Marshall Lerner Condition for Money Demand

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Abstract:

This article derives a twofold Marshall Lerner condition for money demand such that the current account may increase or decrease upon respective decrements or increments in the real exchange rate. This is noteworthy because the historic Marshall Lerner condition is such that the current account increases on account of a depreciation in the real exchange rate, yet, the current account also increases if the real exchange rate appreciates: the seeming contradiction is resolved by realizing that the current account respectively increases on account of a fundamental increment in (i) the real money supply and (ii) money demand, which increments respectively affect the real exchange rate in terms of a depreciation and an appreciation, concomitantly. The explanation advanced by the historic Marshall Lerner condition is therefore incomplete or superficial, if not ultimately misleading: the problem is hereby rectified.

Keywords: current account; exchange rate; Marshall Lerner condition; money demand; money supply; prices.

Introduction

The primary result of this article is the derivation of a Marshall Lerner condition for changes in money demand. The first derivative of the current account with respect to money demand is positive if and only if the sum of (i) the elasticity of exports to the real exchange rate, (ii) the absolute elasticity of imports to the real exchange rate and (iii) the quotient of the elasticity of exports to money demand and the elasticity of the real exchange rate to money demand is greater than one:

$$ca_{m_D} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{m_D}}}{\eta_{e_{m_D}}} > 1.$$

The first derivative of the current account with respect to money demand is negative if and only if the sum of (i) the elasticity of exports to the real exchange rate, (ii) the absolute elasticity of imports to the real exchange rate and (iii-a) the quotient of the absolute elasticity of imports to money demand and the elasticity of the real exchange rate to money demand or (iii-b) the quotient of the elasticity of imports to money demand and the absolute elasticity of the real exchange rate to money demand is smaller than one:

$$ca_{m_D} < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{|\eta_{im_{m_D}}|}{\eta_{e_{m_D}}} < 1 \quad \text{or} \quad \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{m_D}}}{|\eta_{e_{m_D}}|} < 1.$$

Secondary results include: an emphasized distinction between the nominal money supply ratio and the nominal exchange rate; an emphasized distinction between the real exchange rate and the terms of trade; a clear derivation of "Foreign prices in domestic prices", "Domestic prices in foreign prices", the national accounting identity and the real current account; an expression of the real exchange rate in terms of money supply and money demand; a characterization of money demand oriented to trade; a full derivation of the Marshall Lerner condition for changes in real money supply.

1. Exchange Rate and Current Account

1.1 Nominal Exchange Rate

Domestic nominal exchange rate E is the ratio of domestic nominal money supply units x to foreign nominal money supply units x^* :

$E = \frac{x}{x^*},\ \forall x \in M_S \subset \mathbb{R}_{++}$ and $x^* \in M_S^* \subset \mathbb{R}_{++}$.

Nominal money supply ratio $\frac{M_S}{M_S^*}$ is not domestic nominal exchange rate $E: \frac{M_S}{M_S^*} \neq \frac{x}{x^*} = E;$ this is because the nominal exchange rate is a price, affected by both nominal money supply and money demand, in fact, $E = f(\overset{+}{M_S},\ \overset{-}{M_S^*},\ \overset{-}{m_D},\ \overset{+}{m_D^*})$, but more anon. Under domestic financial closure and a domestic single currency ($M_S \equiv M_{SE} = M_{SI}$) domestic nominal exchange rate E is decreed and enforced through domestic balance of payments transactions passing by the domestic central bank; under domestic financial closure and a domestic double currency there are two options: domestic external nominal money supply M_{SE} is retained for domestic imports; domestic external nominal money supply M_{SE} is converted into domestic internal money supply M_{SI} at a set rate $\frac{M_{SE}}{M_{SI}}$. Currency substitutions allow nominal money supplies to be reduced to zero: $M_S \sim M_S^* \longrightarrow M_S,\ M_S^* \subset \mathbb{R}_+$; reserve assets and currency liabilities are reduced at the set nominal exchange rate if carried out at home and gained if carried out abroad (*i.e.* currency accumulation).

1.2. Real Exchange Rate and Terms of Trade

Absolute purchasing power parity (APPP) is the equality between domestic nominal exchange rate E and domestic price ratio $\frac{p}{p^*}$:

$$E = \frac{x}{x^*} = \frac{p}{p^*},\ \forall p,\ p^* \in \mathbb{R}_{++}.$$

Rearranged, it yields domestic real exchange rate e:

$$e = \frac{Ep^*}{p} = \frac{xp^*}{x^*p};$$

it is the ratio of domestic real money supply units $\frac{x}{p}$ to foreign real money supply units $\frac{x^*}{p^*}$ (*i.e.* domestic to foreign commodities). The law of one price (LOP) is the equality between domestic nominal exchange rate E and domestic individual price ratio $\frac{p_i}{p_i^*}: E = \frac{x}{x^*} = \frac{p_i}{p_i^*},\ \forall i \in \mathbb{R}_{++}$.

Rearranged, it yields domestic terms of trade tot: $tot = \frac{Ep_i^*}{p_i} = \frac{xp_i^*}{x^*p_i};$ it is the ratio of domestic real money supply individual units $\frac{x}{p_i}$ to foreign real money supply individual units $\frac{x^*}{p_i^*}$ (*i.e.* domestic to foreign individual commodities).

"*Foreign prices in domestic prices*" are obtained by rearranging APPP and solving for p:

$$E = \frac{x}{x^*} = \frac{p}{p^*} \longrightarrow p = Ep^* = \frac{xp^*}{x^*}.$$

"*Domestic prices in foreign prices*" are obtained by rearranging APPP and solving for p^*:

$$E = \frac{x}{x^*} = \frac{p}{p^*} \longrightarrow p^* = \frac{p}{E} = E^*p = \frac{x^*p}{x}.$$

1.3 National Accounting and Current Account

Domestic demand is household, government and firm domestic expenditure: domestic consumption c; domestic government spending g; domestic investment i. Domestic supply is domestic production y and domestic imports im net of domestic exports ex. Domestic demand equals domestic supply in market clearing and thence stems the national domestic accounting identity:

$D \equiv c + g + i = y + im - ex \equiv S \longrightarrow y = c + g + i + ex - im = c + g + i + ca,$
$\forall y,\ im,\ ex \in \mathbb{R}_+;$ net foreign expenditure $ex - im$ is domestic current account $ca.$

Pricing the national domestic accounting identity domestically yields: $py = p\left(c + g + i + ca\right) = p\left(c + g + i + ex\right) - Ep^* im,$ where domestic imports im are priced through "Foreign prices in domestic prices" Ep^*.

In real terms the national domestic accounting identity becomes:

$$y = c + g + i + ca = c + g + i + ex - \left(\frac{Ep^*}{p}\right) im = c + g + i + ex - e \cdot im,$$

where all variables are divided by domestic prices p. Domestic current account ca expressed in real terms is thus $ca = ex - e \cdot im;$ at efficiency domestic exports ex substitute domestic imports im at domestic real exchange rate e :

$$ca = 0 \longleftrightarrow ex = e \cdot im \longrightarrow \frac{ex}{im} = e.$$

Hak Choi (2018) is wrong, because he argued that in $ca = ex - im$ all three variables are measured in value, rather than in quantity, invalidating the derivation of the elasticity for the Marshall Lerner condition. Yet, domestic current account ca begins in quantities, $ca = ex - im,$ it is transformed in value by domestic prices $p,\ pca = pex - Ep^* im,$ and it is reduced back to real quantities in division by domestic prices $p,\ ca = ex - e \cdot im:$ Choi's blunder would have been clearer if he had derived $ca = ex - e \cdot im$ axiomatically.

1.4 Money Supply and Money Demand

Domestic real exchange rate e is a price; it increases in domestic real money supply m_S and foreign money demand m_D^* and decreases in foreign real money supply m_S^* and domestic money demand m_D :

$$e = f(\overset{+}{m_S},\ \overset{-}{m_S^*},\ \overset{+}{m_D^*},\ \overset{-}{m_D}).$$

More specifically, domestic real exchange rate e decreases in domestic real interest rate r and foreign expected real interest rate er^* and increases in foreign real interest rate r^* and domestic expected real interest rate er (*nb.* expected real interest rates account for over and undershooting):

$$e = f(\overset{-}{r},\ \overset{+}{r^*},\ \overset{+}{er},\ \overset{-}{er^*}).$$

In turn, domestic expected real interest rate er decreases in domestic real interest rate r and domestic real interest rate r decreases in domestic real money supply m_S and increases in domestic money demand $m_D:\ er = f(\overset{-}{r});\ r = f(\overset{-}{m_S},\ \overset{+}{m_D}).$

Domestic real money supply m_S increases in domestic nominal money supply M_S and decreases in domestic prices $p:\ m_S = \frac{M_S}{p}.$ Domestic prices p range from marginal products and technology to supply taxation and varied output, with variations in the nominal money supply, marginal products, technology, supply and demand taxation and output demand, but their characterization is hereby unnecessary. Domestic money demand m_D spans demand and supply taxation, marginal products, technology and output demand and to the end of deriving a Marshall Lerner condition for domestic money demand m_D those of concern are the ones affecting domestic exports ex and domestic imports $im,$ namely, export demand ed (*e.g.* confidence, tariffs, quotas) and import demand id (*i.e.* foreign export demand ed^*), which respectively increase and decrease domestic money demand m_D :

$$m_D = f(\overset{+}{ed},\ \overset{-}{id}).$$

Consequently, domestic exports ex increase in domestic real exchange rate e and export demand ed and domestic imports im decrease in domestic real exchange rate e and increase in import demand id :

$$ex = f(\overset{+}{e},\ \overset{+}{ed});\ im = f(\overset{-}{e},\ \overset{+}{id}).$$

2. Marshall Lerner Conditions

2.1 Marshall Lerner Condition

Following Paul Krugman, Maurice Obstfeld and Marc Mélitz (2018), the Marshall Lerner condition is derived for an increase in domestic current account ca given an increase in real exchange rate e, presupposing an ultimate increase in domestic nominal money supply M_S or a decrease in domestic prices p. Behold it derived given an increase in domestic nominal money supply M_S; for simplicity, $e = f(\overset{+}{m_S},\ \overset{-}{m_S^*},\ \overset{+}{m_D^*},\ \overset{-}{m_D})$, rather than $e = f(\overset{-}{r},\ \overset{+}{r^*},\ \overset{+}{er},\ \overset{-}{er^*})$:

$$ca = ex - e \cdot im$$

$$ca_{M_S} = ex_e e_{m_S} m_{S_{M_S}} - \left(e_{m_S} m_{S_{M_S}} im + e \cdot im_e e_{m_S} m_{S_{M_S}}\right)$$

$$\frac{ca_{M_S}}{e_{m_S} m_{S_{M_S}}} = ex_e - (im + e \cdot im_e) \qquad \frac{ca_{M_S} e}{e_{m_S} m_{S_{M_S}} ex} = \frac{e}{ex}(ex_e - im - e \cdot im_e)$$

$$\frac{ca_{M_S}}{e_{m_S} m_{S_{M_S}} im} = \eta_{ex_e} - \frac{1}{im}(im + e \cdot im_e) \qquad \frac{ca_{M_S}}{e_{m_S} m_{S_{M_S}} im} = \eta_{ex_e} - 1 - \eta_{im_e}$$

$$ca_{M_S} > 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} > 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| > 1.$$

Price elasticity of demand are negative; exports are supplied and imports are demanded, thus, $\eta_{im_e} < 0$ and thereby $|\eta_{im_e}| = -\eta_{im_e}$. The first derivative of domestic current account ca with respect to domestic nominal money supply M_S is positive if and only if the sum of (i) elasticity of domestic exports to domestic real exchange rate η_{ex_e} and (ii) absolute elasticity of domestic imports to domestic real exchange rate $|\eta_{im_e}|$ is greater than one.

The Marshall Lerner condition derived given a decrease in domestic prices p is obtained analogously:

$$ca = ex - e \cdot im \longrightarrow ca_p = ex_e e_{m_S} m_{S_p} - \left(e_{m_S} m_{S_p} im + e \cdot im_e e_{m_S} m_{S_p}\right)$$
$$\longrightarrow \frac{ca_p}{e_{m_S} m_{S_p}} = ex_e - (im + e \cdot im_e) \longrightarrow \frac{ca_p e}{e_{m_S} m_{S_p} ex} = \frac{e}{ex}(ex_e - im - e \cdot im_e)$$
$$\longrightarrow \frac{ca_p}{e_{m_S} m_{S_p} im} = \eta_{ex_e} - \frac{1}{im}(im + e \cdot im_e) \longrightarrow \frac{ca_p}{e_{m_S} m_{S_p} im} = \eta_{ex_e} - 1 - \eta_{im_e}$$
$$\longrightarrow (ca_p < 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} < 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| < 1).$$

2.2 Marshall Lerner Condition for Money Demand

The Marshall Lerner condition derived for an increase in domestic current account ca given a decrease in real exchange rate e is possible through an increase in domestic money demand m_D, presupposing an ultimate increase in export demand ed :

$$ca = ex - e \cdot im$$

$$ca_{ed} = ex_e e_{m_D} m_{D_{ed}} + ex_{ed} - (e_{m_D} m_{D_{ed}} im + e \cdot im_e e_{m_D} m_{D_{ed}})$$
$$\frac{ca_{ed}}{e_{m_D} m_{D_{ed}}} = ex_e + \frac{ex_{ed}}{e_{m_D} m_{D_{ed}}} - (im + e \cdot im_e)$$
$$\frac{ca_{ed} e}{e_{m_D} m_{D_{ed}} ex} = \eta_{ex_e} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - (1 + \eta_{im_e})$$
$$\left(\frac{m_D ed}{m_D ed}\right) \frac{ca_{ed} e}{e_{m_D} m_{D_{ed}} ex} = \frac{m_D ed}{m_D ed}\left(\eta_{ex_e} + \frac{ex_{ed} e}{e_{m_D} m_{D_{ed}} ex} - 1 - \eta_{im_e}\right)$$

$$\frac{ca_{ed}}{e_{m_D}m_{D_{ed}}im} = \eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} - 1 - \eta_{im_e}$$

$$ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} - 1 - \eta_{im_e} > 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}} > 1.$$

The first derivative of domestic current account ca with respect to export demand ed is positive if and only if the sum of (i) elasticity of domestic exports to domestic real exchange rate η_{ex_e}, (ii) absolute elasticity of domestic imports to domestic real exchange rate $|\eta_{im_e}|$ and (iii) quotient of the elasticity of domestic exports to export demand, the elasticity of the domestic real exchange rate to domestic money demand and the elasticity of domestic money demand to export demand $\frac{\eta_{ex_{ed}}}{\eta_{e_{m_D}}\eta_{m_{D_{ed}}}}$ is greater than one. Analogously, the Marshall Lerner condition derived for a decrease in domestic current account ca given an increase in real exchange rate e is possible through a decrease in domestic money demand m_D, presupposing an ultimate increase in import demand id :

$$ca = ex - e \cdot im$$

$$ca_{id} = ex_e e_{m_D} m_{D_{id}} - [e_{m_D} m_{D_{id}} im + e\,(im_e e_{m_D} m_{D_{id}} + im_{id})]$$

$$\frac{ca_{id}}{e_{m_D}m_{D_{id}}} = ex_e - \left[im + e\left(im_e + \frac{im_{id}}{e_{m_D}m_{D_{id}}}\right)\right]$$

$$\frac{ca_{id}e}{e_{m_D}m_{D_{id}}ex} = \eta_{ex_e} - \left(1 + \eta_{im_e} + \frac{e \cdot im_{id}}{e_{m_D}m_{D_{id}}im}\right)$$

$$\left(\frac{m_D id}{m_D id}\right)\frac{ca_{id}e}{e_{m_D}m_{D_{id}}ex} = \frac{m_D id}{m_D id}\left[\eta_{ex_e} - \left(1 + \eta_{im_e} + \frac{e \cdot im_{id}}{e_{m_D}m_{D_{id}}im}\right)\right]$$

$$\frac{ca_{id}}{e_{m_D}m_{D_{id}}im} = \eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}}\eta_{m_{D_{id}}}}$$

$$ca_{id} < 0 \longleftrightarrow \eta_{ex_e} - 1 - \eta_{im_e} - \frac{\eta_{im_{id}}}{\eta_{e_{m_D}}\eta_{m_{D_{id}}}} < 0 \longrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}} < 1$$

or $\eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{im_{id}}}{\eta_{e_{m_D}}|\eta_{m_{D_{id}}}|} < 1.$

The first derivative of domestic current account ca with respect to import demand id is negative if and only if the sum of (i) elasticity of domestic exports to domestic real exchange rate η_{ex_e}, (ii) absolute elasticity of domestic imports to domestic real exchange rate $|\eta_{im_e}|$ and (iii-a) quotient of the elasticity of domestic imports to import demand, the absolute elasticity of the domestic real exchange rate to domestic money demand and the elasticity of domestic money demand to import demand $\frac{\eta_{im_{id}}}{|\eta_{e_{m_D}}|\eta_{m_{D_{id}}}}$ or (iii-b) quotient of the elasticity of domestic imports to import demand, the elasticity of the domestic real exchange rate to domestic money demand and the absolute elasticity of domestic money demand to import demand $\frac{\eta_{im_{id}}}{\eta_{e_{m_D}}|\eta_{m_{D_{id}}}|}$ is smaller than one.

Since domestic money demand m_D is complex to measure elasticity of the domestic real exchange rate to domestic money demand $\eta_{e_{m_D}}$ is not easily calculable, thus, one can specifically adopt $e = f(\overset{+}{m_S},\ \overset{-}{m_S^*},\ \overset{-}{ed},\ \overset{+}{id})$ for empirical testing (*nb.* import demand id is again foreign export demand ed^* and thereby accounts for foreign money demand m_D^*); as suggested, export demand ed can be proxied via confidence, tariffs or quotas. It follows that the twofold Marshall Lerner condition for money demand becomes:

$$ca_{ed} > 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| + \frac{\eta_{ex_{ed}}}{\eta_{e_{ed}}} > 1$$

and

$$ca_{id} < 0 \longleftrightarrow \eta_{ex_e} + |\eta_{im_e}| - \frac{\eta_{im_{id}}}{\eta_{e_{id}}} < 1.$$

Nevertheless, misspecification problems suggest the regression of domestic real exchange rate e on all independent variables of domestic money demand m_D; identical problems in fact suggest the same for the calculation of $m_{D_{ed}}$ in $\eta_{m_{D_{ed}}}$ and $m_{D_{id}}$ in $\eta_{m_{D_{id}}}$.

Conclusion

This article has derived a Marshall Lerner condition for changes in money demand whereby the current account increases or decreases upon respective decrements or increments in the real exchange rate. This is noteworthy because the historic Marshall Lerner condition is such that the current account increases on account of a depreciation in the real exchange rate, yet, the current account also increases if the real exchange rate appreciates: the seeming contradiction is resolved by realizing that the current account respectively increases on account of a fundamental increment in (i) the real money supply and (ii) money demand, which increments respectively affect the real exchange rate in terms of a depreciation and an appreciation, concomitantly. The explanation advanced by the historic Marshall Lerner condition is therefore incomplete or superficial, if not ultimately misleading: the problem has now been rectified.

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