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Comparative Analysis of Financial Market Volatility and Correlation Risk During the Great Recession and the COVID-19 Pandemic

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Abstract:

This paper offers a starting point for reflection on the similarities and differences of the impact on financial markets of the Great Recession of 2008 and of the Covid-19 pandemic of 2020 in terms of volatility and correlation risk among the most significant financial indexes in Europe. More precisely, the dataset employed includes the daily returns of Ftse100, CaC40, Dax30 and FtseMib40, with reference to the two time periods in which the two major crises manifested their effects on the markets. We use two different methodological approaches: the analysis of the daily conditional variance using various families of GARCH models and the study of the weekly realized volatility using HAR models. Furthermore, the estimation of the dependence structure of the GARCH residuals using copula functions is performed.

Keywords: volatility, GARCH processes, HAR models, copula functions, correlation risk.

JEL Classification: G01, G15, C22.

Introduction

In the last 15 years, the world economy has been put under pressure by two major and dramatic shocks that occurred in 2008 and 2020. The first of these, known to history as the Great Recession, brought down the economies of the main nations of the world and originated in the United States. The second is represented by the Sars-Cov-2 pandemic, initially spread to China, a virus that caused the respiratory disease known as Covid-19 from which we still suffer some aftermath but which exhausted its disruptive force in 2021. Of course, these are two completely different crises. However, we can identify at least two points in common: their severity in terms of the effects they have caused on the markets and on the economic-industrial sector in general, and their global nature, since both crises concerned all developed and developing countries.

In this paper we intend to study the two crises in terms of volatility and correlation risk on the markets of four of the main important European countries, taking into consideration the stock market indexes of the UK, France, Germany and Italy. The objective is to understand, in the period regarding the two crises, if there are similarities or systemic differences in the volatility of the market and in the correlation between the indexes. We will address the problem with a time series of returns covering the period before the crises, the period of the crises and the one after that.

The impact of the Covid-19 pandemic on the financial markets and more in general on the real economy was object of numerous publications in these last three years, and to this day the subject is much debated in the literature. Among others, We and Han (2021) uses event-study methodology to estimate the impact of the pandemic on the transmission of monetary policy to financial markets, Uddin et al. (2021) examine the effect of the pandemic on stock market volatility and whether economic strength, measured by a set of economic characteristics and factors such as economic resilience, intensity of capitalism, level of corporate governance, financial development, monetary policy rate can potentially mitigate the possible detrimental effect of the global pandemic on stock market volatility. Certainly, the pandemic shock has increased the severity of stock market volatility confirming the arguments in Engle and Ng (1993) who found that negative return shocks influence volatility more than positive return shocks. Moreover, as argued in Anderson et al. (2001) the asymmetry between stock market returns and volatility has become stronger with negative returns rather than positive returns. For a report on the impact of the pandemic on the financial markets the reader can consult Zhanga et al. (2020) who show that the pandemic has created an unprecedented level of risk, causing investors to suffer significant loses in a very short period of time. For example, the Ftse100 index dropped more than 10% on 12 March, 2020, in its worst day since 1987. The US index S&P 500 closed down -9.51% for its worst day since October 19, 1987. Liu et al. (2020) evaluate the short-term impact of the coronavirus outbreak on 21 leading stock market indices in major affected countries including Japan, Korea, Singapore, the USA, Germany, Italy, and the UK. According to Morales and Callaghan (2012) the global stock markets were becoming more interdependent and important crises, like the Covid-19 pandemic or the Great Recession of 2008, even if are originated in one country they soon spread to many others. Stock market movements become increasingly correlated.

We will not mention the countless publications relating to the great recession of 2008 of the last 14 years because it would go beyond the scope of this work. We would only quote the popular book of the Nobel Prize Paul Krugman (2008) who lays bare the financial crises of 2008 tracing it to the failure of regulation to keep pace with an out-of-control financial system. Furthermore, Farmer (2012 and 2015) establishes that the fall in the stock market in the autumn of 2008 provides a plausible causal explanation for the magnitude of the Great Recession and Anagnostidis et al. (2016) empirically investigate the impact of the 2008 financial crises on the weak-form efficiency of twelve Eurozone stock markets.

In this paper we adopt two approaches which are intensively used in the financial econometrics literature: the generalized autoregressive conditional heteroskedasticity (GARCH) model proposed by Bollerslev (1986) as a generalization of ARCH models introduced by Engle (1982) and the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV) suggested by Corsi (2009). The class of GARCH models is particularly suitable to describe the typical behaviour of financial time series, namely the fact that large (small) price changes tend to be followed by large (small) price changes of either sign; however, this kind of dependency can be exploited only to improve interval or density forecasts, but not point forecasts. However, their popularity grew when they were applied to high frequency data which revealed some of their critical issues such as the phenomena of volatility clustering, leverage effect and asymmetry. This prompted the researchers to find adequate generalizations of the two models that took into account these characteristics of the time series. Nelson (1991) introduces the exponential GARCH model (EGARCH) where the natural logarithm of the conditional variance h_t^2 is a function of ε_{t-1}^2, ε_{t-1} and of the natural logarithm of the lagged variance h_{t-1}^2. The model is more suitable to capture the negative correlation between lagged returns and the current conditional variance since positive or negative residuals of the same magnitude will have a different impact on the level of the volatility. Glosten, Jagannathan & Runkle (1993) propose the GJR-GARCH model where the standard GARCH(1,1) is augmented by $I_{t-1}\varepsilon_{t-1}^2$ where I_{t-1} is an indicator function which takes value 1 if ε_{t-1} is negative and zero otherwise. In this study we consider GARCH, EGARCH and GJR-GARCH models for the selected market stock returns in order to effectively and efficiently estimate volatility dynamics in both crises' periods.

On the other hand, HAR-RV, introduced for the first time by Corsi (2009), was designed to parsimoniously capture the strong persistence typically observed in return volatility and has become the landmark in the study of realized variance due to its consistently good forecasting performance. Furthermore, HAR-RV models are particularly suitable to capture some stylized facts about financial data such as the strong persistence of autocorrelations of square and absolute returns relating to log time periods (months, bi-months, quarters), or the fact that the return probability densities have a shape which depends on the time scale. In our application we use HAR-RV models to estimate the weekly evolution of the realized volatility in both crises' periods.

The aspect of correlation risk is investigated starting from the GARCH residuals by using copula functions to recover their joint distribution. An extensive and detailed discussion of copulas can be found in Nelsen (1994) and Cherubini et al. (2012). The original result is the Sklar theorem (1959) which showed that any n-dimensional joint distribution function may be decomposed into its n marginal distributions, and a copula, which completely describes the dependence between the n variables. In time series framework, we need to handle conditioning variables and their joint conditional distributions. For this reason, Patton (2006a, 2006b) extends the Sklar's theorem and introduces the notion of conditional copula (definition 1 and theorem 1 in Patton, 2006b) which will be intensively used in this paper in the part of the study relating to the correlation analysis.

The plan of the paper is the following. Section 1 presents the dataset under study. Section 2 introduces both volatility models, the dependence structure and comments the estimation results.

1. The Dataset

The study is based on a dataset made up of four stock market indexes relating to the four major European countries: Ftse100 (Great Britain), Cac40 (France), Dax30 (Germany) and FtseMib40 (Italy). We use time series of daily log-returns of the indexes in two distinct periods covering the two economic crises under consideration. The first period, relating to the Great Recession of 2008, runs from January 2, 2007 to December 31, 2009 for a total of 781 observations. The second period, relating to the Covid-19 pandemic, covers the period from January 2, 2019 to December 31, 2021 for a total of 783 observations. As we can observe from the time intervals, we have selected the periods considering three different moments that can somehow overlap. The first observation period covers the performance of the indexes in the phase preceding the two crises, therefore in 2007 and part of 2008 for the Great Recession and 2019 and the beginning of 2020 for the pandemic. The second period represents the heart of the two crises when they brutally manifested their effects (second half of 2008 for the Great Recession and all of 2020 and the beginning of 2021 for the pandemic). Finally, the third period consists of the post-crises period, until their effects are exhausted, at least as regards the financial markets.

Already from a first look at Table 1, which collects the descriptive statistics of the indexes in the two periods, we can extract some rather significant considerations. It seems at first glance that the pandemic has had a more violent impact on the lows of market returns. In fact, much larger values of the kurtosis are noted for all four indexes, with particular relevance for the FtseMib40, thus denoting the more leptokurtic form of the distribution of returns in the pandemic period. Not only that, even the skewness that was positive for three out of four indexes during the Great Recession, becomes very negative during the pandemic, signalling a greater tendency for returns to assume negative values. This is compensated by a lower volatility but considering the whole period. It should be noted that on average the returns of the indexes were negative during the period we considered concerning the Great Recession and positive during the period we considered concerning the Covid-19 pandemic. Looking also at the graphs of the log-returns shown in Figures 1 to 4, we can in fact notice some temporal congruences. All the indexes undergo a strong shock near the period in which the two crises manifest themselves in all their drama. A strong increment in volatility is observed in the last quarter of 2008 (the announcement of the bankruptcy of the Lehmann Brothers was on 15 September) and in the first quarter of 2020, coinciding with the arrival of restrictive measures, such as the lockdown, which was announced by the various governments between 9th of March and 23rd of March. However, some differences are not negligible. For example, it is quite evident by observing the fluctuations of returns that the impact of the pandemic was stronger but shorter than that of the Great Recession, a period in which the turbulence (and consequently the volatility) had a greater temporal span. The reason may be that most economic agents feared the Great Crises of 2008 more than the pandemic. As if he had more fears about the consequences. In fact, the Great Recession originated in the heart of finance and therefore can be considered a structural crisis destined to change paradigms. Vice versa, the pandemic was sudden, violent and profound but did not originate from the economy but from a completely external event.

Table 1. Stock market indexes: descriptive statistics relating to log-returns in percentage form for both periods

Great Recession period				
	Ftse100	Cac40	Dax30	FtseMib40
N. of obs.	781	781	781	781
Mean	-0.01777	-0.04369	-0.00816	-0.07381
Maximum	9.38434	10.59459	14.49533	10.87690
Minimum	-9.26577	-9.47154	-6.78616	-8.59811
Std. dev.	1.70305	1.84432	1.71142	1.84104
Skewness	-0.06005	0.15208	0.75193	0.06753
Excess Kurtosis	5.60791	5.89951	10.37753	5.30209
Covid-19 pandemic period				
	Ftse100	Cac40	Dax30	FtseMib40
N. of obs.	783	783	783	783
Mean	0.01189	0.05280	0.04249	0.05113
Maximum	8.66681	8.05611	9.32503	8.54946
Minimum	-11.51243	-13.0983	-13.03668	-18.54115
Std. dev.	1.22666	1.36826	1.37768	1.50243
Skewness	-1.29624	-1.48465	-1.29965	-3.05551
Excess Kurtosis	16.32931	16.85040	15.54943	36.59369

Figure 1. Log-returns of the Ftse100 stock market for both periods

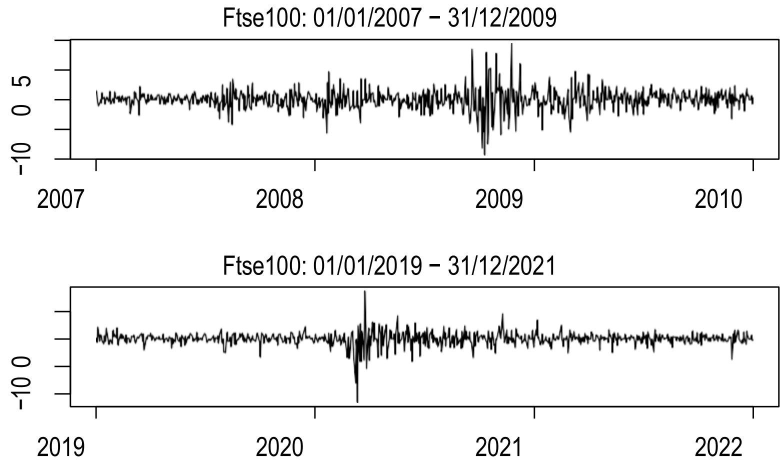


Figure 2. Log-returns of the Cac40 stock market for both periods

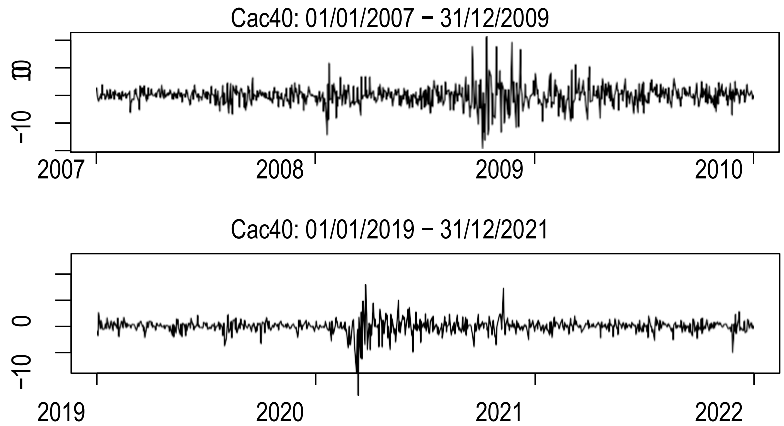


Figure 3. Log-returns of the Dax30 stock market for both periods

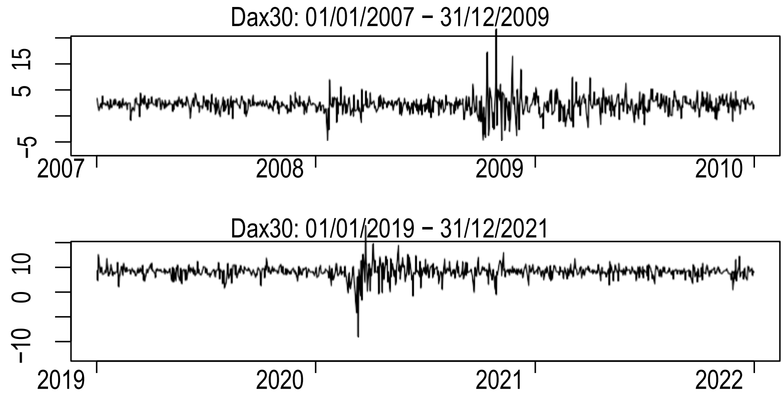
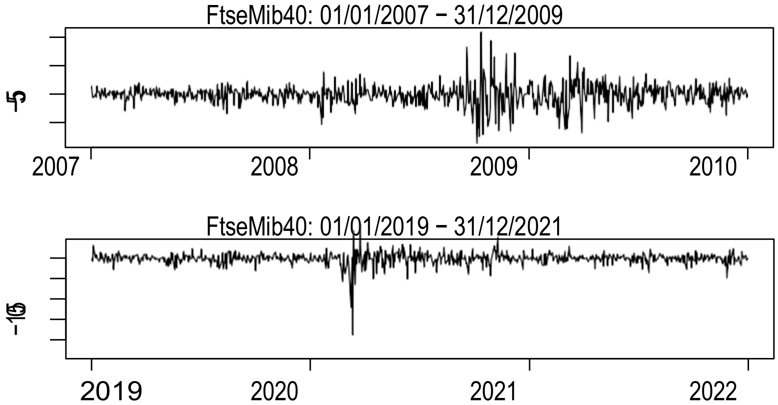


Figure 4. Log-returns of the FtseMib40 stock market for both periods.



2. Volatility Analysis

2.1. The GARCH Models' Approach

We proceed to the analysis of the volatility of the log-returns of the indexes using GARCH models. Particularly, in equity return time series empirical investigations have detected a negative correlation between lagged returns and the current conditional variance. In other words, volatility seems to increase in bear markets, which is exactly the case under study in this paper. The mechanism is often called "leverage effect". In general, standard GARCH models (Bollerslev, 1986) are not fit for capturing this effect. A number of alternative models have been proposed and we consider two of them: exponential GARCH (Nelson, 1991) and GJR-GARCH models (Glosten et al., 1993). We denote by F_{t-1} the information set available in the market up to time t − 1. Let $(\varepsilon_t)_t$ be a sequence of i.i.d. random variable which represents the innovation of the model. We will consider two different types of conditional distributions: the Student's t distribution, $\varepsilon_t|F_{t-1} \sim$ i.i.d.t(v) and the Skewed-t distribution, $\varepsilon_t|F_{t-1} \sim$ i.i.d.skt(v,η), *where v* is the parameter degrees of freedom and η is the skewness parameter. The log-returns x_t are characterized by the following equation:

$x_t = c + h_t \varepsilon_t$, *where* h_t is the conditional volatility of x_t.

In this paper, we use three different specifications for the conditional variance h^2. The first one is the standard GARCH(1,1) model introduced by Bollerslev (1986):

$$h_t^2=\omega+\alpha x_{t-1}^2+\beta h_{t-1}^2 \quad (3.1)$$

where α is the arch parameter and β is the GARCH parameter.

The second specification is given by the exponential GARCH (EGARCH) model proposed by Nelson (1991), where the equation of the conditional variance is:

$$\ln(h_t^2)=\omega+\alpha\varepsilon_{t-1} + \gamma(|\varepsilon_{t-1}| - E[|\varepsilon_{t-1}|]) + \beta \ln(h_{t-1}^2) \quad (3.2)$$

where the coefficient α captures the sign effect and γ the size effect.

The two components $\alpha\varepsilon_{t-1}$ and $\gamma(|\varepsilon_{t-1}| - E[|\varepsilon_{t-1}|])$ have both mean zero and when $\varepsilon_{t-1}>0$ the slope of the linear component of $\ln(h_t^2)$ is α + γ whereas if $\varepsilon_{t-1}< 0$ the slope is α − γ. Thus, the conditional variance process responds asymmetrically to rises and falls in stock price. The third specification is the GJR-GARCH model introduced by Glosten, Jagannathan and Runkle (1993) in which the variance equation is:

$$h_t^2=\omega+\alpha\varepsilon_{t-1}^2 + \delta I_{t-1}\varepsilon_{t-1}^2 + \beta h_{t-1}^2 \quad (3.3)$$

where α and *β* have the same meaning as in standard GARCH models whereas *δ* represents the leverage term since the indicator function I_{t-1} takes value 1 if ε_{t-1} is negative and zero otherwise.

It is a question of selecting the appropriate model for each of the four market indexes considering the different structure of the conditional variance and the different properties of the distribution of innovations. In order to select the correct models to investigate the dependence structure among stock market returns within the Great Recession and Covid pandemic periods, we use the Akaike Information Criterion (AIC), introduced by Akaike (1998) which estimates the amount of information lost by a model considering the trade-off between the goodness of fit and the number of parameters of the model.

2.2. Results

In this section we present the estimation results within the models considered above in the two periods under observation. In particular, several issues significant from an economic and financial point of view are discussed: (i) the asymmetry between positive and negative returns and the conditional variance; (ii) the persistence of shocks to variance; (iii) fat tails and asymmetry in the conditional distribution of innovations. The estimated parameters for each model are reported in Tables 2 - 9. Next, we examine the findings implied by the parameter estimates.

The asymmetry relation between returns and volatility is represented by α in the EGARCH model and δ in the GJR-GARCH model: we note that they are highly significant in both periods. In all EGARCH models α is negative which indicates that volatility tends to increases when stock returns are negative (bad news). However, some differences between the two crises periods can be appreciated. Particularly, the estimate of α is higher (e.g., more negative) in the Covid-19 pandemic period for all stock indexes, indicating a higher leverage effect induced by the pandemic than the Great Recession period. This finding is particularly clear in the case of the FtseMib40 where the estimate of α goes from about -0.11 during the pandemic period to about -0.19 during the Great Recession. In the case of the GJR-GARCH models we see that the parameter δ (which measures the sign effect) is highly significant in both periods, while the parameter α (size effect) is never significantly different from zero indicating that the good news has no impact on the conditional volatility in either of the two crises periods. As regards δ, only in the case of the Ftse100 it falls from about -0.19 during the Great Recession to about -0.09 during the Covid pandemic, whereas in all other three market indexes δ raises, signalling that bad news have more impact on volatility. (ii) Tables 2 - 9 report the persistence of shocks for each model denoted by $p̂$. Note that the persistence is measured by α + β for standard GARCH, β for EGARCH and α + β + δκ, where κ is the expected value of the standardized residuals below zero, for GJR-GARCH models. The persistence concerns how fast large volatilities decay after a shock. In all estimated models p̂ is very high and in some cases very close to one. However, volatility of the Dax30 and of the FtseMib40 presents persistence values lower or close 0.98 during the Covid pandemic. (iii) As regards the parameters of the distributions of innovations we can distinguish the case Student's *t* with *v* d.o.f. from the case skew-t, with v d.o.f. and η as skewness parameter.

Table 2. Marginal models: estimation of parameters* Ftse100. Period: Great Recession

Great Recession				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c:	0.061716	0.012177	0.024621
		(0.034693)	(0.035490)	(0.040603)
	ω:	0.030705	0.010288	0.039651*
		(0.019867)	(0.005356)	(0.016287)
	α:	0.130211*	-0.142017*	0.000000
		(0.028873)	(0.021489)	(0.024587)
	γ		0.141505*	
			(0.009655)	
Ftse100	δ:			0.191429*
				(0.043051)
	β:	0.864594*	0.978873*	0.888238*
		(0.055543)	(0.002013)	(0.021921)
	v:	6.7904*	8.441907*	8.048084*
		(1.763425)	(2.663707)	(2.427168)
		p̂=0.994805	p̂=0.978873	p̂=0.983953
		LB=0.1005	LB=0.1408	LB=0.1220
		AIC=3.4880	AIC=3.4513	AIC=3.4588
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c:	0.043043	-0.014997	c: -0.002673
		(0.041056)	(0.023578)	(0.041540)
	ω:	0.033693	0.014160*	0.039643*
		(0.018113)	(0.004739)	(0.015864)
	α:	0.127258*	-0.144862*	0.000000
		(0.026538)	(0.019949)	(0.023129)
	γ:		0.139517	
			(0.010562)	
Ftse100	δ			0.195640*

Great Recession				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
				(0.042576)
	β:	0.866852*	0.977693*	0.887810*
		(0.024397)	(0.002103)	(0.021174)
	v:	7.5636*	9.8855*	9.7292*
		(2.146346)	(3.633106)	(3.550384)
	η:	0.918901*	0.882925*	0.882272*
		(0.046477)	(0.045023)	(0.046359)
persistence		pˆ= 0.994109	pˆ = 0.977693	pˆ = 0.981497
		LB = 0.1005	LB = 0.1527	LB = 0.1334
		AIC = 3.4868	AIC = 3.4463	AIC = 3.4538

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 3. Marginal models: estimation of parameters*. Ftse100. Period: Covid-19 pandemic

Covid-19 pandemic				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	0.058058*	0.041243*	0.040620
		(0.026251)	(0.000004)	(0.026855)
	ω	0.024625	-0.013169*	0.012963
		(0.014617)	(0.000001)	(0.007128)
	α	0.092324*	-0.194141	0.000000
		(0.038061)	(0.000009)	(0.015828)
	γ		-0.069993*	
			(0.000021)	
Ftse100	δ			0.088987*
				(0.028647)
	β	0.890833*	0.996750*	0.939363*
		(0.042590)	(0.000075)	(0.021117)
	v	4.2408*	4.1430*	4.3109*
		(0.646002)	(0.000391)	(0.648831)
		pˆ=0.983157	pˆ=0.996749	pˆ=0.983857
		LB=0.6875	LB=0.9619	LB=0.5649
		AIC=2.6752	AIC=2.6367	AIC=2.6643
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	0.030407	-0.007153*	0.006453
		(0.028994)	(0.000001)	(0.029820)
	ω	0.024276	-0.003712*	0.014499*
		(0.013951)	(0.000002)	(0.007694)
	α	0.091632*	-0.186057*	0.000000
		(0.036425)	(0.000297)	(0.015681)
	γ		-0.056627*	
			(0.000028)	
Ftse100	δ			0.095884*
				(0.031180)
	β	0.889737*	0.993759*	0.936074*
		(0.041368)	(0.000450)	(0.022908)
	v	4.4896	4.6700*	4.5507*
		(0.727836)	(0.001225)	(0.730324)
	η	0.896207*	0.829042*	0.878161*
		(0.044271)	(0.011729)	(0.043911)
persistence		pˆ = 0.981369	pˆ= 0.993759	pˆ= 0.981235
		LB = 0.6865	LB = 0.7716	LB = 0.5588
		AIC = 2.6713	AIC = 2.6256	AIC = 2.6580

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 4. Marginal models: estimation of parameters*. Cac40. Period: Great Recession

Great Recession				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	0.041853	-0.016234	-0.002592
		(0.041554)	(0.039414)	(0.043685)
	ω	0.041774*	0.017010*	0.044805*
		(0.020734)	(0.006620)	(0.017484)
	α	0.12667*	-0.167502*	0.000000
		(0.026225)	(0.025115)	(0.024044)
	γ		0.145699*	
			(0.011103)	
Cac40	δ			0.210255*
				(0.047379)
	β	0.87049*	0.974111*	0.882315*
		(0.023092)	(0.000793)	(0.021762)
	ν	7.5191*	8.892091*	8.920602*
		(2.271002)	(2.852622)	(2.797807)
		p^=0.992715	p^=0.974110	p^=0.987442
		LB=0.0581	LB=0.0957	LB=0.0944
		AIC=3.6562	AIC=3.6100	AIC=3.6169
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	0.027795	-0.035123	-0.022717
		(0.045534)	(0.039125)	(0.044763)
	ω	0.039919	0.020325*	0.045558
		(0.020621)	(0.006809)	(0.017353)
	α	0.118666*	-0.169169*	0.000000
		(0.025334)	(0.024288)	(0.023535)
	γ		0.143905*	
			(0.012724)	
Cac40	δ			0.211304
				(0.046942)
	β	0.873442*	0.972711*	0.882521
		(0.023615)	(0.000751)	(0.021633)
	ν	7.9304*	9.5045*	9.9480
		(2.222908)	(3.233203)	(3.453095)
	η	0.943127*	0.912274*	0.911442
		(0.046150)	(0.045352)	(0.046471)
persistence		p^= 0.992108	p^= 0.972711	p^= 0.984841
		LB = 0.0573	LB = 0.1027	LB = 0.1002
		AIC = 3.6569	AIC = 3.6082	AIC = 3.6151

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 5. Marginal models: estimation of parameters*. Cac40. Period: Covid-19 pandemic

Covid-19 pandemic				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	0.134891	0.115647*	0.107026*
		(0.024865)	(0.019830)	(0.025762)
	ω	0.048739*	-0.018364*	0.029415*
		(0.019838)	(0.005064)	(0.013700)
	α	0.194028*	-0.207246*	0.000000
		(0.043325)	(0.025143)	(0.020890)
	γ		0.051230*	
			(0.018911)	
Cac40	δ			0.272240*
				(0.076222)
	β	0.804974*	0.994085*	0.852833*
		(0.034060)	(0.000270)	(0.038711)
	ν	3.5844*	3.6616*	3.7669*

Covid-19 pandemic				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
		(0.390712)	(0.463760)	(0.515426)
		pˆ=0.998999	pˆ=0.994085	pˆ=0.988953
		LB=0.8646	LB=0.8506	LB=0.7774
		AIC=2.8090	AIC=2.7573	AIC=2.7759
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	0.082768*	0.044885*	0.046325
		(0.029603)	(0.022258)	(0.029037)
	ω	0.040002*	-0.002826	0.028296
		(0.016184)	(0.006111)	(0.015182)
	α	0.188767*	-0.204696*	0.00000
		(0.045041)	(0.022752)	(0.019976)
	γ		0.072472*	
			(0.011403)	
Cac40	δ			0.274873
				(0.071222)
	β	0.810233*	0.987515*	0.858516
		(0.034414)	(0.000016)	(0.035656)
	ν	3.8998*	4.1097*	4.1599
		(0.564476)	(0.584156)	(0.618022)
	η	0.834306*	0.782270*	0.794225
		(0.042228)	(0.038924)	(0.041323)
persistence		pˆ = 0.998999	pˆ= 0.987515	pˆ = 0.982005
		LB = 0.8774	LB = 0.7882	LB = 0.7316
		AIC = 2.7947	AIC = 2.7315	AIC = 2.7518

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 6. Marginal models: estimation of parameters* Dax30. Period: Great Recession

Great Recession				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	0.075871 0.038901	0.041317	
		(0.042397)	(0.039770)	(0.041410)
	ω	0.038474*	0.010048	0.040410*
		(0.018008)	(0.006056)	(0.016132)
	α	0.111013*	-0.125168*	0.004322
		(0.029529)	(0.022956)	(0.021905)
	γ		0.149733*	
			(0.011576)	
Dax30	δ			0.168518*
				(0.041015)
	β	0.879478*	0.978583*	0.893706*
		(0.025625)	(0.000843)	(0.021929)
	ν	6.4465*	7.297017*	7.328084*
		(1.639115)	(1.971554)	(1.993729)
		pˆ=0.990491	pˆ=0.978583	pˆ=0.982287
		LB=0.5888	LB=0.7433	LB=0.6471
		AIC=3.4482	AIC=3.4504	AIC=3.4560
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	0.054332	0.009539	0.012041
		(0.043335)	(0.038847)	(0.043164)
	ω	0.036305*	0.013249*	0.040443*
		(0.018234)	(0.006138)	(0.015419)
	α	0.106965*	-0.127816*	0.000000
		(0.024639)	(0.021862)	(0.020581)
	γ		0.144185*	
			(0.010546)	
Dax30	δ			0.171761*

Great Recession				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
				(0.039873)
	β	0.882906*	0.977521*	0.897645*
		(0.023642)	(0.000992)	(0.020745)
	v	6.6201*	7.5737*	7.7529*
		(1.620166)	(2.118300)	(2.233117)
	η	0.930272*	0.900234*	0.901499*
		(0.043472)	(0.042295)	(0.043190)
persistence		p^= 0.989871	p^= 0.977520	p^= 0.980299
		LB = 0.5856	LB = 0.7787	LB = 0.6688
		AIC = 3.4777	AIC = 3.4466	AIC = 3.4525

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 7. Marginal models: estimation of parameters* Dax30. Period: Covid-19 pandemic.

Covid-19 pandemic				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	0.114608*	0.081520*	0.083927*
		(0.023751)	(0.026708)	(0.030235)
	ω	0.065942*	-0.007983	0.057167*
		(0.025630)	(0.004493)	(0.022707)
	α	0.151198*	-0.160201*	0.005893
		(0.044921)	(0.024155)	(0.026290)
	γ		0.048261*	
			(0.020719)	
Dax30	δ			0.226836*
				(0.069490)
	β	0.827881*	0.993921*	0.848135*
		(0.038825)	(0.000226)	(0.039977)
	v	3.7282*	3.9283*	3.9437*
		(0.465947)	(0.596723)	(0.611340)
		p^=0.979078	p^=0.993921	p^=0.967446
		LB=0.5485	LB=0.2778	LB=0.5295
		AIC=2.9720	AIC=2.9455	AIC=2.9494
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	0.063773*	-0.033572	0.021497
		(0.031012)	(0.031516)	
	ω	0.054098*	0.190765	0.050713
		(0.022155)	(0.014386)	(0.026989)
	α	0.138626*	-1.717237	0.000002
		(0.039113)	(0.274943)	(0.027724)
	γ		1.115753	
			(0.065187)	
Dax30	δ			0.230145*
				(0.082042)
	β	0.839888*	0.963240	0.862791*
		(0.036714)	(0.001039)	(0.052899)
	v	4.0668*	2.0153	4.1711*
		(0.656903)	(0.002201)	(0.555200)
	η	0.856972*	0.827922	0.826871*
		(0.041697)	(0.033391)	(0.041010)
persistence		p^= 0.978514	p^= 0.963240	p^= 0.967968
		LB = 0.5563	LB = 0.0643	LB = 0.5917
		AIC = 2.9613	AIC = 2.9687	AIC = 2.9323

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 8. Marginal models: estimation of parameters*. FtseMib40. Period: Great Recession

Great Recession				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	-0.007071	-0.055443	-0.044366
		(0.043009)	(0.038296)	(0.043873)
	ω	0.036067*	0.009654	0.029745*
		(0.018069)	(0.004217)	(0.013695)
	α	0.130173*	-0.110524*	0.014293
		(0.026624)	(0.017592)	(0.022575)
	γ		0.164537*	
			(0.038868)	
FtseMib40	δ			0.164632*
				(0.037802)
	β	0.865370*	0.985833*	0.893443*
		(0.024104)	(0.005358)	(0.020181)
	v	8.3886*	10.266683*	10.073052*
		(2.5879)	(3.761368)	(3.698645)
		pˆ=0.995543	pˆ=0.985833	pˆ=0.990052
		LB=0.4160	LB=0.4460	LB=0.4992
		AIC=3.6160	AIC=3.5893	AIC=3.5939
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	-0.039872	-0.094475*	-0.080624
		(0.043428)	(0.044342)	(0.043725)
	ω	0.028345	0.011903	0.026401*
		(0.015893)	(0.007813)	(0.012239)
	α	0.124085*	-0.109736*	0.011633
		(0.024111)	(0.019770)	(0.020370)
	γ		0.159996*	
			(0.031370)	
FtseMib40	δ			0.164058*
				(0.035386)
	β	0.872813*	0.987323*	0.899732*
		(0.021986)	(0.003885)	(0.018205)
	v	9.3153*	12.7351*	12.2265*
		(3.053593)	(5.644610)	(5.247733)
	η	0.859960*	0.829509*	0.837429*
		(0.044076)	(0.043874)	(0.044375)
persistence		pˆ= 0.996898	pˆ= 0.987323	pˆ= 0.988845
		LB = 0.3896	LB = 0.4375	LB = 0.5002
		AIC = 3.6068	AIC = 3.5748	AIC = 3.5811

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

Table 9. Marginal models: estimation of parameters*. FtseMib40. Period: Covid-19 pandemic

Covid-19 pandemic				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
	c	0.125327*	0.074370*	0.098272*
		(0.028081)	(0.000020)	(0.031675)
	ω	0.064943*	-0.006674*	0.038341
		(0.026309)	(0.000004)	(0.052727)
	α	0.135662*	-0.203473*	0.000000
		(0.039310)	(0.000105)	(0.097932)
	γ		-0.081995*	
			(0.000015)	
FtseMib40	δ			0.178940*
				(0.078045)
	β	0.840461*	0.989992*	0.884180*
		(0.034810)	(0.000046)	(0.134873)
	v	3.8795*	4.3906*	4.2205*
		(0.542437)	(0.001137)	(0.715032)

Covid-19 pandemic				
Stock market index		t-GARCH(1,1)	t-EGARCH(1,1)	t-GJR-GARCH(1,1)
		$\hat{p}$=0.976123	$\hat{p}$=0.989991	$\hat{p}$=0.973650
		LB=0.8386	LB=0.7895	LB=0.9707
		AIC=3.0492	AIC=2.9724	AIC=3.0138
		skt-GARCH(1,1)	skt-EGARCH(1,1)	skt-GJR-GARCH(1,1)
	c	0.067071*	-0.009884*	0.032832
		(0.034055)	(0.000035)	(0.072635)
	ω	0.054194*	0.008849*	0.032236
		(0.022053)	(0.000014)	(0.504810)
	α	0.137843*	-0.185316*	0.00000
		(0.035181)	(0.000240)	(1.224455)
	γ		-0.069205*	
			(0.000318)	
FtseMib40	δ			0.189746
				(0.098309)
	β	0.841064*	0.985877*	0.890407*
		(0.032722)	(0.004101)	(0.173320)
	v	4.3335*	5.1001*	4.6181*
		(0.694712)	(0.004340)	(1.141546)
	η	0.825332*	0.7556035*	0.792305*
		(0.042898)	(0.003655)	(0.059188)
persistence		$\hat{p}$ = 0.978907	$\hat{p}$= 0.985877	$\hat{p}$= 0.976112
		LB = 0.8008	LB = 0.8393	LB = 0.9607
		AIC = 3.0261	AIC = 2.9424	AIC = 2.9907

Note: * The table also reports the p-value of the Ljung-Box test on residuals and the AIC value relating to GARCH, eGARCH and GJR-GARCH models with different distribution of innovations.

All these parameters are significant. There is a clear direction about the parameter v. For all stock market indexes v drops drastically from the Great Recession period to the Covid-19 pandemic period indicating thicker tails in the conditional distributions in the second period. In the case of FtseMib40 the estimates fall more than half, from about 10-12 to 4-5. The same can be said of η, the skewness parameter. Notice that for the parametrization of the skew-t distribution that we have used, there is no asymmetry in the distribution when η = 1, therefore the more η is less than 1 the more skewness there is in the data. The effect of the two crises is clear: all stock market returns are characterized by increased asymmetry during the Covid-19 pandemic (values of η are close to 0.8 or lower) than during the Great Recession (values of η close to 0.9 or upper).

2.3. Conditional Volatility Dynamics

The AIC values allow for a selection of the optimal model by period and by market index. We have highlighted the model with the lowest AIC in tables 2-9. It is observed that regardless of the period and the market index, the optimal models are characterized by an innovation with skew-t distribution since the asymmetry plays a decisive role. The Ljung-Box test (Ljung and Box, 1973), whose p-value is shown in tables 2-9, suggests the absence of serial autocorrelation in the residuals thus ensuring the goodness of the selected models.

Figures 5 - 8 compare the volatility of the market indexes in the two periods. A substantial difference lies in the fact that in the period preceding the Great Recession, we can say from the first months of 2008, there was an increase in market volatility before the peak which occurred in the last quarter of 2008 after the announcement of the use of the chapter 11 of Lehman Brothers on September 15th. This is due to the fact that the crises was ”announcing”, the signals were already clear before the summer, it was not sudden. Conversely, the pandemic crises were unexpected, and in fact volatility undergoes a dramatic increase in early 2020 without having grown in the previous months. Covid was initially undervalued and even if the news from China arrived at the end of 2019, the markets did not believe in such a devastating impact. Furthermore, we observe that volatility remains high after the Great Recession throughout 2009 which was a year of strong contraction of economies around the world. On the contrary, in the Covid period, immediately after the peak due to the arrival of the pandemic and the closures decided by all European governments, there is a rapid reduction with new peaks at the end of 2020 due to the new widespread lockdowns, albeit less stringent. Within the countries we note that during the period of the Great Recession, volatility remained higher in the Euro countries than in Great Britain: this is partly attributable to the different attitude held by the Central Banks. While the Bank of England, in the footsteps of the US Federal Reserve, began the quantitative easing plan in March 2009, the European Central Bank was much less reactive.

Figure 5: Daily GARCH volatility of the Ftse100 stock market for both periods

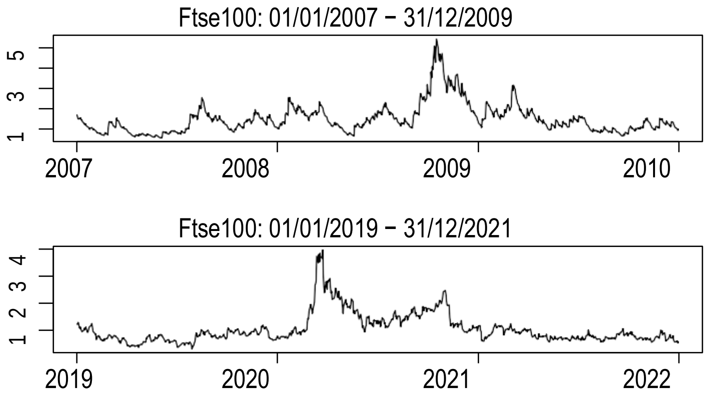


Figure 8 shows that the volatility of the FtseMib40 returns has reached the highest peak among the four indices under observation in the Covid period. A possible explanation can be found in the fact that Italy was the first Euro- pean country in which the Sars-Cov2 virus was detected and that the Italian economy is undoubtedly the weakest in terms of growth and fundamentals, a situation that has not changed between the two crises we are studying. In fact, even in 2009 volatility remained higher following the Great Recession precisely due to a greater structural weakness perceived by the financial markets.

For a more in-depth understanding of the dynamics of conditional volatility in the two crises periods analysed, we compute the average volatility in three sub-periods identified as before, during and after the crises. More precisely, in the case of the Great Recession we split the sample dates as follows: before the crises, from January 1, 2007 to February 29, 2008; during the crises, from March 1, 2008 to December 31, 2008; after the crises, January 1, 2009 to December 31, 2009. On the other hand, regarding the Covid-19 pandemic we split the sample dates as follows: before the crises, from January 1, 2019 to January 31, 2020; during the crises, from February 1, 2020 to March 31, 2021; after the crises, April 1, 2021 to December 31, 2021. Table 10 collects the average values of the estimated volatility in different sub-periods expressed in percentage form. There are at least two considerations that emerge.

Table 10: Conditional volatility: average values in the sub-periods (percentages).

Great Recession period				
Stock market index	Ftse100	Cac40	Dax30	FtseMib40
before	1.25	1.31	1.14	1.13
during	2.03	2.18	1.92	2.14
after	1.37	1.55	1.58	1.83
Covid-19 pandemic period				
Stock market index	Ftse100	Cac40	Dax30	FtseMib40
before	0.77	0.90	1.01	1.06
during	1.51	1.67	1.65	1.73
after	0.74	0.92	1.04	0.96

The increase in the market volatility determined by the two crises is remarkable for each of the indexes but does not have a single direction. For example, in the case of the Ftse100 and the Cac40, the pandemic has caused an increase in volatility of around 90% while the Great Recession only by 60%. The opposite happens for the Dax30 and the FtseMib40: in the first case the impact of the two crises was approximately the same, whereas in the second it was the Great Recession that caused the most consistent raise in market volatility. A significant difference between the two crises is noticeable if we consider the reduction in volatility following the end of the most intense period of the two crises. In fact, it is definitely more marked after the pandemic where volatility was reduced by approximately 40-50%, while after the Great Recession the reduction, albeit significant, reached 20-30%.

Figure 6: Daily GARCH volatility of the Cac40 stock market for both periods

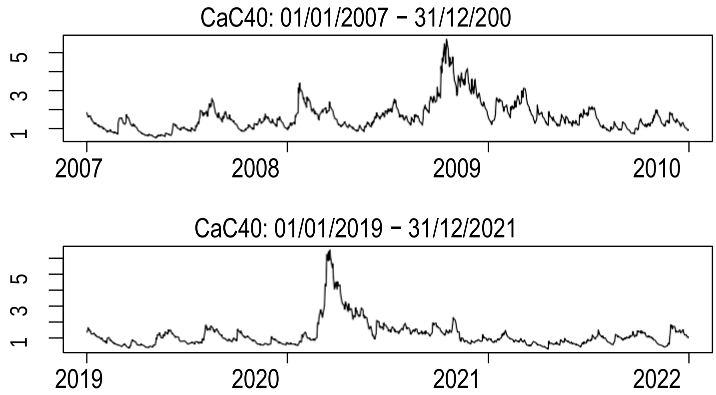


Figure 7: Daily GARCH volatility of the Dax30 stock market for both periods.

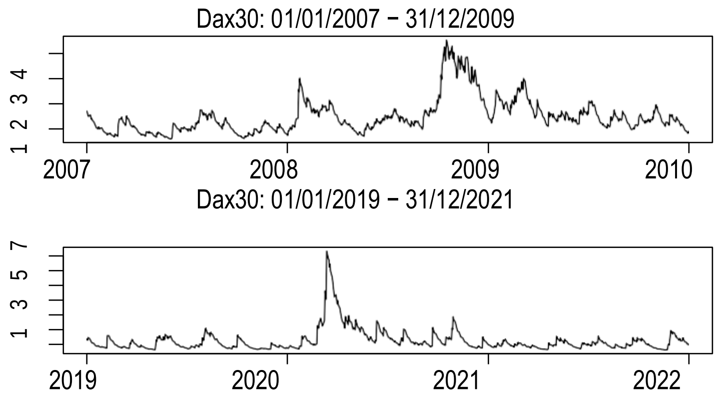
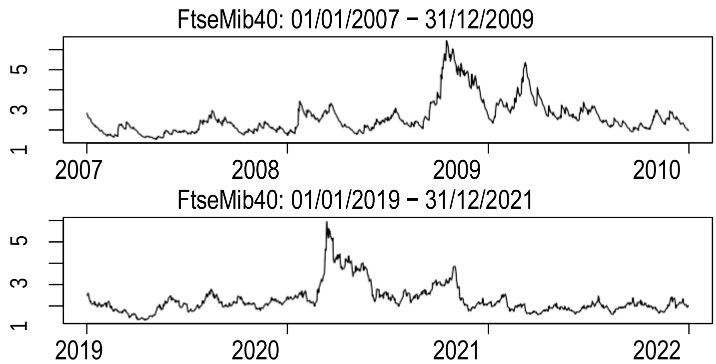


Figure 8: Daily GARCH volatility of the FtseMib40 stock market for both periods



2.4. HAR-RV Models Approach

The Heterogeneous Autoregressive model of realized Volatility (HAR-RV) pro- posed by Corsi (2009) is a very popular model to estimate and forecasting realized volatility. The formulation is flexible and simple and consists in a linear function of lagged squared returns over the same horizon together with the squared returns over longer horizons. Some extension of these models can be found in Buccheri and Corsi (2021). The original methodology was applied to high-frequency data in order to obtain a daily realized variance from the aggregation of squared residuals available within one trading day. In our application we work with weekly realized variance obtained from daily squared returns. So, denote by RV_t the weekly realized volatility obtained by summing the squared daily returns:

$RV_t = \sqrt{\frac{1}{5}\sum_{i=1}^{5} x_{t-i}^2}$ *where* the subscript t indicates the week, while i indicates the day within the week.

The HAR-RV model we use in this paper is an additive cascade of three realized volatility lagged of one period corresponding to three different time horizons, weekly, monthly and bi-monthly. In equation

$$RV_t = \theta_0 + \theta_1 RV_{t-1} + \theta_2 RV_{t-1}^{m}\ {}^{m} + \theta_3 RV_{t-1}^{2m} + u_t, \qquad (3.4)$$

where RV_t^m is the monthly lagged realized volatility and RV_t^{2m} is the bi-monthly lagged realized volatility.

Both of them are determined by aggregating the weekly realized volatilities:

$RV_t^m = \sqrt{\frac{1}{4}\sum_{i=1}^{4} RV_{t-i}}$ and $RV_t^{2m} = \sqrt{\frac{1}{8}\sum_{i=1}^{8} RV_{t-i}}$

As Corsi (2009) explains, the model focuses on the heterogeneity that originates from the difference in the time horizons since financial markets are populated by economic agents having a large spectrum of trading frequency. On one side, we have intraday speculators, who work with a very short time horizon and on the other side we have institutional investors (hedge funds, banks, insurance companies) who trade much less frequently. Such different types of market participants cause and perceive different types of volatility components. Therefore, in this application, we identify three volatility components: weekly (which is the reference time), monthly and bi-monthly.

2.5. Results

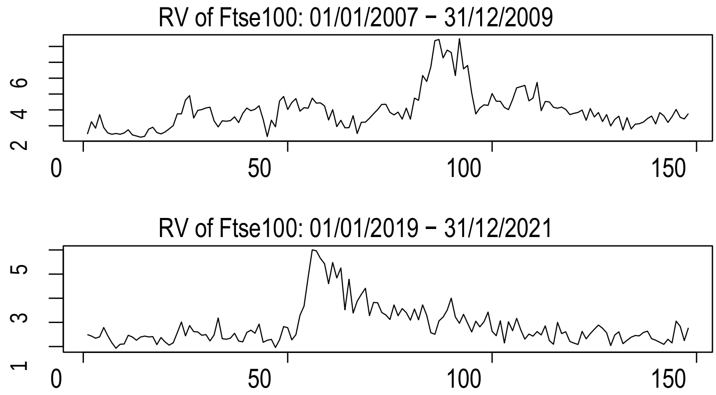
Table 11 reports the results of the estimation of the HAR-RV model for each stock market index for both crises periods. From the value of standard errors, we deduce that not all the three realized volatilities aggregated over the three different horizons are significantly different from zero. The exceptions are the coefficients of bi-monthly realized volatility for Dax30 and FtseMib40 during the Great Recession and the coefficients of monthly realized volatility for Ftse100, Cac40 and FtseMib40 during the pandemic. This implies a different reaction of agents with respect to various types of volatilities in the two crises. It can be noted that a substantial difference in the weights of the volatility components concerns the realized bi-monthly volatility. For each index the coefficients θ_3 are definitely higher in the pandemic period than in the Great Recession period, where even in two cases they are not significantly different from zero. A possible interpretation lies in the fact that the pandemic had a resurgence between the end of 2020 and the beginning of 2021 creating further uncertainty, while the Great Recession, after its terrible impact at the end of 2008, was better managed thanks, above all, to the interventions of Central Banks in 2009. Surprisingly, the monthly volatility component represented by the coefficient θ_2 is absent in the pandemic period whereas is strong in the Great Recession period and its weight is greater for Dax30 and FtseMib40.

Table 11. Estimation results for HAR(3) models

Great Recession period				
Stock market index	Ftse100	Cac40	Dax30	FtseMib40
θ_0	0.2324021*	0.2728977*	0.24355341*	0.2544113*
θ_1	(0.007711) 0.3911141*	(0.0101272) 0.4070375*	(0.006478) 0.32630034*	(0.008294) 0.3360613*
θ_2	(0.011743) 0.1494234*	(0.009789) 0.1233975*	(0.009752) 0.39528274*	(0.006121) 0.3683461*
	(0.061361)	(0.058137)	(0.098708)	(0.044623)
θ_3	0.2363191*	0.2275717*	0.04102855	0.0646104
	(0.041042)	(0.031425)	(0.059307)	(0.0646104)
Covid-19 pandemic period				
Stock market index	Ftse100	Cac40	Dax30	FtseMib40
θ_0	0.125099*	0.180715*	0.181150*	0.238831*
θ_1	(0.002284) 0.370981*	(0.003588) 0.452361*	(0.003414) 0.493653*	(0.004241) 0.424093*
	(0.023231)	(0.016854)	(0.012565)	(0.013684)
θ_2	0.037725	-0.043056	-0.155282*	0.006084
θ_3	(0.045500) 0.361818*	(0.025749) 0.308019*	(0.037950) 0.409326*	(0.018063) 0.252013*
	(0.061569)	(0.030807)	(0.030329)	(0.017776)

A graphical inspection of the realized volatilities is provided by figures 9-12. On the other hand, figures 13 and 14 summarize the character of the realized weekly volatility distributions estimated with the HAR-RV models for each stock market index in both periods. The right tail of the distributions is very thick in the Great Recession period without exception: this means that it has reached higher peaks more frequently confirming the greater severity of the first crises compared to the pandemic. Furthermore, the peak of densities is, for all four indexes, more to the left in the pandemic period (less than 2%) than in the period of the Great Recession (above 2%).

Figure 9: Weekly realized volatility estimated with the HAR-RV model. Ftse100 stock market for both periods



2.6. Dependence structure among GARCH residuals

A further interesting feature is the dependence structure among GARCH residuals to investigate how the correlation risk has changed between the two crises. In particular, we employ copula functions, which represent the joint distribution of a random vector given the marginal distributions (Nelsen, 1994; Cherubini et al., 2012) but, since in our application we work with time series, copula functions must be considered in their conditioned versions in the spirit of GARCH models. This extension was introduced and studied in Patton (2006a, 2006b). Therefore, assuming that the time series represent the log-returns of a set of n financial instruments at time *t*, e. g., $(X_{1,t}, ..., X_{n,t})_t$, adapted to the filtration $(F_t)_t$ with F_{t-1} - conditional marginal distributions $G_{1,t}, ..., G_{n,t}$ respectively, the F_{t-1}-conditional joint distribution H_t of the vec- tor $(X_{1,t}, ..., X_{n,t})$ can be expressed in terms of an F_{t-1}-conditional copula C_t: $H_t(x_1, ..., x_n) = P(X_{1,t} \leq x_1, ..., X_{n,t} \leq x_n) = C_t(G_{1,t}(x_1), ..., G_{n,t}(x_n))$, $(x_1, ..., x_n) \in R^n$.

Figure 10: Weekly realized volatility estimated with the HAR-RV model. Cac40 stock market for both periods

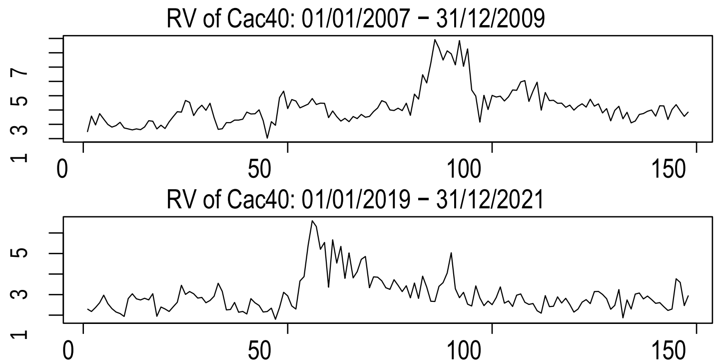


Figure 11. Weekly realized volatility estimated with the HAR-RV model. Dax30 stock market for both periods

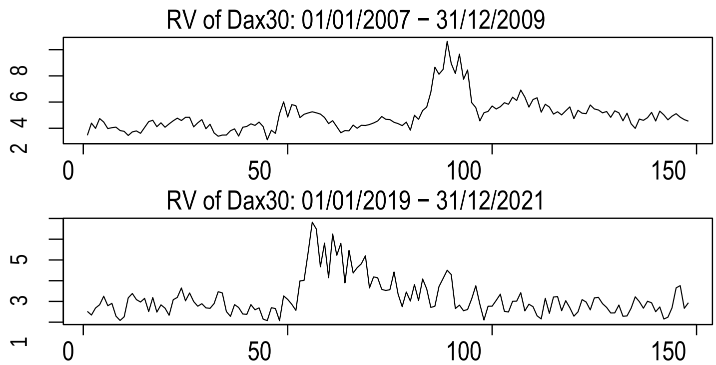


Figure 12. Weekly realized volatility estimated with the HAR-RV model. FtseMib40 stock market for both periods

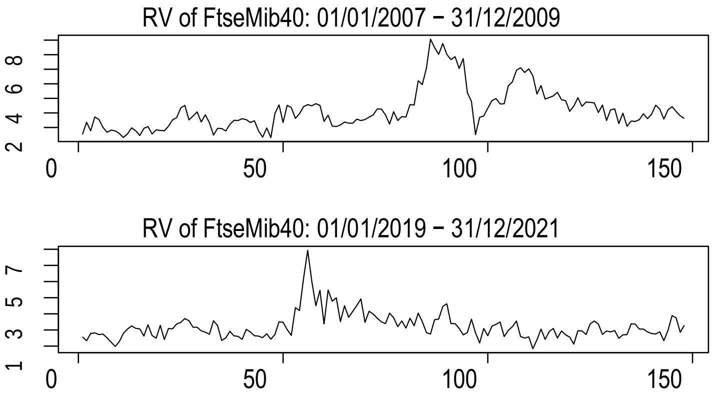


Figure 13. Comparison of distributions of weekly realized volatility within the two periods. Ftse100 (bottom) and Cac40 (down)

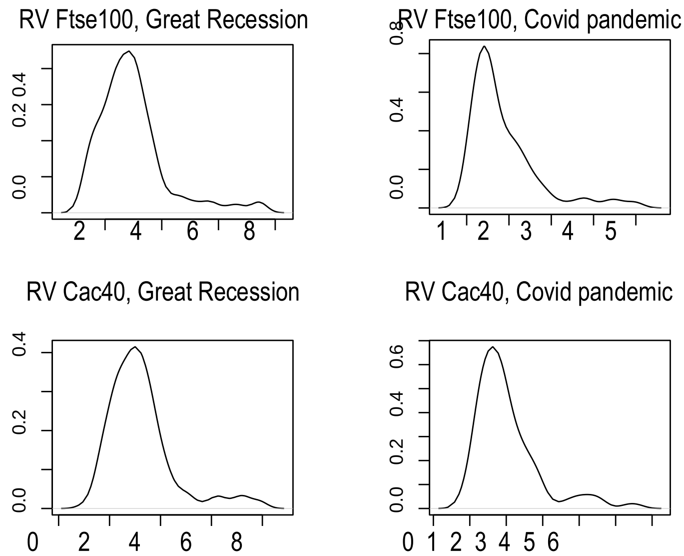
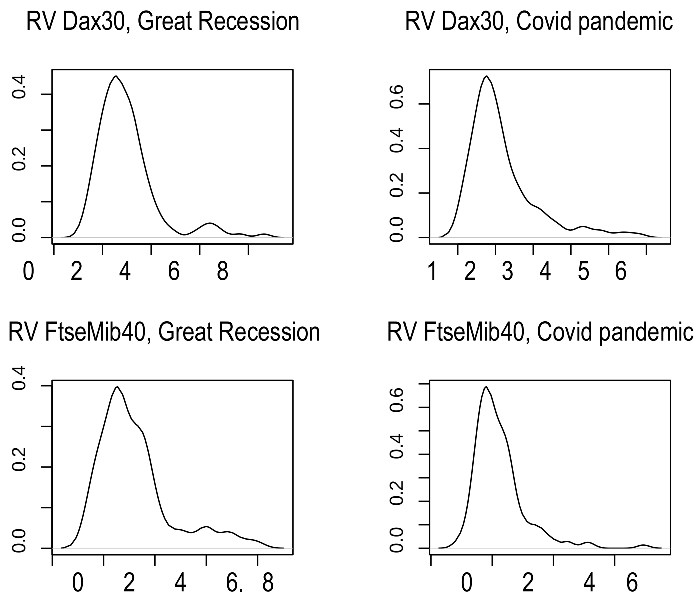


Figure 14: Comparison of distributions of weekly realized volatility within the two periods. Dax30 (bottom) and FtseMib40 (down)



The Copula function C_t is characterized by a functional form and a set of parameters. In this paper we estimate the dependence structure among residuals filtered by GARCH models selected by the estimation procedure. More formally, let $(R_{1,t}, R_{2,t}, R_{3,t}, R_{4,t})|F_{t-1}$ be the vector of GARCH residuals be parameterized as

$H_t(\Theta) = C_t (G_{1t}(r_{1t}; \theta_1), G_{2t}(r_{2t}; \theta_2), G_{3t}(r_{3t}; \theta_3), G_{4t}(r_{4t}; \theta_4); \Theta))$,

where F_{it}, i = 1, ..., 4, is the conditional cdf of residuals R_{it} of the marginal GARCH models selected in section 2.2., and Θ is the parameter (or vector of parameters) of the copula.

The functional form of C_t is known and so the conditional multivariate density of $(R_{1,t}, R_{2,t}, R_{3,t}, R_{4,t})$ is given by:

$$h_t(r_{1t}, r_{2t}, r_{3t}, r_{4t}; \Theta) = =[\prod_{i=1}^{4} g_{it}(r_{it}; \hat{\theta}_i)] c_t(G_{1t}(r_{1t}; \hat{\theta}_1), G_{2t}(r_{2t}; \hat{\theta}_2), G_{3t}(r_{3t}; \hat{\theta}_3), G_{4t}(r_{4t}; \hat{\theta}_4); \Theta))$$

where g_{it}, i = 1, ..., 4, is the marginal density function of residuals and c_t is the copula density.

In particular, we use the two main elliptic copulas: the gaussian copula and the t-copula, considering that this is the most appropriate choice in the presence of market returns. Actually, in light of the literature on this topic the gaussian copula does not seem the best choice. In fact, many studies of equity returns have reported deviations from multivariate normality, in the form of asymmetric dependence. One example of asymmetric dependence is where two returns exhibit greater correlation during market declines than market upturns, as reported in Erb et al. (1994), Longin & Solnik (2001), and Ang & Chen (2002). Ribeiro & Veronesi (2002) suggest correlations between international stock markets increase during market downturns as a consequence of investors having greater uncertainty about the state of the economy. This is exactly the case under observation in this work. For completeness we report the density function of a gaussian copula associated to a random vector $(X_1, ..., X_n)$ is given by:

$$c_N(u_1, \dots, u_n; \Sigma) = \frac{1}{[\det(\Sigma)]^{\frac{1}{2}}} \exp\left(-\frac{1}{2} u^T(\Sigma^{-1} - I)u\right)$$

where: $u = (\Phi^{-1}(u_1), ..., \Phi^{-1}(u_n))$, Σ is the n × n correlation matrix, e.g., $\Sigma = (\rho_{ij})_{i,j=1,\dots,d}$ with $\rho_{ii} = 1$ for all *i* and $\Phi^{-1}\cdot$ is the inverse *cdf* of standard univariate normal random variable. Notice that, in this case, the parameters are given by the correlation matrix Θ = Σ.

The density function of a t-copula is given by:

$$c_T(u_1, \dots, u_n; \Sigma, \nu) = \frac{1}{[\det(\Sigma)]^{\frac{1}{2}}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)}\right)^{d} \frac{\left((1+\nu^{-1}\mathbf{u}^T\Sigma^{-1}\mathbf{u})\right)^{-\frac{\nu+n}{2}}}{\prod_{i=1}^{d}\left(1+\frac{u_i^2}{\nu}\right)^{-\frac{\nu+1}{2}}},$$

where: $u = \left(t_{(\nu)}^{-1}(u_1), \dots, t_{(\nu)}^{-1}(u_n)\right)$ and $t_{(\nu)}^{-1}(.)$ is the inverse *cdf* of a Student's t random variable with *v* degrees of freedom.

Notice that, in this case, the parameters are given by the correlation matrix and the degrees of freedom parameter Θ = (Σ, v). The t copula generalizes the gaussian copula by providing a non-zero dependence in extreme tails of the joint distribution. In particular, we can have an upper or a lower tail dependence as in Joe (2015). Suppose that returns of two market stock indexes exhibit a lower (upper) tail dependence: this implies a positive probability of observing an extremely large depreciation (appreciation) of the first index together with an extremely large depreciation (appreciation) of the second index. There exist two coefficients which measures lower or upper tail dependencies, τ^U and τ^L, whose computation depends on the copula characteristics we use. Being the t copula a symmetric copula, we have that $\tau^U = \tau^L$ and we say that two variables exhibit a lower or upper tail dependence if $\tau^U, \tau^L > 0$.

Observe that the use of copulas in constructing multivariate models allows for the partitioning of the parameter vector into elements relating only to a marginal distribution, and elements relating to the copula. In this direction we use a two-stage maximum likelihood estimator as in White (1994) and Patton (2006a).

Tables 12 and Table 13 summarize the estimation results for both copulas. Al- though the values are high, a correlation risk behaviour can still be observed between the two periods. It is undoubtedly a fact that there is dependence in the tails and that the correlations, while remaining high, present significant differences. In fact, we can extract some indications about the two crises. (i) In both periods under observation the t-copula is the one that offers the best fit to the dependence structure among the stock market indexes, as evidenced by a higher likelihood value. The parameter degrees of freedom, *v*, is decidedly lower in the Great Recession period (5.831 against 11.7955) and this is a clear signal of a greater severity of the tails of joint distribution during the Great Recession rather than in the Covid-19 pandemic period. (ii) A general decline in the correlations from the period of the Great Recession to the period of the Covid-19 pandemic is appreciated: in fact, the average coefficient is 0.9246 in the first period and 0.8591 in the second. This can be interpreted as a reduction in the risk of correlation in the pandemic period. The economic crises induced by the Great Recession was more invasive and showed greater interdependence between European economies, at least in terms of market indexes. (iii) As regards the reciprocal dependencies between the indexes, a very strong dependence is observed in the Cac40-Dax30 pair, while the least correlated pair is the Ftse100-FtseMib40.

A further consideration concerns the tail dependence. There exists a link between the parameters *ρ* and v of a bivariate t-copula and the tail dependence coefficients τ^U and τ^L is given by the following relationships: $\tau^U = \tau^L = 2t_{(\nu+1)}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)$, *where* t(v+1) is the *cdf* of a Student's t random variable with v+1 degrees of freedom.

Table 12. Estimation of the dependence structure given by a Gaussian Copula

Great Recession period	LogLik=2106			
	Ftse100	Cac40	Dax30	FtseMib40
Ftse100	1	0.9317	0.8895	0.8658
		(0.003467)	(0.005617)	(0.006587)
Cac40	-	1	0.9312	0.9081
			(0.003506)	(0.004652)
Dax30	-	-	1	0.8553
				(0.007210)
FtseMib40	-	-	-	1
Covid-19 pandemic period	LogLik=1631			
	Ftse100	Cac40	Dax30	FtseMib40
Ftse100	1	0.8684	0.8113	0.7946
		(0.006747)	(0.009818)	(0.010111)
Cac40	-	1	0.9062	0.9010
			(0.004824)	(0.004995)
Dax30	-	-	1	0.8583
				(0.007152)
FtseMib40	-	-	-	1

In our case we can calculate the average tail dependence coefficient using the average value of the correlations among the market indexes in the two periods. Results are $\tau^U = \tau^L = 0.6239$ in the Great Recession period and $\tau^U = \tau^L = 0.3445$ in the Covid-19 period. This means that, on average, during the Great Recession period if the return of one market index takes an extreme value (positive or negative) there is about a 62% chance of the return of other market index taking an extreme value, whereas the same probability drops to about 34% during the Covid-19 pandemic period.

Table 13. Estimation of the dependence structure given by a t-copula.

Great Recession period	LogLik=2217		v: 5.8531	
	Ftse100	Cac40	Dax30	FtseMib40
Ftse100	1	0.9336	0.8988	0.8800
		(0.004065)	(0.006156)	(0.007083)
Cac40	-	1	0.9414	0.9139
			(0.003595)	(0.005274)
Dax30	-	-	1	0.8797
				(0.007645)
FtseMib40	-	-	-	1

Covid-19 pandemic period	LogLik=1648		v: 11.7955	
	Ftse100	Cac40	Dax30	FtseMib40
Ftse100	1	0.8704	0.8161	0.7971
		(0.012612)	(0.016684)	(0.018047)
Cac40	-	1	0.9083	0.9017
			(0.008261)	(0.008805)
Dax30	-	-	1	0.8611
				(0.011397)
FtseMib40	-	-	-	1

Concluding remarks

This paper analyses two major financial crises that have occurred in the last two decades from the stock market volatility point of view: the Great Recession of 2008 and the Covid-19 pandemic. The analysis is conducted using log- returns of market indexes relating to four European countries, Great Britain, France, Germany and Italy. Our approach is twofold. We analyse differences and similarities between the two crises through the daily conditional volatility estimated using some families of GARCH models and through the weekly realized variance estimated using HAR-RV models. Furthermore, the dependence structure among GARCH residuals is studied using copula functions in order to capture variations in terms of correlation risk in the two crises. From the volatility point of view, we observe that the estimates lean towards a greater persistence in the period of the Great Recession but with more important extreme peaks during the first phase of the pandemic. Although not clear-cut, some differences between the two crises can be glimpsed. The Great Recession was certainly deeper and more lasting than the pandemic and it was more expensive to overcome it. Probably, Central Bank intervention during the pandemic was prompter and more effective thanks to the lessons of the past.

Credit Authorship Contribution Statement

Gobbi, F. conducted the data collection and analysis, utilizing advanced econometric and machine learning techniques to investigate financial market volatility and correlation risk during the Great Recession and the COVID-19 Pandemic. He also developed the theoretical framework and performed the interpretation of the results.

Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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