1-q	0.16	0.1	1	1	0.09	0.13	0.05	0.1
France	O f	0.62	0.29	0.39	0.63	1	1	0.24	0.22
	N 1-f	0.17	0.11	0.16	0.22	1	1	0.19	0.39
USA	O a	0.27	0.33	0.42	0.57	0.43	0.25	1	1
	N 1-a	0.45	0.29	0.39	0.25	0.06	0.15	1	1

Note: Yellow Marks are coefficients accepted at 5% of significance level

Table 7. Summary of equilibrium strategies

		H/A		ONP		France		USA	
		O t	N 1-t	O Q	N 1-q	O F	N 1-f	O a	N 1-a
H/A	O t	1	1	0.39 (0.32, 0.30, 0.19, 0.17; 0)	0.22 0.25, 0.52, 0.02, 0.21; 0)	0.55 (0.37, 0.25, 0.16, 0.22; 0)	0.14 (0.35, 0.19, 0.39, 0.07, 0)	0.2 (0.36, 0.35, 0.2, 0.09: 0)	0.16 (0.39, 0.18, 0.19, 0.22; 1)
	N 1-t	1	1	0.1 (0.01, 0.11, 0.53, 0.35: 0)	0.07 0.04, 0.06, 0.28, 0.62; 0)	0.16 (0.23, 0.22, 0.45, 0.1; 0)	0.38 (0.4, 0.53, 0,03, 0.04; 0)	0.13 (0.4, 0.31, 0.28, 0.01; 1)	0.15 (0.56, 0.38, 0.04, 0.02; 1)
ONP	O q	0.46 (0.30, 0.30, 0.29, 0.09; 0)	0.31 (0.29, 0.37, 0.12, 0.22 ; 0)	1	1	0.37 (0.33, 0.28, 0.25, 0.14; 0)	0.55 (0.29, 0.31, 0.22, 0.18; 0)	0.015 (0.34, 0.3, 0.2, 0.16; 0)	0.38 (0.53, 0.15, 0.18, 0.1; 0)
	N 1-q	0.16 (0.25, 0.44, 0.18, 0.13; 0)	0.1 (0.47, 0.19, 0.29, 0.03; 0)	1	1	0.09 (0.25, 0.44, 0.3, 0.01; 0)	0.13 (0.29, 0.31, 0.18, 0.22; 0)	0.05 (0.21, 0.25, 0.14, 0.4; 0)	0.1 (0.63, 0.14, 0.12, 0.11; 1)
France	O f	0.62 (0.36, 0.3, 0.24, 0.1; 0)	0.29 (0.22, 0.39, 0.11, 0.25; 0)	0.39 (0.25, 0.28, 0.31, 0.16; 0)	0.63 (0.29, 0.39, 0.22, 0.1; 0)	1	1	0.24 (0.48, 0.22, 0.25, 0.05; 0)	0.22 (0.55, 0.2, 0.15, 0.15; 1)
	N 1-f	0.17 (0.29, 0.25, 0.2, 0.24; 0)	0.11 (0.34, 0, 45, 0.2, 0.01; 1)	0.16 (0.36, 0.34, 0.27, 0.03; 0)	0.22 (0.25, 0.22, 0.22, 0.31; 1)	1	1	0.19 (0.25, 0.35, 0.22, 0.18)	0.39 (0.2, 0.39, 0.25, 0.16; 0)
USA	O a	0.27 (0.3, 0.3, 0.2, 0.2; 0)	0.33 (0.3, 0.3, 0.15, 0.25; 0)	0.42 (0.33, 0.24, 0.26, 0.07; 0)	0.57 (0.62, 0.25, 0.1, 0.03; 0)	0.43 (0.29, 0.35, 0.2, 0.16 ; 0)	0.25 (0.15, 0.33, 0.23, 0.29; 0)	1	1
	N 1-a	0.45 (0.35, 0.26, 0.22, 0.17; 1)	0.29 (0.65, 0.11, 0.12, 0.13; 1)	0.39 (0.29, 0.22, 0.33, 0.16; 0)	0.25 (0.2, 0.33, 0.32, 0.15; 1)	0.06 (0.4, 0.21, 0.3, 0.09; 0)	0.15 (0.63, 0.14, 0.12, 0.11; 1)	1	1

Note: Yellow Marks are coefficients accepted at 5% of significance level

3. Interpretation of Results under Equilibrium Solution for All Players

- *MOF*: Both H/A and ONP have a significant interest in control over MOF as indicated by the high weighting each player assigns to this parameter under the equilibrium solution, though H/A weighs it slightly more. Finding an equilibrium solution thus requires a cooperative approach to find a solution that satisfies both players. For example, H/A maintains nominal controls MOF while ONP shares in some manner in the nomination. An analysis of the sub-optimal solution – strategy (N, N) – provides a useful insight: in a scenario whereby ONP overly prioritizes its control over MOF, H/A responds by de-prioritizing the MOF & Regime parameters, both of which converge to zero, resulting in Regime instability;
- *Regime*: All players seek regime stability regime with no interest to change the regime in the short-term;
- *Economic reforms*: France plays a larger role in the Economic Reforms as these reforms are a higher relative priority for it;
- *Regional*: France is the largest relative winner in respect of its Regional interests;
- *Sanctions*: Escalation of sanctions would impede an equilibrium solution (and could result in escalation of conflict).

The USA does not achieve equilibrium with all other players under the mixed strategies that achieve equilibrium between the other players. This suggests that the "game" should be expanded to include additional national parameters regional players against which the USA can achieve relative gains such that a global equilibrium can be found.

3.1. Interpretation of Equilibrium Solutions between Pairs of Players

H/A and ONP Equilibrium Solution: Strategy (O, O)

When H/A plays strategy O, ONP player response by its own strategy O.

- *MOF*: Both players highly weigh control over MOF and, as a result, an equilibrium solution requires a cooperative approach whereby, for example, one player nominally controls MOF while the other shares in some manner in the nomination. Given that H/A weighs control over MOF slightly more highly than ONP under Strategy (O, O), an equilibrium solution may be one whereby H/A maintains nominal control over MOF while ONP shares in some manner in the nomination. Importantly, H/A's response to an ONP strategy that overly prioritizes control over MOF could result in it de-prioritizing Regime stability;
- *Regime*: Stability maintained;
- *Economics reform*: Short-term reforms implemented;
- *Regional*: Non-significant impact for H/A and ONP;
- *Sanctions*: No sanctions in the equilibrium solution.

The results further indicate that, under an equilibrium solution, the MOF issue must be resolved between H/A and ONP, alone, in a cooperative manner. This cooperative game ensures the stability of Regime (equal-weighted parameter μ), and, as a result, certain short-term aspects of the economic reforms could be facilitated.

It is noteworthy that under the sub-optimal strategy (N, N), which is a scenario in which player ONP gains control over MOF, the MOF & Regime parameters in the N strategy of player H/A converge to zero, meaning that Regime stability and the global system are disturbed. It is also striking that, under this strategy (N, N), player H/A plays the Economics Reform parameter to a greater extent in the face of ONP's strategy (N, N).

Under all strategies wherein either of players H/A or ONP attempt unilaterally to modify and play the Regime or Economic Reforms variables – strategies (N, O) (O, N) & (N, N) – the system is made unstable (Regime variable) and no equilibrium solution is found.

Summary: A solution to the MOF issue requires a cooperative approach between H/A and ONP whereby, for example, one player controls MOF while the other shares in some manner in the nomination. Given that H/A weighs control over MOF slightly more highly ONP under Strategy (O, O), an equilibrium solution may be one whereby H/A maintains nominal control over MOF while ONP shares in some manner in the nomination.

H/A and France Equilibrium: Strategy (O, O)

- *MOF*: France shows less relative interest in MOF and allows H/A to prioritize its own interest in MOF. If France exerts an interest in control over MOF, H/A responds using a strategy that prioritizes its own Regional interests, which could result in conflict;

- *Regime*: Both players have an interest in regime stability;
- *Economics reform*: France has a greater relative interest in Economic Reform than H/A, though H/A does not oppose economic reforms (the parameter is positive and significant for it);
- *Regional*: Both players have an interest in the Regional parameter but, in this particular game, France has a higher relative interest than H/A;
- *Sanctions*: No sanctions (sanctions result in no equilibrium solution).

The estimated parameters indicate that France is better off giving H/A player the MOF. When both France and H/A maintain a priority interest in controlling MOF (strategy O, N), H/A's interest shifts to the Regional parameter (Strategy O, N). Both action and reaction lead to non-equilibrium.

Moreover, it is notable that when France plays the Sanction parameter in the pure strategy N, H/A responds by showing a higher relative interest in MOF and Regime (*i.e.,* responds with strategy N).H/A's higher interest in the Regime parameter under this scenario implies a greater motivation for it to seek changes to the regime system.

Summary: H/A maintains greater control over the MOF, and France, in return, benefits from regime stability and execution of certain short-term economic reforms facilitated by such stability. France also achieves regional gains in respect of its role and credibility as a regional player.

ONP and France Equilibrium: Strategy (N, N)

- *MOF*: France shows less relative interest in MOF and allows ONP to prioritize its own interest in MOF;
- Regime: Both players have an interest in regime stability;
- *Economics reform*: Both players have an interest in the Economic Reform parameter, but such interest is relatively greater for France compared to ONP;
- *Regional*: Both players have Regional interest but such interest is relatively greater for France;
- *Sanctions*: Sanctions lead to non-equilibrium (in Strategy (N, N)).

Summary: France permits ONP to reach equilibrium solution with H/A in respect of MOF. In exchange, France plays a larger role in the economic reform program, benefits from the resulting regime stability and improved regional credibility.

ONP and USA Equilibrium: Strategy (ON, ON):

- *MOF*: USA has no interest in MOF whereas ONP does;
- *Regime*: Both players have an interest in regime stability;
- *Economics reform*: USA has no interest in Economic Reforms variable;
- *Regional*: USA's interests are purely in respect of its Regional interests;
- *Sanctions*: Sanction lead to non-equilibrium (strategy N, N).

Summary: USA unlikely to leverage the Sanctions parameter to the extent it would risk destabilizing the Regime system, neither is the USA likely to insist on or facilitate the Economic Reforms. The USA's interests are purely Regional.

Further Considerations

We question whether the USA would permit France to gain strategically in Lebanon and in the region without any payoff for the USA, which is the consequence of the equilibrium solutions described above.

We observed two schema/strategies for each player, one which leads to a Nash Equilibrium solution and another which provides information on important characteristics of the game. The first schema/strategy combination is based on France's interest in a local equilibrium solution; that is, to achieve regime stability and to gain regional credibility. On the other hand, the USA's schema leads strictly to gains in respect of its regional interests. As a result, both countries are in regional competition. To answer the question posed above, we must measure the stability of both strategies/schema and the equilibrium solution between H/A, ONP, and France which results in a government formation without the USA. In addition, we must better understand how the USA can use its influence over certain variables (national and regional) to impact the final equilibrium or to force a new equilibrium that respects its interest relative to the other players.

Table 8. Sensitivity of the convergence to optimal Nash equilibrium G-Best response

Best Response Nash Equilibrium	Z-score	Sensitive Z-score
(O_{HA}, O_{ONP})	1.12	0.904
(O_{HA}, O_{France})	0.96	0.209
(N_{ONP}, N_{France})	1.04	0.598
(ON_{USA}, OV_{ONP})	0.50	1.96

Estimated Mean = 0.906; Standard Deviation is = 0.2.

We estimate the smooth convergence/divergence of each equilibrium strategy pair using the z-score equation. Then, we calculate the sensitivity of the convergence of the equilibrium strategies of each player to understand the equilibrium consistency. The results suggest that the distribution between the three Nash Equilibrium best response strategies of H/A (O, O), ONP (O, O), and France (N, N) is very small (between 0.05 and 0.21) compared to the spread for the USA, which is concentrated around 0.41. This concentration of the z-score distribution suggests that the equilibrium between players is consistent and profitable to the group and to all players. The divergence of the USA's score indicates that the USA's payoff strategy is not in equilibrium, and the USA's focus is on one single objective: to maximize its regional payoff (or impose sanctions otherwise).

Moreover, the sensitivity estimations indicate that France's actions have a relatively small impact on the global equilibrium (0.2 with A/H and 0.6 with ONP). In other words, France is not the main player in this game. H/A, ONP, and the USA's estimated sensitivities shows that H/A and ONP are more sensitive to the global equilibrium, and both players have a greater impact (negatively or positively) than France on the global equilibrium. Notably, the analysis suggests that the USA is the most influential player in the game as its actions/reactions have a greater impact on the equilibrium solution (via its interaction with ONP).

Conclusion

In this study, we attempted to find the optimal best response of each player in a political game comprising Lebanon's government formation initiative. Based on four preferential interests assumed for each player, the results indicate that there is a consistent Nash Equilibrium between H/A, ONP, and France, but this equilibrium is very fragile. However, the USA's payoff function is not clear related to the other players and their preferential interests. This may be due to the USA's main interests residing in other national and regional considerations not reflected in the assumptions of this game.

Notably, the game results indicate that France is not the main player. Its contribution was the least of all players, contrary to the role of the USA, which was the greatest.

In order to develop a more comprehensive understanding of the USA's contribution to the government formation game, an extension of this study is needed which integrates additional national and regional interests and additional regional players, such as Saudi Arabia, Iran, etc. This game expansion could form the basis of a future analysis.

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ANNEX 1. Expected Best Response of Players in a Mixed Strategy Game

The expected best response of player H/A in a mixed strategy game is:

For Strategy O: $U_{AH}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{U_{AH}}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = q * \beta_{t,q} + 1 - q * \beta_{t,1-q} + f * \beta_{t,f} + 1 - f * \beta_{t,1-f} + a * \beta_{t,a} + 1 - a * \beta_{t,1-a}$

Strategy N: $U_{AH}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{U_{AH}}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = q * \beta_{1-t,q} + 1 - q * \beta_{1-t,1-q} + f * \beta_{1-t,f} + 1 - f * \beta_{1-t,1-f} + a * \beta_{1-t,a} + 1 - a * \beta_{1-t,1-a}$

The expected best response of player ONP in a mixed strategy game is:

For Strategy O: $U_{ONP}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{U_{ONP}}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = t * \beta_{q,t} + 1 - t * \beta_{q,1-t} + f * \beta_{q,f} + 1 - f * \beta_{q,1-f} + a * \beta_{q,a} + 1 - a * \beta_{q,1-a}$

Strategy N: $U_{ONP}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{U_{ONP}}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = t * \beta_{1-q,t} + 1 - q * \beta_{1-q,1-t} + f * \beta_{1-q,f} + 1 - f * \beta_{1-q,1-f} + a * \beta_{1-q,a} + 1 - a * \beta_{1-q,1-a}$

The expected best response of player France in a mixed strategy game is:

For Strategy O: $U_{Fr}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{Fr}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = t * \beta_{f,t} + 1 - t * \beta_{f,1-t} + q * \beta_{f,q} + 1 - q * \beta_{f,1-q} + a * \beta_{f,a} + 1 - a * \beta_{f,1-a}$

Strategy N: $U_{Fr}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{U_{Fr}}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = t * \beta_{1-\mathbb{I},t} + 1 - t * \beta_{1-f,1-t} + q * \beta_{1-f,q} + 1 - q * \beta_{1-f,1-q} + a * \beta_{1-f,a} + 1 - a * \beta_{1-f,1-a}$

The expected best response of player USA in a mixed strategy game is:

For Strategy O: $U_{USA}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{USA}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = t * \beta_{\mathbb{I},t} + 1 - t * \beta_{a,1-t} + q * \beta_{a,q} + 1 - q * \beta_{a,1-q} + f * \beta_{a,f} + 1 - f * \beta_{a,1-f}$

Strategy N: $U_{USA}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = E_{USA}(p_{AH}, p_{ONP}, p_{Fr}, p_{USA}) = \left(t * \beta_{1-a,t} + 1 - t * \beta_{1-a,1-t} + q * \beta_{1-a,q} + 1 - q * \beta_{1-a,1-q} + f * \beta_{1-a,f} + 1 - f * \beta_{1-a,1-f}\right)$