# Spurious Trend in Stationary Series and its Implications

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**Abstract:**

Nelson and Kang (1984) showed that regression of a unit root time-series on a linear time trend provides significant results even if there is no forecast able association amongst the path of the time-series and linear trend. Using Monte Carlo simulations, this paper shows that phenomenon also exists in stationary time series and regression of a stationary time series on linear trend also produces significant results without the existence of any predictable relationship between the time -series and linear trend. The spurious trend is observable in most of the moderate sample sizes and sometimes in sufficiently large samples of size over 500 observations. The implications of these findings for unit root test procedures are discussed briefly.

**Keyword:** spurious trend; spurious regression; deterministic regressors; equations; time series; Monte Carlo simulations.

**JEL Classification:** B23, C01, C15, C18, C51, C53.

## Introduction

A regression between two variables is termed as spurious if the regression shows a very strong relation between the variables whereas actually there is no or very weak predictable relationship between them. Yule (1926) observed that spurious regression is often found in the relationship between time series data. After many decades, two studies provided clues about the reasons behind the spurious regression by Monte Carlo experiment. The first study was due to "Granger and Newbold (1974)" who found spurious regression among two independent unit root series having no direct or indirect predictable relationship. Second study is due to Nelson and Kang (1984) who observed this phenomenon of spurious-trend in unit root series *i.e.* if a unit root series is regressed on the linear deterministic trend, the regression statistics shows the trend to be significant, whereas, in fact, absence of predictable relationship between the unit root series and the deterministic trend.

The contribution of "Granger and Newbold (1974)" focused on the relation amongst both stochastic time-series while seminal work by Nelson and Kang (1984) focused on relationship between a stochastic series and linear deterministic trend. The two studies imply that existence of unit root may cause spurious regression. These studies imply that unit root is one of possibly many reasons of spurious regression and by no way these studies imply that every spurious relation must be because of the unit root. But unfortunately, it was falsely perceived by many of econometricians that spurious regression phenomenon emerges because of unit roots only. Therefore, the word spurious regression became a synonym of 'presence of unit root in underlying time-series without co integration'.

Later on, Granger *et al.* (1998) observed that spurious regression could also be observed even if the two series are stationary. Granger *et al.* (1998) observed that the regression between two stationary time series could yield spurious significance even absence of a predictable relation amongst the underlying time-series.

Reconsidering the development in spurious-regression related literature; Granger and Newbold (1974) observed spurious regression between two stochastic unit root series and Granger *et al.* (1998) found that spurious regression could also exist between two non-unit root stochastic stationary series. On the other hand, Nelson and Kang (1984) found that spurious trend exists in stochastic unit root series. A natural question that comes to mind is that whether there is any spurious trend in stationary time series or not. This question has very serious implications in econometrics; however, the question has not been explored so far.

This study attempts to explore the possibility of a spurious trend in stationary series using "Monte Carlo experiments". Extensive simulations had been done to explore the distribution of the conventional t-statistics for significance of linear-trend in stationary time-series. Present study finds that spurious regression does exist between stationary time series and the linear trend and linear trend appears to be significant even if the series does not have any dependence on it. Monte Carlo experiments show that regression of a stationary time series on linear trend is heavily biased toward rejection of the hypothesis of 'no relationship'.

These findings have very serious implications for a number of statistical procedures and tests including procedures of unit root testing. Use of unit root test on a series requires prior specification of a deterministic-trend in the underlying time-series and if deterministic-trend could not be reliably specified, results from unit root tests would also be unreliable. This paper shows the general to simple type procedures that are usually applied for the specification of a deterministic part in a unit root test equation are unreliable due to the spurious trend hence output of unit root tests will also be at stake.

This paper considers three variations of a deterministic trend that are used in unit root test equations and shows that conventional statistical procedures are quite misleading in specifying the deterministic regressors in both stationary and unit root time series. The implications of these finding for unit root testing procedures are discussed in details at the end of paper.

The reminder of paper is ordered as follows: Section 2, 3, 4 and 5 shows reviews pertinent literature; discusses the existence of a predictable relationship between a stationary time series and the linear time trend; discusses how the existence of spurious trend could be tested; describes the simulation and Monte Carlo design used in the present study; Summarizes the results of Monte Carlo simulations and lastly confers the policy implications of the results for unit root tests and testing procedures.

## 1. Literature Review

Though there is a long list of studies on the phenomenon of spurious regression, the research on possibility of 'spurious regression in stationary series' or research on 'existence of spurious trend' is scarce with the exception of some of the recent work, for examples occurred, Agiakloglou (2013), Philips (2014), Santaularia and Noriega (2015), Wang and Han (2015), Vinod (2016) and Kitov, and Kitov, (2008) among others). The 'spurious trend in stationary series', a combination of two scarce research questions is absent from the available literature. The spurious regression in time series was first reported by Yule (1926). Yule noted that a regression between two time-series often appears to reveal significant relationship whereas in fact the two series are theoretically independent of each other. Yule (1926) thought that this phenomenon emerges because of some missing variable. Granger and Newbold (1974) witnessed that if two independent time-series with a unit root, without having any indirect relation through any third variable, appear to highly correlated and dependent on each other. Therefore, Granger and Newbold suggested that it is the unit root that causes spurious regression in the time series models. Philips (1987) provided analytical proofs for the results found by Granger and Newbold (1974).

Though it was not implied by the study of Granger and Newbold (1974), most of the researchers falsely assumed that spurious regression is only unit root related phenomenon. Most of the literature presents spurious regression as a synonym of unit root with no co integration. A quarter century later, Granger *et al.* (1998) showed that it is not true that spurious regression occurs only because of the unit root. They show that spurious regression could also be observed between stationary time series. Rehman and Malik (2014) also show that the spurious regression exist between two time series with autoregressive roots.

Granger and Newbold (1974) and Granger *et al.* (1998) both observed spurious regression among two stochastic time series with first using unit root time series and second using stationary time series. However, Nelson and Kang (1984) extended the discussion of spurious regression to another dimension. They observed that spurious regression also exists when a unit root series is regressed on deterministic linear trend. This means that a stochastic time series shows a spurious trend when there is unit root. A natural question arises as to what will happen if a stationary time series serial dependence is regressed on linear deterministic trend?

Agiakloglou (2013) uses a Monte Carlo analysis to illustrate that estimation of the spurious regression in with lagged dependent variable eliminates the spurious problem. However, Choi (2013) found that spurious results can also be found fixed effects models with weak time-series variation. Agiakloglou *et al.* (2015) analyze the spurious regression phenomenon for two independent stationary spatially autoregressive processes of order 1, and find that the spurious behavior is not detected nor the presence of spatially auto-correlated errors. Fernandez (2015) states that wavelet-correlation-based approach should not be used to test for co integration. However, Leong (2015) argues that this statement is dismissive since the existing residual-based tests for co integration are themselves tests of correlation.

Jin *et al.* (2015) considered the case where the deterministic components of the processes generating individual series are independently heavy-tailed with structure changes. The paper found that spurious phenomenon is present regardless of the sample size and structural breaks. Santaularia and Noriega (2015) introduced a method to distinguish a genuine relationship from a spurious one among integrated processes and find that their procedure does not find (spurious) significant relationships. Wang and Han (2015) studied the asymptotic properties of least squares estimators and related test statistics in some spurious regression models that are generated by stationary or non-stationary fractionally integrated processes. Khan et al. (2019) reported non-stationarity even in the presence of structural breaks in time series for Pakistan that might be due to spurious trends in deterministic and stochastic components. Perron and Rodriguez (2015) provide generalized least-squares (GLS) de trended versions of single-equation static regression or residuals-based tests for testing whether or not non-stationary time series are co integrated. Their paper finds that its GLS tests provide substantial power improvements over the OLS counterparts. Pesaran *et al.* (2013) examine the cross section augmented panel unit root test to the case of a multifactor error structure. This paper applies Monte Carlo experiments for the properties of the small sample size and finds that the test exhibits higher power than the alternative tests for large T and N, both in the case of an intercept only, and intercept and a linear trend.

In the meantime, Bacallado *et al.* (2015) use Metropolis-Hastings, Langevin, and Hamiltonian Monte Carlo to compute posterior distributions for test statistics relevant for testing independence. Wang and Jasra (2016) introduce a new adaptive sequential Monte Carlo (SMC) algorithm for approximating permanents of $n \times n$ binary matrices and establish the convergence of the estimate. Vinod (2016) proposes new confidence intervals (CIs) based on the Maximum Entropy bootstrap and shows that it can provide more reliable conservative CIs than traditional band block bootstrap intervals and robust linear regression analysis using Monte Carlo Simulations (Mishra 2008).

However, utilizing all available researched resources, we could not find research on stationary time series with temporal dependence is regressed on linear deterministic-trend model which addresses the above question. Therefore, this study attempts to fill the gap and analyze the distribution of regression output when a stationary series is regressed on the linear deterministic trend.

## 2. Deterministic Trend in Autoregressive Series

Consider the following three autoregressive models:

| | | | |
|---|---|---|---|
| M1 | Autoregressive model without drift, trend | $y_t = d y_{t-1} + e_t$ | |
| M2 | Autoregressive model with drift, but no trend | $y_t = a + d y_{t-1} + e_t$ | (1) |
| M3 | Autoregressive model with drift and trend | $y_t = a + b t + d y_{t-1} + e_t$ | |

where: $e_t \sim iid(0, s^2)$.

These are the three models that were used by Dickey and Fuller (1979) in designing the tests for a unit root. Each of these models gives a unit root series if $d$=1 and a stationary series if $d < 1$. Let us discuss the predictable relation between these models and the linear deterministic trend.

We see that model M3 explicitly contains the linear deterministic trend. If a series generated by this model provides significant coefficient when regressed on a linear trend, that significance would be considered genuine. However, the relation of M1 and M2 with a linear trend needs to be discussed. Therefore, considering the following model:

$$y_t = \alpha + \delta y_{t-1} + \varepsilon_t \,;\; \varepsilon_t \sim iid(0,\sigma^2) \tag{2}$$

This model is equivalent to M2 and could be simplified to M1 by specifying $\alpha = 0$. The successive substitution yields:

$$y_t = \delta^t y_0 + \alpha \sum_{i=0}^{t} \delta^i + \sum_{i=0}^{t} \delta^i \varepsilon_{t-i} \tag{3}$$

$$E(y_t) = \delta^t E(y_0) + \alpha \sum_{i=0}^{t} \delta^i + \sum_{i=0}^{t} \delta^i E(\varepsilon_{t-i}) \tag{4}$$

Since $\varepsilon_t \sim iid(0,\sigma^2)$ therefore, $E(\varepsilon_t) = 0$; Also suppose $E(y_0) = 0$, therefore,

$$E(y_t) = E\,[\alpha \textstyle\sum_{i=0}^{t} \delta^i\,] \tag{5}$$

Therefore, for large value of t,

$$E(y_t) = \begin{cases} \frac{\alpha}{1-\delta} & if\ \delta < 1 \\ \alpha\ t & if\ \delta = 1 \end{cases} \tag{6}$$

If $\alpha = 0$, *i.e.* the data is actually generated from M1 having no deterministic part. In this case, both expressions on the RHS of (6) would become zero. This means regardless of the value of $\delta$ the expectation of series are independent of linear trend. Therefore, a regression of series generated by M1 on time-trend should be insignificant, and if it is significant, the significance must be spurious. "On the other hand, consider the case when $\alpha \neq 0$ (series is generated by M2) and $\delta < 1$; the expectation of the series would be $\alpha(1-\delta)^{-1}$. This shows that there is no relation between the series and linear trend. But if $\alpha \neq 0$ and $\delta = 1$ *i.e.* the process would become random walk with drift, and the expectation of the series would be $\alpha\ t$. In this case, the series would have a predictable relationship with thee time trend. This analysis shows that if the data generating process is of the form of M2 with autoregressive coefficient $\delta < 0$, there is no predictable relationship between the trend and the observations of the time" series. This "derivation is valid if the time series length is large enough. This is because we approximate $\sum_{i=0}^{t} \delta_i$ by $\frac{1}{1-\delta}$, whereas, $\sum_{i=0}^{t} \delta_i = \frac{1-\delta^t}{1-\delta}$ and $\delta^t$ should be close to zero to make the approximation work. Thus, for smaller *t*, E($y_t$), may not be independent of time".

The discussion could be summarized as following:

- If data is generated by M1, it is independent of time irrespective of the value of autoregressive root and if the trend appears significant, it must be spurious.
- Considering data generated by M2, it is independent of time when the value of autoregressive root is less than unity and if the regression appears to be significant, it must be spurious.

## 2.1. Testing Spurious Trend and Monte Carlo Design

In this study, two types of equations are used for testing the existence of spurious trend. First equation is the one used by Nelson and Kang (1984) for testing the spurious trend which is as follows:

$$y_t = a + b\,t + e_t \tag{7}$$

*where:* $y_t$ is a series generated by M1 or M2, described in equation (1).

This equation is referred as NK Equation hereafter. The t-statistics for testing significance of linear-trend is referred as $t_b$.

Second equation we consider is the equations used by Dickey and Fuller (1979) which is as follows:

$$y_t = a + b\,t + \delta y_{t-1} + e_t \tag{8}$$

Again $y_t$ is a series generated by M1 or M2, described in equation (1). This equation be referred as DF Equation hereafter. The *t-statistics* for testing significance of linear-trend is referred as $t_\beta$ . The coefficients of linear terms in the two equations would be indicative of existence of spurious trend, if any.

Extensive Monte-Carlo experiments were carried out to see existence of spurious trend which consists of following steps:

- Choose a DGP *i.e.* one of the models described in equation (1);
- Generate a series $y_t$ according to the chosen model;
- Estimate DF Equation and NK Equation and record $t_b$ and $t_\beta$ ;
- Repeat (b) – (c) large number of times;
- Analyze the distribution of $t_b$ and $t_\beta$ using different descriptive tools.

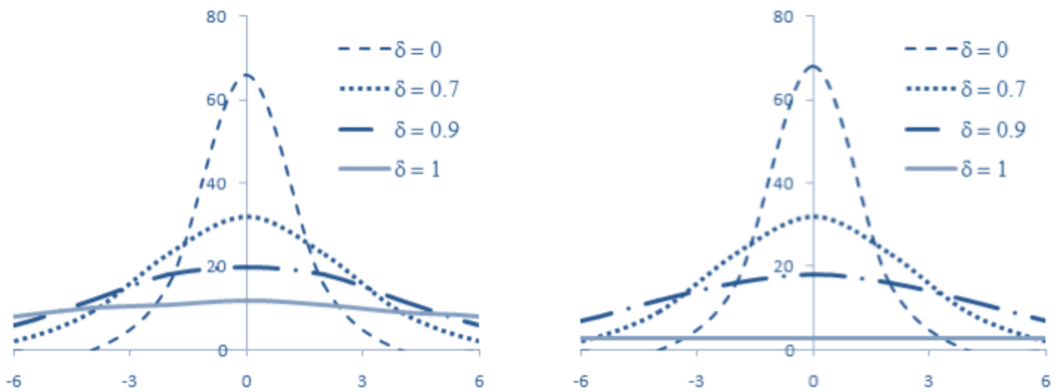
## 3. Results

### *a. Distribution of $t_b$ for M1*

Figure 1 summarizes the distribution of t-statistics for coefficient of linear-trend in NK Equation when the data is autoregressive without a constant in the auto-regression i.e. generated by model M1. As discussed in section 3, the data generated by M1 has no predictable relationship with linear trend for stationary roots as well as for unit root. However, the distribution of t-statistics gives much different pictures. The Left panel in Figure 1 corresponds to sample size 30 whereas the right panel corresponds to sample size 500. For sample size 30, the dashed curve represents the distribution of $t_b$ when $d = 0$ . We see that the distribution of $t_b$ in this case is similar to that of normal distribution.

However, distribution of $t_b$ become flatter when $d \pi 0$ even for the stationary values of $d$ . The conventional statistical analysis assumes a coefficient to be significant if absolute value of corresponding *t-statistics* is higher than 2 and probability of getting significant coefficient is close 5% when the distribution is approximately normal. The flatter distributions of *t-statistics* indicate that the chances of getting significant results exceed from the nominal 5% size whenever the auto-regression is non-zero, even if the root is stationary. This implies that the spurious trend is observable even if there is no unit root.

Figure 1. The distribution of $t_b$ for various values of $d$ and various sample sizes, DGP: M1




Sample Size = 30 Sample size = 500

For data generating process M1, if $d = 1$ than the process becomes unit root and the distribution of should be similar to that observed by Nelson and Kang (1984). Thus the distributions represented by solid lines in the two panels in Figure 1 represent the distributions observed by Nelson and Kang. The distributions that correspond to $d = 0$ are for IID series.

However, assuming asymptotic distribution of t-statistics for stationary series is normal with chances of spurious regression shall reduce to zero when sample size is large. The right panel in Figure 1 summarizes the distribution of for sample size 500. This is sufficiently large sample for the practitioner and practitioners rarely find such a large sample in the macroeconomic time series. For this sample, it is evident from the Figure 1 that the distribution of $t_b$ at $d = 0.7$ , is not very different from the distribution from the distribution observed for sample size 30. The Table 1 which summarizes the distribution probability of getting significant $t_b$ shows that the probability of getting significant coefficient is 40 even at sample size 500 which implies that chances of spurious regression are 35% which are still very high from the nominal significance level.

We noted percentage rejection of null of no-relationship with linear-trend for various samples sizes ranging 30 to 500 which are summarized in Figure 2. We see that probability of significant $t_b$ for d = 0.7 remains closer to 40% for all sample sizes. If the value of autoregressive coefficient increases, the probability of false rejection of hypothesis b = 0 also increases.

Figure 2. Percentage rejection of b= 0 in NK-Equation: GDP: M1

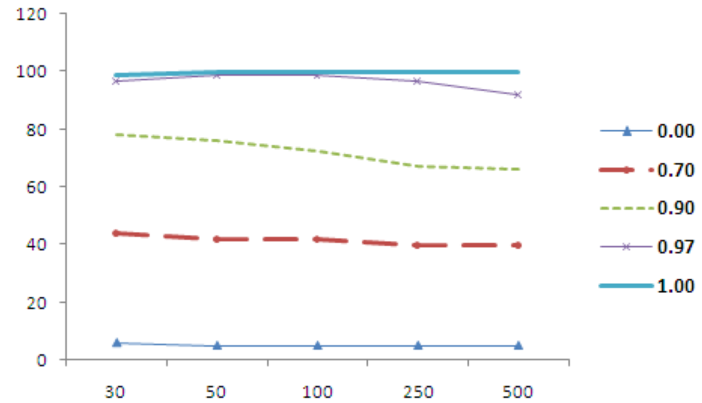


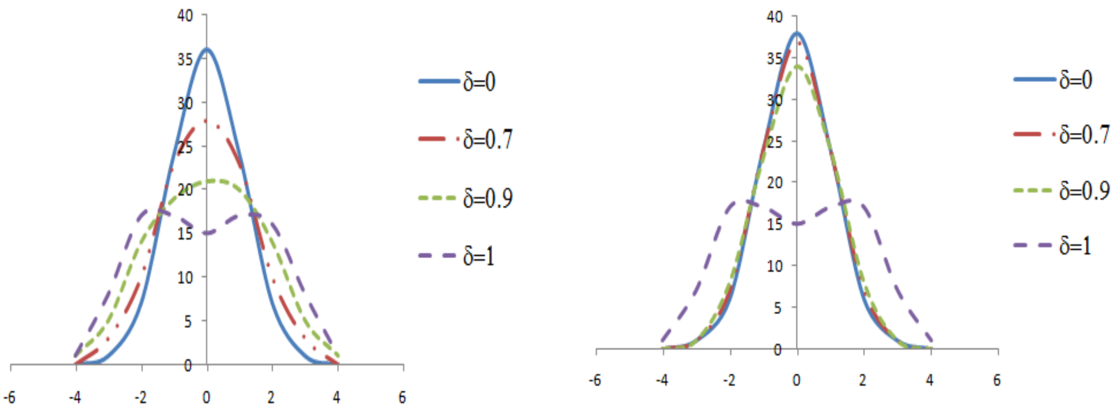

Figure 2 also shows that even though for stationary time series, the probability of spurious significance of linear-trend decreases with increase in sample size, with such small decrease, for any reasonable sample size, there will be very high probability of spurious trend. The Figure also show that the actual size of the t-test for significance of linear trend is closer to nominal size only when series is IID and whenever the series has serial dependence, there are chances of finding spurious results.

### *b. Distribution of $t_b$ for M1*

Figure 3 summarizes the distribution of the "t-statistics" for linear trend in DF-Equation *i.e.* $t_b$ for the data generated by M1 for sample size 30 and 250. The Figure 3 again illustrates that the distribution of *t-statistics* looks closer to "standard-normal" only when value of auto-regressive coefficient is $d$ zero.

In DF Equation and DGP M1, the distribution of $t_b$ converges to standard normal distribution at a faster speed. The right panel in Figure 3 summarizes the distribution of for sample size 250. One can see that the distributions of $t_b$ become indistinguishable for all stationary roots. Though for the unit root, the distribution is still far from normal distribution and is bimodal, for the stationary roots, the distribution of $t_b$ has approximate normal distribution. This means that the chances of spurious regression shall reduce to zero. However, in smaller sample sizes, there are heavy chances of spurious regression.

Figure 3. The distribution of $t_b$ in DF-Equation, DGP is M1




Sample size = 30　　　　Sample size = 250

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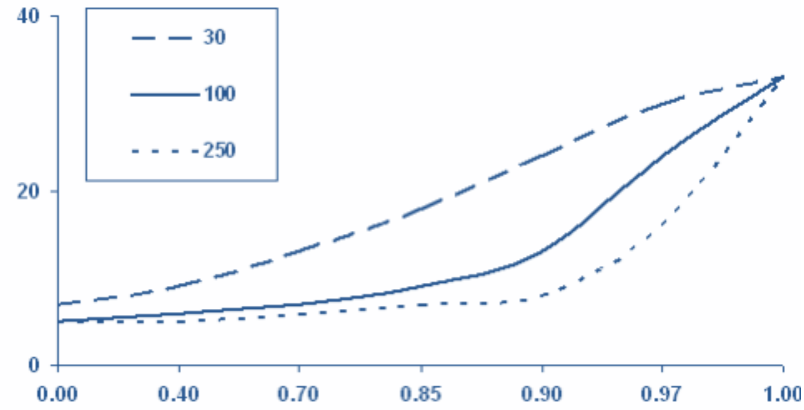
Figure 4 provides the rejection probability of the null hypothesis of no relation among the trend and the time path of series". This figure tells that the probability of getting significant coefficient of trend is close to 40% when the value of auto-regressive coefficient close to 1 for all three sample sizes summarized in Figure 4. This implies that the chances of spurious regression are close to 35% given the stationary roots close to 1.

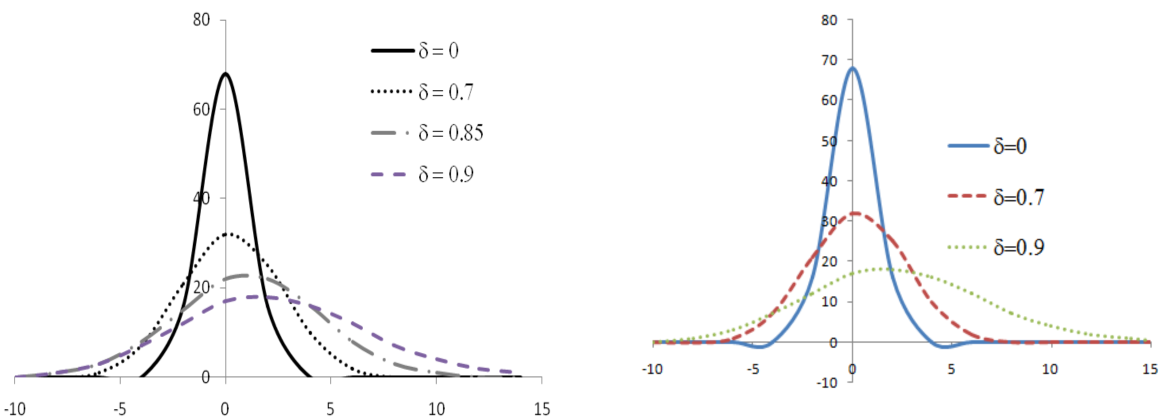
Figure 4. Percentage rejection of null b = 0 in DF-Equation, DGP is M1




*c. Distribution of $t_b$ for M2*

Figure 5 plots distribution of t-statistics in NK-Equation when M2 used as DGP. Left panel gives distribution of $t_b$ for sample size 30 and the right panel shows the distribution for the time series length 500. As we have shown in section 3, the series generated through M2 is independent of time whenever $d < 1$, therefore the probability of getting significant coefficient of linear trend should be close to 5%. However it was observed "that if drift parameter $a$ is positive, the distribution of t-statistics is skewed towards right. The skewness increases with increase in value of auto-regressive parameter" but decreases as the length of time series increases.

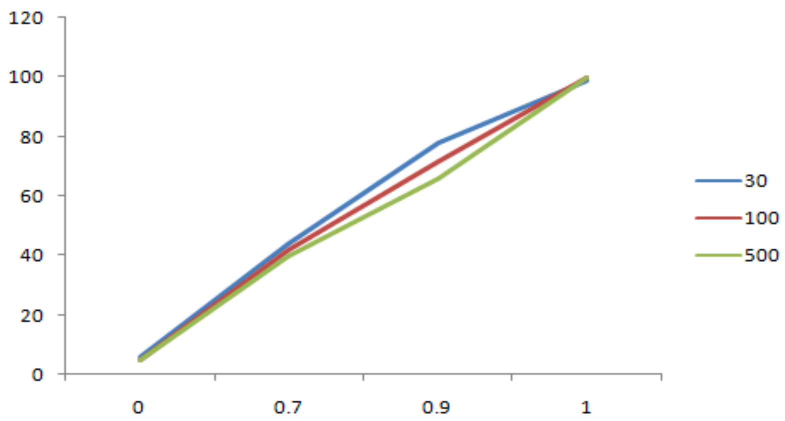
Figure 5. The distribution of $t_b$ in NK-Equation, DGP: M2




Sample size = 30 Sample size = 500

However, getting spurious significance in this case is very high for all reasonable sample sizes. As shown in Figure 6, the probability of getting significant linear trend converges to 100% as value of autoregressive parameter becomes close to zero. Further, it was observed that the sample size does not make significant difference in the probability of getting spurious regression. We see that for large sample the probability of getting significant linear trend is smaller; however the difference is not a considerable. Changing the sample size from 30 to 500, there is only 3% reduction in the probability of spurious significance of the linear trend when auto-regressive coefficient is 0.9.

Figure 6. Percentage rejection of b = 0 in NK Equation, GDP: M2




### d. Distribution of $t_b$ for M2

Figure 7 gives the distribution of t-statistics in DF-Equation when data generating process is M2. Like NK Equation, the DF Equation also gives skewed distribution when the series are generated by M2. The distribution looks closer to standard normal only when there is no auto-regression in the underlying series *i.e.* $d = 0$ and the distribution becomes biased toward right as the value of $d$ exceeds 0. By this variation in distribution of t-statistics, the probability of getting significant trend term also changes and exceeds the nominal significance level by a huge percentage.

Figure 7. The distribution of $t_b$ in DF equation, DGP: M2

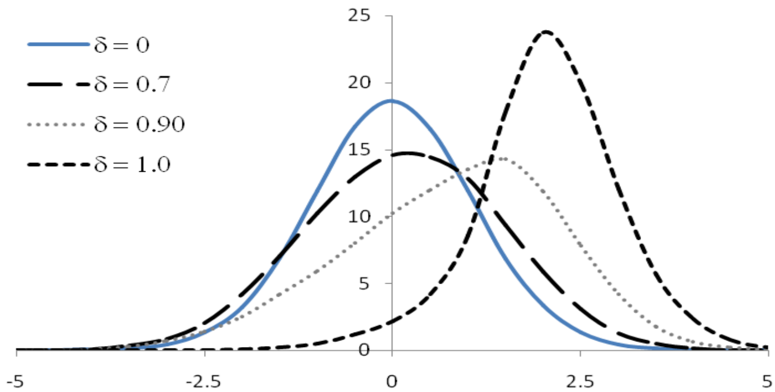
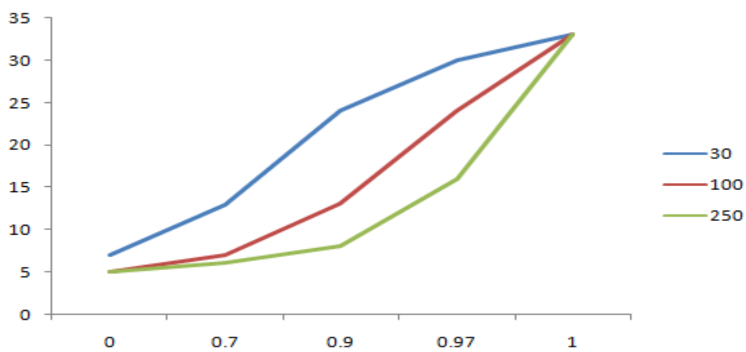



Figure 8 summarizes the probability of getting significant coefficient of linear trend when DF Equation is used in combination with M2. This figure shows that the sample size creates a significant difference in the probability of getting spurious trend, however, even in moderate sample sizes, the probability of getting spurious regression remains sufficiently high. The actual size of the test for significance of linear trend exceeds the nominal size.

Figure 8. Percentage rejection of null b = 0 in DF equation", DGP is M2



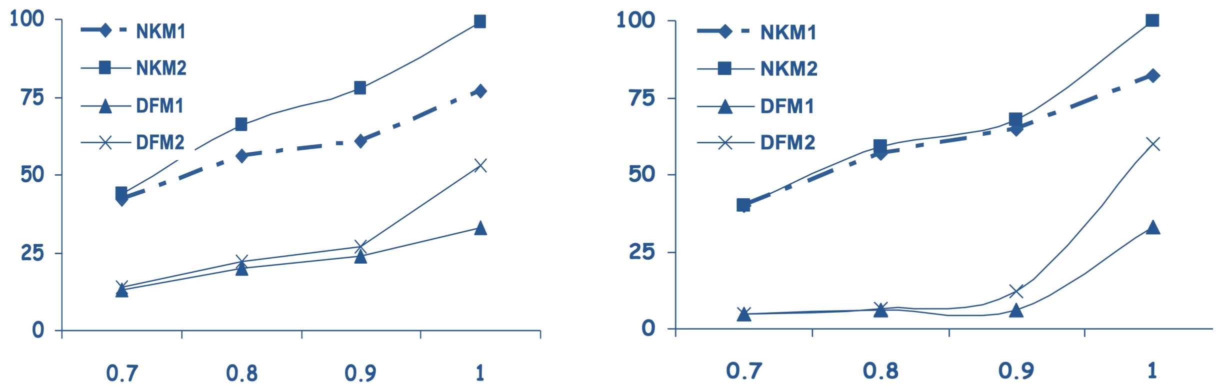

Therefore, "distribution is positively skewed, if positive drift coefficient is present in the DGP. Figure 8 illustrates probability of rejection of null hypothesis for different sample sizes. The Figure also illustrates that the probability of rejection of b = 0 matches with the nominal size, if $d \in [0, 0.8]$. This suggests that in DF- Equation, conventional Student's *t-statistis* can be used to specify the deterministic regressor provided the auto-regression is weak and the autoregressive root $d$ is closer to zero. On contrary, if the auto-regressive root is closer to 1 (but < 1 *i.e.* stationary series), the probability of false rejection of null b = 0 increases" significantly.

### *e. Comparison of NK-equation with DF-equation for M1 and M2*

The Figure 9 presents a comparison of the rejection rates of coefficient of linear trend for model M1 and M2. We observe following from the Figure.

- Rejection rates for M2 are always higher that the rejection rates for M1. This remains valid for DF equation and for NK equation;
- The rejection rates for NK Equation are always higher than the rejection rates in DF equation. This is valid for both M1 and M2;
- Increasing the sample size reduces the chances of spurious trend. However, the reduction is not as fast and even for a sample of 400, all of the possible combinations of DGP and test equations give very high probability of false rejection of null hypothesis of no linear- trend.

Figure 9. A comparison of Rejection Rate by Nelson and Kang and DF equations




Sample size = 100 | Sample size = 400

Similarly, it is also observed that by increasing the sample size, the probability of spurious trend reduces significantly when DF Equations are used and there is very small reduction when NK equations are used.

In short we have seen huge probabilities of spurious trend for the model with drift (M2) and for model without trend (M1) and by using DF Equation as well as NK equation. The procedures which are used for specification of deterministic part in a series before application of unit root depends on DF type Equations. We have seen that DF equation is also heavily biased toward rejection when the value of autoregressive is non-zero. Thus the asymptotic theory that is assumed valid for stationary series is actually valid only for IID series with no auto-regressions. As we discussed, the distribution of t-statistics is biased when the series in not IID, this means that procedures for specification of deterministic trend in unit root test equation are not valid and hence the output of unit root test is at stake.

## Conclusion and Policy Implications

Modern days' time series analysis starts with the unit root testing. One very important decision is use of unit root test is specification of deterministic-part. Many of the unit root tests are designed taking a specific form of deterministic trend *e.g.* Westerlund (2014), Kruse (2009), Jentsch *et al.* (2011), Otero and Smith (2012), Darné and Diebold (2002), Diebold and Giraud (2005) etc.

However, assuming this kind of model is not valid for real data. The second option in this case is specifying the deterministic part on the basis of the data to be tested. The details discussed in this paper show that the procedures based on normal approximations do not work for specification of deterministic trend whether the data is stationary or whether it is unit root. Therefore, there is still dire need of extensive research on the methodology of specifying the deterministic part in any model before conducting unit root tests.

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